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AMERICAN SOCIETY OF CIVIL ENGINEERS

INSTITUTED 1852

PAPERS AND DISCUSSIONS

UPWARD PRESSURES UNDER DAMS: EXPERIMENTS BY THE UNITED STATES BUREAU OF RECLAMATION

BY JULIAN HINDS,* M. AM. SOC. C. E.

SYNOPSIS

Uplift measurements have been made by the United States Bureau of Reclamation on three gravity diversion dams and one gravity storage dam. Two of the diversion dams, the Colorado River Dam, on the Grand Valley Project, in Colorado, and the Percha Dam, on the Rio Grande Project, in New Mexico, are founded on gravel. The other diversion dam, the Willwood Dam, on the Shoshone Project, in Wyoming, is founded on sandstone and shale, thoroughly grouted. The gravity storage dam, the recently completed American Falls Dam on the Minidoka Project of Idaho, is built on a columnar basalt foundation, also thoroughly grouted.

The measurements at the Colorado River and Percha Dams indicate that the materials deposited above a dam built on porous foundation act somewhat as a filter. When the filter surface is thoroughly seasoned, it offers considerable resistance to the passage of water, and the uplift pressure is partly relieved. When the filter surface is disturbed, as in time of flood, the uplift pressures are greater. The results are too indefinite to permit an attempt to prove, or disprove, any of the theories of flow under dams of this type.

The worst uplift conditions occur with a freshly disturbed filter surface, and experimental data taken under any other circumstances should be used with care. A limited area of escape at any point below the filter surface as at the Colorado River Dam, causes an increase in pressure above the constriction, and a decrease below.

NOTE.—The Special Committee on Irrigation Hydraulics selected the subject of "Water Movement and Pressure Under Dams" as one of ten for study and research. This paper was submitted to the Committee by its author and the Committee has recommended its publication in *Proceedings* in order to elicit discussion of the subject (see Progress Report of the Committee, *Proceedings*, Am. Soc. C. E., March, 1927, Society Affairs, p. 121). Written discussion on this paper will be closed in August, 1928.

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At the Willwood Dam there is, generally, no evidence of appreciable uplift for that portion of the dam founded on shale. For sandstone the uplift pressures vary with the up-stream and down-stream water levels, in a reasonably regular way, and the uplift appears to vary, roughly as a straight line, from full tail-water depth at the toe to full tail-water plus one-fourth the difference in tail-water and head-water elevations at the heel.

The results at American Falls are generally similar to those at Willwood. Although there are no shale areas at this site there are a few apparently water-tight holes. The uplift pressure intensities are on the average slightly higher, relatively, than at Willwood.

Colorado River Dam

The type of design and general conditions at the Colorado River Dam are shown in Fig. 1, while the general appearance of the structure is shown in the photograph, Fig. 5. The design contemplated an open gravel foundation throughout, although it was known that rock could be reached at the ends of the structure. When the cut-off trench was opened, the gravel in a large portion of the river bed was found to be fairly tightly cemented, but not of sufficient hardness to justify changing to a standard type of gravity dam. The dam was finally founded on the upper strata of loose gravel, the up-stream cut-off being carried to tighter material except for a short distance in the vicinity of Pier *B*, where the open gravel extended to an undetermined depth, and across the sluice-way, where the percolation distance is such that a deep cut-off was not considered essential. The gravel in the river bed is mostly rather coarse with only a small percentage of sand.

The ogee crest of the dam is divided into six parts, each controlled by a 70 ft. by 10 ft. 3-in. roller crest. An additional 60 ft. by 15 ft. 3-in. roller controls a sluice-way at the west end of the dam. The total height from the top of the down-stream apron to the top of the rollers in the closed position is 18.25 ft., which is the maximum possible unbalanced head. The dam was completed in 1915.

Pier *B* is over the deep gravel portion of the foundation and is provided with two pipes for determining uplift pressures. These pipes are situated as shown in the pier detail, Fig. 1. Details of the pipe and its setting are shown in Fig. 2. One pipe was also installed in Pier *G*, its position being intermediate between the upper and lower pipe locations shown for Pier *B*.

A large number of readings have been recorded from these pipe wells, daily observations having been made throughout several irrigation seasons. Some of the results are given in Table 1. The data shown cover the full range of variations in head, and are typical.

Fig. 3 shows graphically the daily fluctuations in water levels from April 1 to October 30, 1922. A study of the relations shown on this diagram is interesting. The up-stream water level, E_1, is controlled by the roller gates, and is more or less independent of other factors. The tail-water level, E_4, depends on the flow passing over the dam. The fluctuations in the water

level in the lower well, E_3, closely follow the changes in the depth of tail-water. The depth in the upper well, E_2, changes promptly with a change in the up-stream water level, and appears to respond more slowly to general changes in the height of tail-water.

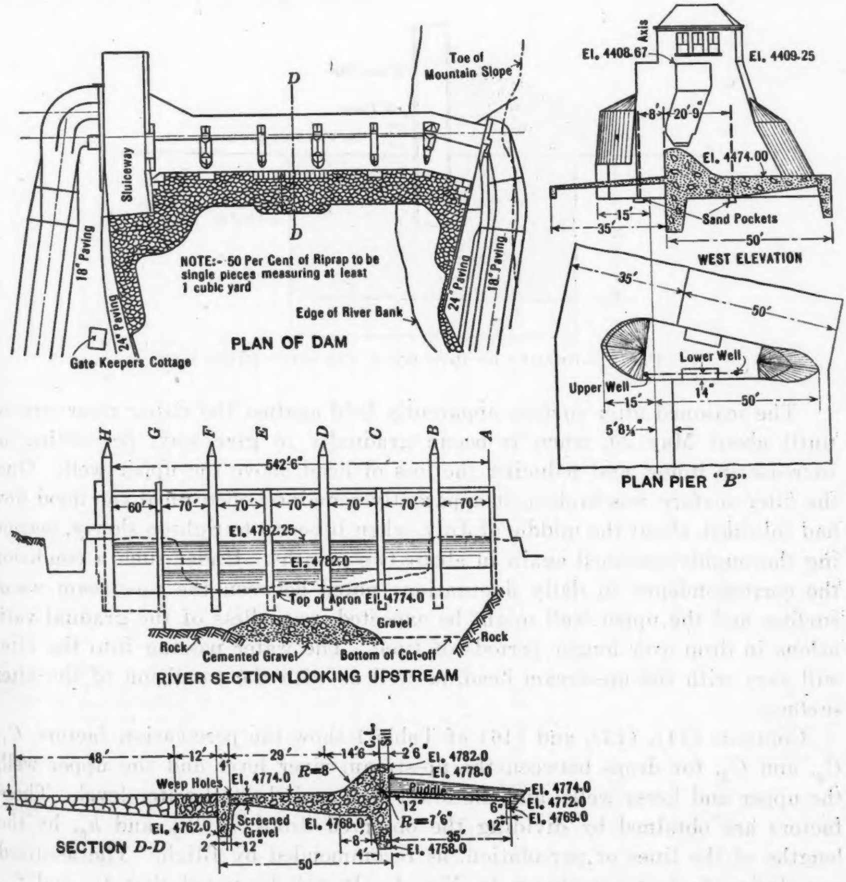

FIG. 1.—GENERAL PLAN OF COLORADO RIVER DAM.

It will be noted that on April 20 a gradual rise in tail-water depth began. The depth in the lower well immediately increased and followed the fluctuations in tail-water levels in detail. The upper well continued to respond in detail to changes in the up-stream level, but did not reflect the change in tail-water level until about 30 days later, when a general rise began. A similar delay occurred when the tail-water started to lower about June 15. Although the general relation of the two upper curves in Fig. 3 vary widely, the daily fluctuations correspond in a remarkable way.

No entirely adequate explanation of the apparently delayed action of the upper well has been offered. It is probable that this effect is produced by changes in the mechanical or bacterial condition of the up-stream silt

blanket, rather than by any hydraulic consideration. Prior to April 20 the discharge had been moderate and steady for a time and the silt bed above the dam, acting as a filter for the percolating water, had probably become well seasoned, causing an appreciable loss of head at the point of entrance.

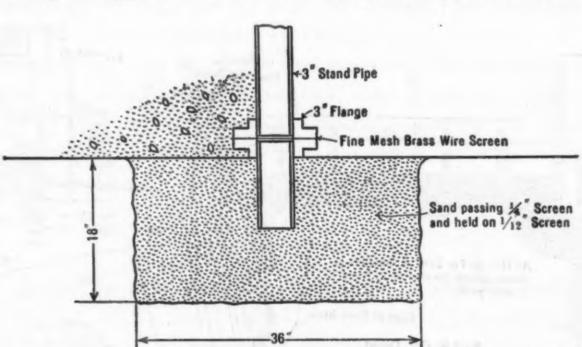

Fig. 2.—Details of Pipe Well, Colorado River Dam.

The seasoned filter surface apparently held against the rising river current until about May 20, when it began gradually to give way, permitting an increase in inflow and reducing the loss of head above the upper well. Once the filter surface was broken, it apparently remained open until the flood flow had subsided, about the middle of July, when it began to tighten slowly, becoming thoroughly seasoned again in about two months. Under such a condition the correspondence in daily fluctuations noted between the up-stream water surface and the upper well might be expected, regardless of the gradual variations in drop over longer periods of time. The water passing into the filter will vary with the up-stream head, as well as with the condition of the filter surface.

Columns (14), (15), and (16) of Table 1 show the percolation factors, C_1, C_2, and C_3, for drops between the up-stream river level and the upper well, the upper and lower wells, and the lower well and the tail-water level. These factors are obtained by dividing the observed drops, h_1, h_2, and h_3, by the lengths of the lines of percolation, as recommended by Bligh. The assumed percolation factors are shown in Fig. 4. It will be noted that C_1 and C_3, which are for the upper and lower reaches, respectively, are very irregular. The value of C_2, which applies between the upper and lower wells, is reasonably consistent, except for measurements made before the rollers were put into service, and when the drop from head-water to tail-water was low. Because the total drop from head-water to tail-water varies, with the percolation distances constant, it is not logical that the percolation factors remain constant, but they might ordinarily be expected to remain proportional to each other, which they fail to do.

The variations in these factors can be accounted for in a general way by considering the physical conditions in connection with Fig. 3. Water can flow under the up-stream apron of the dam across the full width of the river channel, but can escape in appreciable quantity only through the narrow gap

TABLE 1.—Measured Uplift Under Colorado River Dam.
(See Figs. 3 and 4.)

Year.	Month and day.	E_1.	E_2.	E_3.	E_4.	h_1.	h_2.	h_3.	H.	$\frac{h_1}{H}$.	$\frac{h_2}{H}$.	$\frac{h_3}{H}$.	C_1 $40 \div h_1$.	C_2 $43 \div h_2$.	C_3 $22 \div h_3$.	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	
1916	2- 8	83.4	82.3	79.6	78.7	1.1	2.7	0.9	4.7	0.23	0.58	0.19	36	16	24	....
1916	3-30	83.4	81.8	80.1	79.3	1.6	1.7	0.8	4.1	0.39	0.41	0.20	25	25	28	....
1916	4-22	84.6	83.1	81.0	80.2	1.5	2.1	0.8	4.4	0.34	0.48	0.18	27	20	28	....
1916	4-26	85.3	83.5	81.6	80.7	1.8	1.9	0.9	4.6	0.39	0.41	0.20	22	23	24	....
1916	5- 3	85.9	83.9	82.1	81.4	2.0	1.8	0.7	4.5	0.44	0.40	0.16	20	24	31	....
1916	5- 5	85.7	83.7	81.7	81.3	2.0	2.0	0.4	4.4	0.45	0.46	0.09	20	22	55	....
1916	5- 8	87.0	85.0	83.2	82.9	2.0	1.8	0.3	4.1	0.49	0.44	0.07	20	24	73	....
1916	5-10	88.0	86.5	84.4	83.7	1.5	2.1	0.7	4.3	0.35	0.49	0.16	27	20	31	....
1916	5-25	84.7	84.1	82.6	82.0	0.6	1.5	0.6	2.7	0.22	0.56	0.22	67	29	37	....
1916	5-27	85.0	84.5	83.1	82.5	0.5	1.4	0.6	2.5	0.20	0.56	0.24	80	31	37	....
1916	5-30	85.2	84.8	83.5	83.0	0.4	1.3	0.5	2.2	0.18	0.59	0.23	100	33	44	....
1916	6- 2	85.7	85.5	84.4	84.2	0.2	1.1	0.2	1.5	0.13	0.73	0.13	200	39	110	....
1916	6- 4	85.8	85.6	84.6	84.2	0.2	1.0	0.4	1.6	0.12	0.63	0.25	200	43	55	....
1916	6- 9	85.6	85.4	84.2	83.9	0.2	1.2	0.3	1.7	0.12	0.70	0.18	200	36	73	....
1916	6-11	86.3	86.3	85.4	85.2	0.0	0.9	0.2	1.1	0.0	0.82	0.18	8	48	110	....
1916	6-20	86.7	86.5	85.6	85.4	0.2	0.9	0.2	1.3	0.15	0.70	0.15	200	48	110	....
1916	6-24	85.6	85.4	84.0	83.7	0.2	1.4	0.3	1.9	0.11	0.73	0.16	200	31	73	....
1917	6-16	92.0	89.1	88.7	88.1	2.9	0.4	0.6	3.9	0.75	0.10	0.15	14	110	36	....
1917	7-15	87.7	87.2	83.9	82.7	0.5	3.3	1.2	5.0	0.10	0.66	0.24	80	13	18	....
1917	8-15	87.1	85.9	82.0	78.7	1.2	3.9	3.3	8.4	0.14	0.47	0.39	33	11	7	....
1917	9-15	87.3	86.9	81.7	79.2	0.4	5.2	2.5	8.1	0.05	0.64	0.31	100	8	9	....
1917	10-15	86.8	85.3	81.0	78.6	1.5	4.3	2.4	8.2	0.18	0.53	0.29	27	10	9	....
1917	11-15	87.0	85.8	81.1	78.6	1.2	4.7	2.5	8.4	0.14	0.56	0.30	33	9	9	....
1917	12-10	86.0	84.5	80.4	78.3	1.5	4.1	2.1	7.7	0.19	0.54	0.27	27	11	10	....
1918	4- 1	87.5	86.9	81.6	78.9	0.6	5.3	2.7	8.6	0.07	0.62	0.31	67	8	8	....
1918	6- 1	87.6	87.3	84.0	83.0	0.3	3.3	1.0	4.6	0.07	0.71	0.22	133	13	22	....
1918	7- 1	87.5	87.1	83.6	81.7	0.4	8.5	1.9	5.8	0.07	0.60	0.33	100	12	12	....
1918	8- 1	86.9	85.9	81.2	80.3	1.0	4.7	0.9	6.6	0.15	0.71	0.14	40	9	24	....
1918	9- 2	87.6	86.8	81.1	78.2	0.8	5.7	2.9	9.4	0.09	0.60	0.31	50	8	8	....
1918	10- 1	87.8	85.3	80.9	78.7	2.5	4.4	2.2	9.1	0.27	0.49	0.24	16	10	10	....
1918	11- 1	87.1	84.8	80.8	78.8	2.3	4.0	2.0	8.3	0.28	0.48	0.24	17	11	11	....
1918	12-28	85.9	83.3	79.9	78.1	2.6	3.4	1.8	7.8	0.33	0.44	0.23	15	13	12	....
1920	4-20	86.8	83.6	79.4	76.2	3.2	4.2	3.2	10.6	0.30	0.40	0.30	13	10	7	10:00 A. M.
1920	4-20	89.4	84.3	78.3	76.2	5.1	6.0	2.1	13.2	0.39	0.45	0.16	8	7	10	11:00 A. M.
1920	4-20	91.8	85.4	78.6	76.2	6.4	6.8	2.4	15.6	0.41	0.44	0.15	6	6	9	11:45 A. M.
1920	4-20	92.8	85.8	80.6	76.2	7.0	5.2	4.4	16.6	0.42	0.32	0.26	6	8	5	12:15 P. M.
1920	4-20	89.3	86.0	81.3	76.2	3.3	4.7	5.1	13.1	0.25	0.36	0.39	12	11	4	1:00 P. M.
1922	4- 2	88.3	85.9	80.9	79.4	2.4	5.0	1.5	8.9	0.27	0.56	0.17	17	9	5	....
1922	4-10	91.6	88.3	81.7	79.8	3.3	6.6	1.9	11.8	0.28	0.56	0.16	12	6	12	....
1922	4-20	91.6	88.0	81.1	79.0	3.6	6.9	2.1	12.6	0.29	0 55	0.16	11	6	10	....
1922	4-30	91.4	87.3	82.4	81.2	4.1	4.9	1.2	10.2	0.40	0.48	0.12	9	9	18	....
1922	5- 9	91.4	89.0	84.1	82.9	2.4	4.9	1.2	8.5	0.28	0.58	0.14	17	9	18	....
1922	5-17	91.9	87.8	83.0	81.9	4.1	4.8	1.1	10.0	0.41	0.48	0.11	9	9	20	....
1922	5-29	91.7	90.6	85.9	85.6	1.1	4.7	0.3	6.1	0.18	0.77	0.05	36	9	73	....
1922	6- 5	89.7	89.3	84.4	83.6	0.4	4.9	0.8	6.1	0.07	0.80	0.13	100	9	23	....
1922	6-10	91.5	90.3	85.6	84.9	1.2	4.7	0.7	6:6	0.18	0.71	0.11	33	9	31	....
1922	6-20	91.0	90.5	84.2	83.4	0.5	6.3	0.8	7.6	0.07	0.83	0.10	80	7	22	....
1922	6-27	91.4	90.9	83.0	82.2	0.5	7.9	0.8	9.2	0.05	0.86	0.09	80	6	22	....
1922	7-10	91.1	90.4	82.6	80.9	0.7	7.8	1.7	10.2	0.07	0.76	0.17	57	6	13	....
1922	7-18	92.0	91.1	81.9	80.0	0.9	9.2	1.9	12.0	0.08	0.76	0.16	44	5	12	....
1922	7-26	89.7	88.9	81.3	79.7	0.8	7.6	1.8	10.2	0.08	0.74	0.18	50	5	12	....
1922	8- 5	92.2	89.6	81.5	79.8	2.6	8.1	1.7	12.4	0.21	0.65	0.14	15	5	13	....
1922	8-16	89.7	87.2	80.8	79.2	2.5	6.4	1.6	10.5	0.24	0.61	0.15	16	7	14	....
1922	8-18	91.7	88.6	81.4	79.7	3.1	7.2	1.7	12.0	0.26	0.60	0.14	13	6	13	....
1922	8-28	90.5	86.9	80.8	79.5	3.6	6.1	1.3	11.0	0.33	0.55	0.12	11	7	17	....
1922	9- 4	92.2	87.1	81.2	79.8	5.1	5.9	1.4	12.4	0.41	0.48	0.11	8	7	16	....
1922	9- 7	90.7	85.9	80.5	79.2	4.8	5.4	1.3	11.5	0.42	0.47	0.11	8	8	17	....
1922	9-13	87.0	84.9	80.5	79.1	2.1	4.4	1.4	7.9	0.26	0.56	0.18	19	10	16	....
1922	9-22	91.8	86.6	80.5	78.9	5.2	6.1	1.6	12.9	0.41	0.47	0.12	8	7	14	....
1922	10-10	92.3	86.8	80.5	79.0	5.5	6.3	1.5	13.3	0.41	0.47	0.12	7	7	15	....
1922	10-18	91.4	86.3	80.5	78.9	5.1	5.8	1.6	12.5	0.41	0.46	0.13	8	7	14	....
1922	10-27	91.9	86.5	80.3	78.8	5.4	6.2	1.5	13.1	0.41	0.48	0.11	7	7	15	....

of open gravel under Pier *B*. Likewise, after passing the restricted open gravel area under the cut-off, the flow is free to spread laterally, thus reducing the resistance to outflow. It seems logical, therefore, that the greatest loss should occur at the point of passing the cut-off wall. At times like June and July, 1922 (see Fig. 3), the great area open to easy infiltration causes the

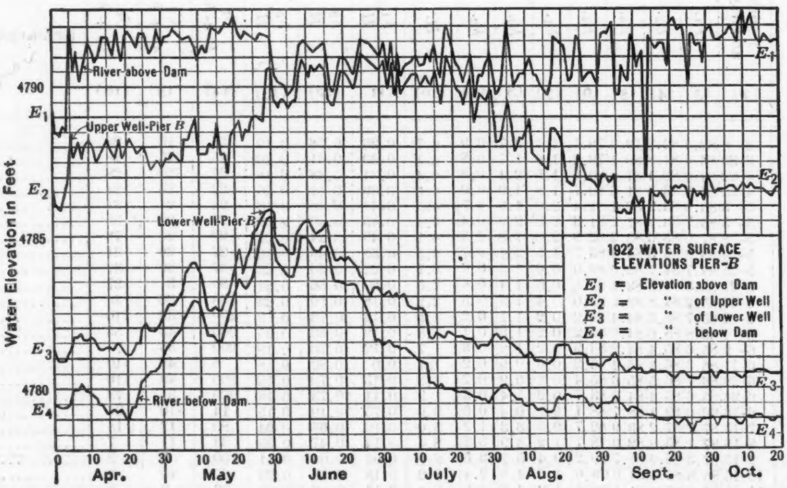

FIG. 3.—OBSERVATIONS OF WATER ELEVATIONS, AT PIER *B*, APRIL TO OCTOBER, 1922, COLORADO RIVER DAM.

entrance loss to be small in comparison with the loss under the cut-off. In September and October the filter bed had tightened until, notwithstanding its relatively large extent, it offered practically as much resistance as the cut-off wall. The reason for the high values of C_3 in May and June is not equally apparent.

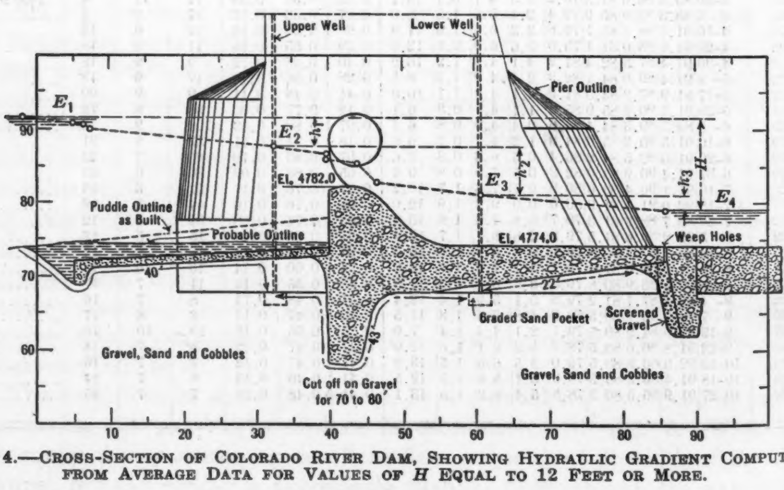

FIG. 4.—CROSS-SECTION OF COLORADO RIVER DAM, SHOWING HYDRAULIC GRADIENT COMPUTED FROM AVERAGE DATA FOR VALUES OF *H* EQUAL TO 12 FEET OR MORE.

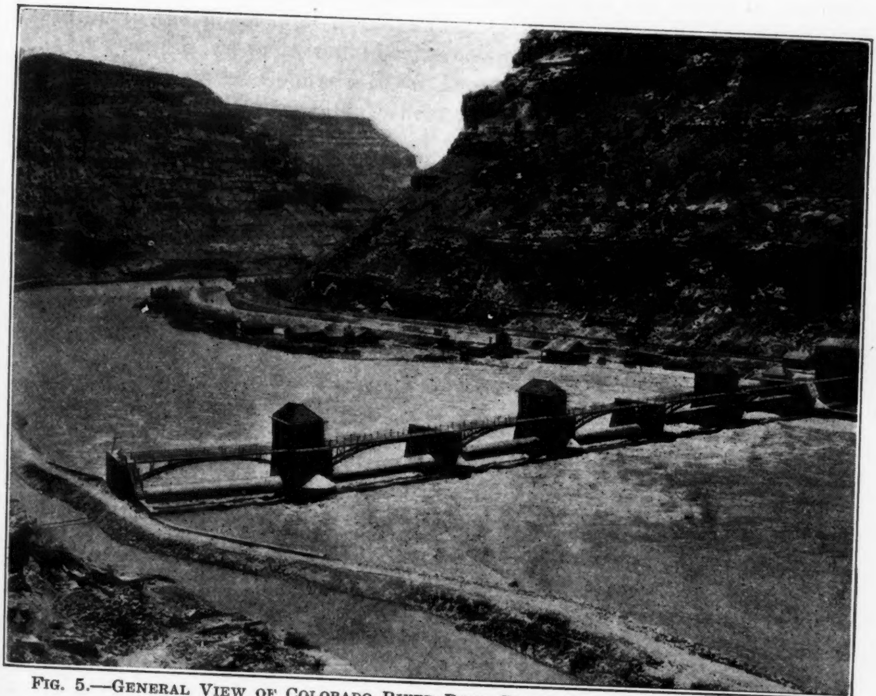

FIG. 5.—GENERAL VIEW OF COLORADO RIVER DAM, GRAND VALLEY PROJECT, COLORADO.

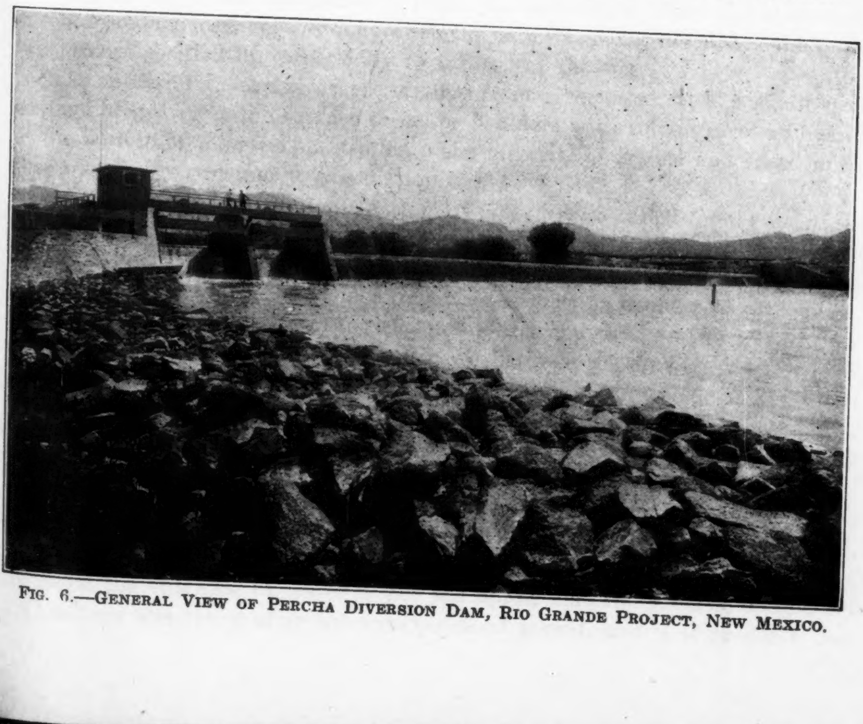

FIG. 6.—GENERAL VIEW OF PERCHA DIVERSION DAM, RIO GRANDE PROJECT, NEW MEXICO.

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If open gravel existed below the cut-off wall for the full length of the dam, the drop to the upper well would undoubtedly be materially increased, and that between the two wells would be decreased. Because of the irregular conditions existing at the Colorado River Dam, the data in Table 1 and Fig. 3 neither prove nor disprove Bligh's theory of a uniform pressure drop along the line of creep.

The one pressure well at Pier *G*, is just down stream from the cut-off wall. Observations indicate drops corresponding roughly with the drop to the downstream well at Pier *B*. Conditions at Pier *G* are too complicated to permit an analysis of the data there. The cut-off wall is founded on rock on one side of the pier, and not on the other, and the long sluice structure introduces further uncertainty in the path of percolation.

A view of the site of the Colorado River Dam, looking down stream, is shown in Fig. 5.

Percha Dam

The Percha Dam is more strictly a floating type, no rock or impervious material being in the vicinity. Its general appearance is shown in Fig. 6 and its dimensions in Fig. 7. An overflow section, in the river channel, is flanked on each end by an earth dike. The weir is of the ogee gravity type, with free overflow crest. Heavy paving and rip-rap below the dam prevent scour and piping. Upward pressure is relieved a short distance below the toe of the gravity section by weep-holes through the rubble concrete apron. Percolation is retarded, and uplift reduced by an up-stream concrete apron, two concrete cut-off walls, and a line of steel sheet-piling under the up-stream cut-off. Construction was completed in 1917. No back-filling above the up-stream apron, to reduce leakage, was done when the dam was constructed, but the river bed promptly silted up, practically to the top of the weir.

The material in the river bed grades from river sand with a small percentage of gravel at the surface to heavy boulders with some gravel and sand at the bottom of the steel sheet-piles. The material is graded and silted until it is reasonably resistant to percolation.

At the time of construction a system of pipes was installed for the measurement of uplift, as illustrated in Figs. 8 and 9. Uplift pressures have since been observed, as recorded in Table 2. The various notations, E_1, E_2, etc., and h_1, h_2, etc., have the same meanings (see Fig. 9) as in the case of Table 1. Inspection of Table 2 shows that except the first and the last two sets of data approximately one-half the total drop occurs between the up-stream water level and Well No. 1, which is just inside the up-stream apron cut-off. A large part of the remainder appears in h_3, which is the drop produced by the sheet-piling cut-off. Losses along the flat surfaces, between Wells Nos. 1 and 2, and Nos. 2 and 4, are small.

The first set of readings, and the last two, Table 2, show practically no loss at entrance. The first readings were taken soon after the dam was placed, and perhaps before the channel had completely silted and seasoned sufficiently to resist the entrance of water under the apron materially. The second set of readings was taken after the summer flood period, and it is probable that

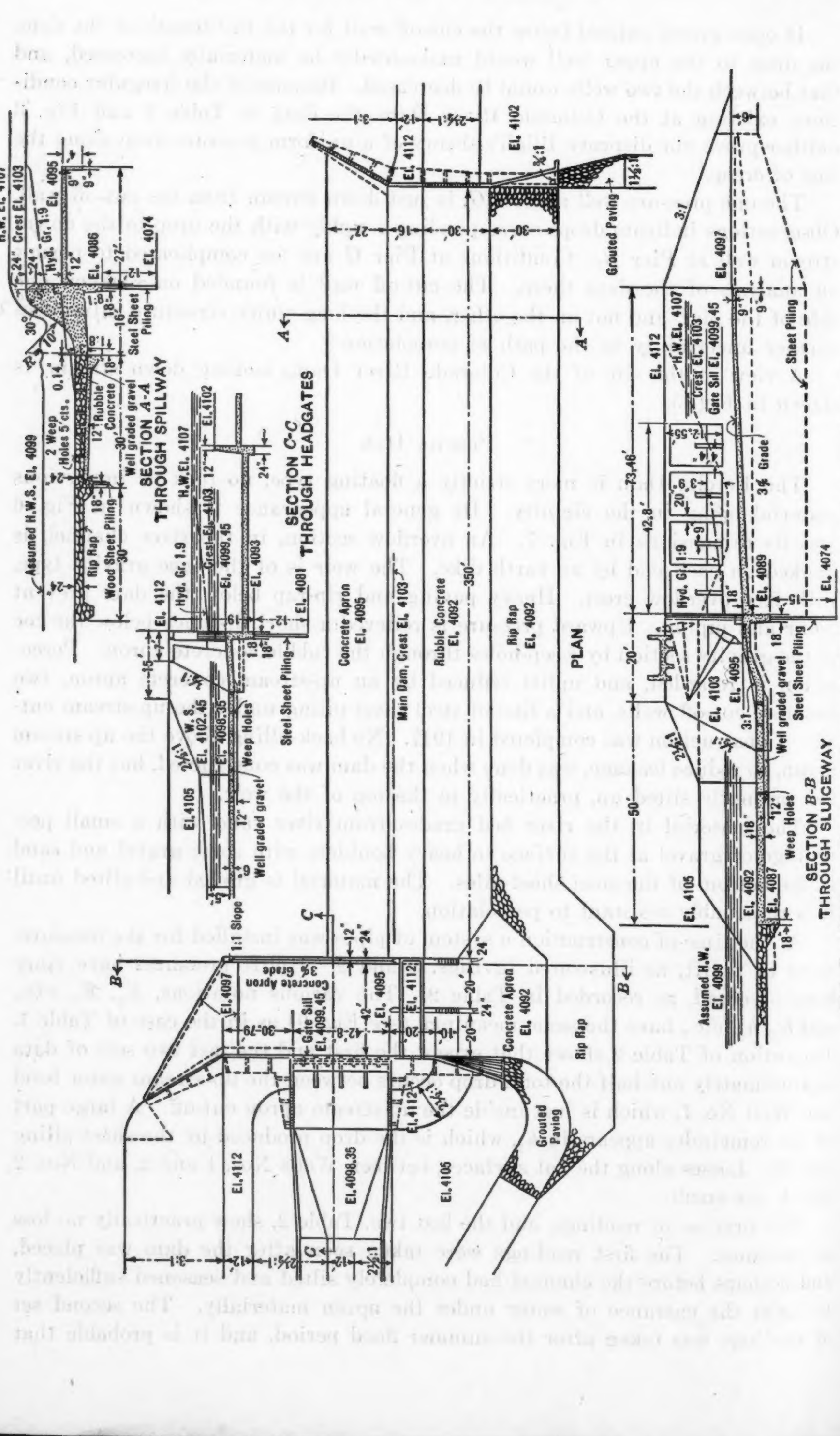

FIG. 7.—GENERAL PLAN AND SECTIONS, PERCHA DAM, RIO GRANDE PROJECT.

the up-stream silt bed had been disturbed or partly removed. The entrance drop, h_1, was accordingly small at these times, the other drops being thereby materially increased. The first reading shows the loss to occur almost entirely at the two cut-off walls, the drop along the horizontal surfaces being small. The last two sets of data correspond reasonably well to Bligh's theory of a uniform drop along the line of creep, considering the steel sheet-piling to be water-tight.

TABLE 2.—Measured Uplift Under Perçha Dam (See Fig. 9).

Year.	Month and day.	E_0.	E_1.	E_2.	E_3.	E_4.	E_5.*	E_6.	h_1.	h_2.	h_3.	h_4.	h_5.*	h_6.*	Total drop.	Remarks.
1918	3-10	03.80	03.25	03.25	99.25	98.75	97.65	96.45	0.55	—0.00	4.00	0.50	1.10	1.20	7.35	....
1920	4-25	03.72	97.85	98.00	96.20	96.05	96.55	.....	5.87	0.15	1.80	0.15	—0.50	....	....	....
1920	3-..	03.93	98.36	98.36	96.59	96.37	96.56	.....	5.57	0.00	1.77	0.22	—0.19	....	....	....
1920	.....	04.25	99.46	99.56	97.56	97.26	97.06	.....	4.79	—0.10	2.00	0.30	—1.20	....	....	....
1921	10-25	04.10	99.41	99.51	96.41	96.36	98.11	94.74	4.69	—0.10	3.10	0.05	—1.75	3.37	9.36	....
1921	11-19	04.01	99.26	99.36	96.26	96.05	96.77	94.42	4.75	—0.10	3.10	0.21	—0 72	2.35	9.59	....
1921	11-20	03.72	99.16	99.16	96.16	95.75	96.67	93.86	4.56	0.00	3.00	0.41	—0.92	2.81	9.86	....
1921	11-21	03.71	99.01	99.01	96.71	95.60	96.57	93.84	4.70	0.00	2.30	1.11	—0.97	2.73	9.87	....
1921	11-22	04.34	99.56	99.56	96.61	96.50	97.17	95.10	4.78	0.00	2.95	0.11	—0.67	2.07	9.24	Plotted.
1921	11-23	04.13	99.36	99.46	96.46	96.35	97.17	94.72	4.77	—0.10	3.00	0.11	—0.82	2.45	9.41	....
1923	10-30	03.39	03.01	99.91	96.06	95.01	95.91	94.17	0.38	3.10	3.85	1.05	—0.90	1.74	9.22	....
1923	10-31	03.39	03.21	00.06	96.16	95.01	95.91	94.17	0.18	3.15	3.90	1.15	—0.90	1.74	9.22	....

* Possible leak in Well No. 5.

The data presented are too limited to form the basis of any definite conclusion, but support, in a general way, the tentative conclusions drawn from the data for the Colorado River Dam.

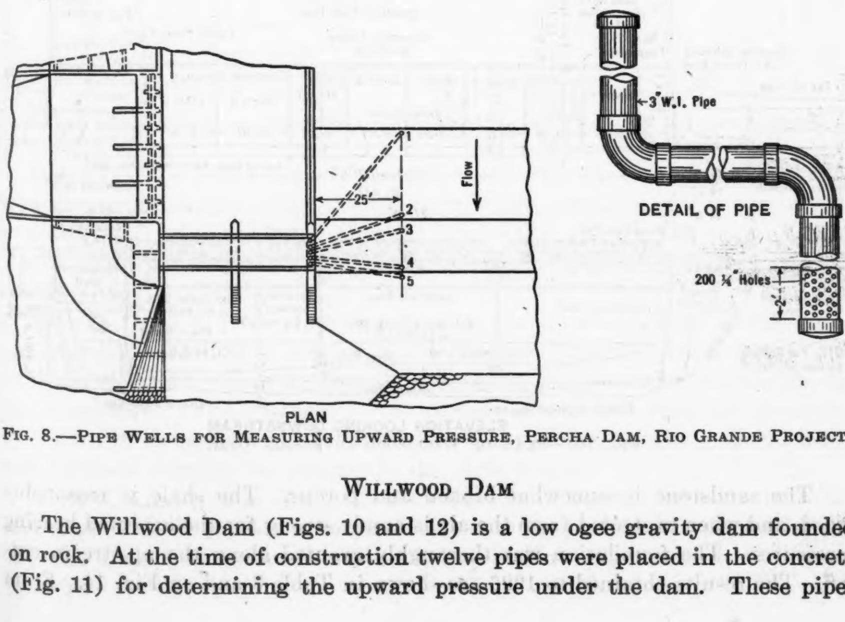

Fig. 8.—Pipe Wells for Measuring Upward Pressure, Percha Dam, Rio Grande Project.

Willwood Dam

The Willwood Dam (Figs. 10 and 12) is a low ogee gravity dam founded on rock. At the time of construction twelve pipes were placed in the concrete (Fig. 11) for determining the upward pressure under the dam. These pipes

are arranged along three lines, relatively close together, near the center of the stream bed. The foundation is partly sandstone and partly shale. The area covered by the wells is divided by the line of contact between these two rock types.

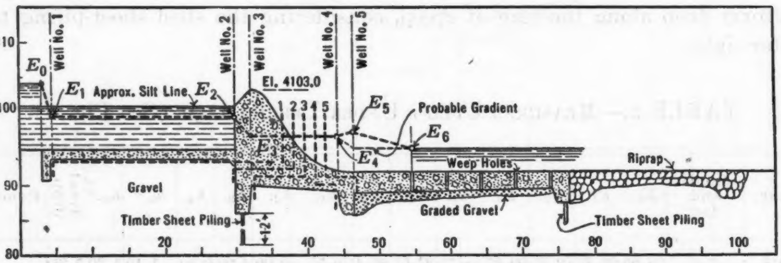

FIG. 9.—UNDERFLOW, PERCHA DAM, RIO GRANDE PROJECT.

The dimensions of the dam, and the general location of the inspection gallery, are shown in Fig. 10, while the character of the sandstone at the north end of the dam is evident from Fig. 13. The exact location and spacing of the pipes, and the relation of the bottoms of the wells to the shale and sandstone areas, are shown in Fig. 11. All the pipes were made straight, to facilitate cleaning, and to make it possible to observe the lowest water level without difficulty. The pipes end at the line of contact between the dam and the foundation.

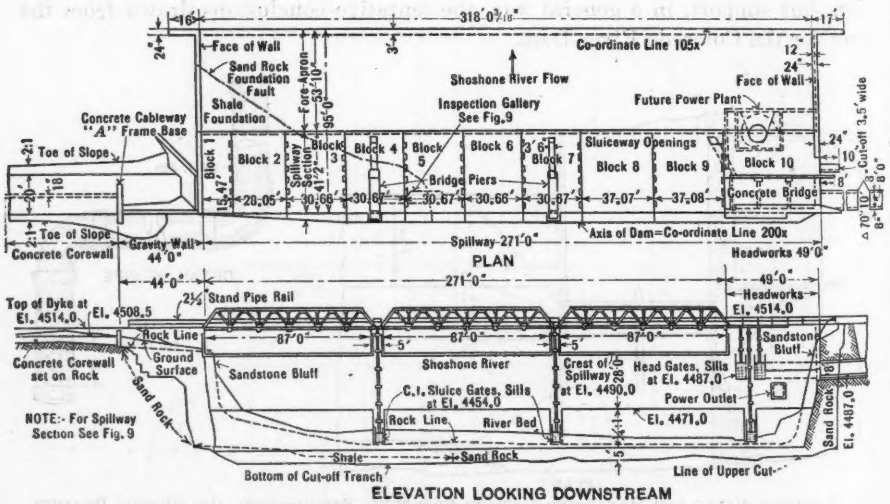

FIG. 10.—PLAN OF WILLWOOD DIVERSION DAM.

The sandstone is somewhat broken and porous. The shale is reasonably good, and when protected from the air is amply strong for the expected bearing pressures. The foundation was thoroughly grouted along the up-stream cut-off. The results obtained to 1926 are shown in Table 3 and on Fig. 14. From

a study of Fig. 14, it appears that in the sandstone areas the uplift pressures vary in a regular way with the heights of the head- and tail-waters. Pressures in the shale areas are erratic.

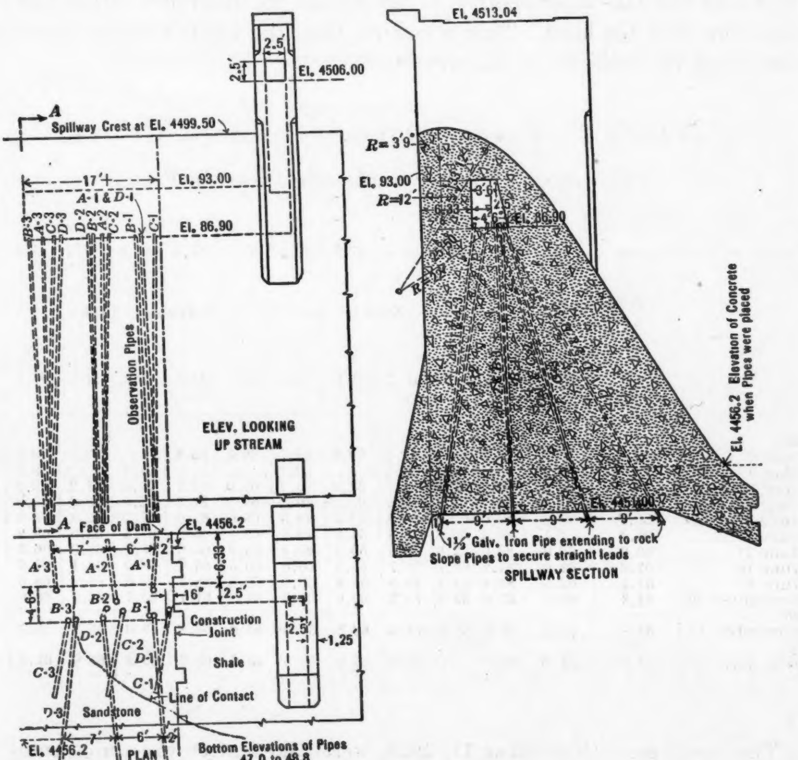

FIG. 11.—PLAN SHOWING LOCATION OF PRESSURE PIPES, WILLWOOD DIVERSION DAM.

Holes *B*, Line 1 and Line 2, are well within the shale area, and seem to have no communication with the reservoir water pressure. All the observed levels in these holes are below both the head-water and the tail-water, which can be accounted for only on the assumption that the holes are entirely or almost entirely water-tight.

Hole *A*, Line 1, is in shale, but is near the up-stream face of the dam, and may be reached by a crevice, which accounts for the evident connection with the reservoir. Hole *A*, Line 2, is in shale, and shows no fluctuation in the water level. The absence of fluctuations in this hole lead the observers to believe it to be obstructed. However, this was not definitely determined, and tightness may be due entirely to the shale bottom.

Hole *C*, Line 1, is in shale, but near the line of contact between the shale and sandstone. It appears to communicate with the pressures in the sandstone. Hole *C*, Line 3, is reported to be plugged, and no measurements for it are recorded.

Line 3 is entirely in sandstone, and shows a reasonably definite variation in uplift pressure as the head- and tail-water levels vary. The uplift pressure appears to vary, approximately as a straight line, from full tail-water depth at the toe to full tail-water depth plus one-fourth the difference in head-and tail-water levels at the head. This is smaller than the uplift pressure assumed in computing the stability of the structure.

TABLE 3.—Willwood Diversion—Inspection Gallery.

(Elevations of Water Surface in Pipes, in Feet.)

Elevation, top of pipes			87.2	87.1	87.2	87.0	87.2	87.2	87.1	87.4	87.4	87.5	87.4
Date.	Water-Surface Elevation.		Elevation of Water Surface in Pipes.										
	Reservoir.	Tail-water.	*A*-1.	*A*-2.	*A*-3.	*B*-1.	*B*-2.	*B*-3.	*C*-1.	*C*-2.	*D*-1.	*D*-2.	*D*-3.
1923:													
May 2	69.3	60.3	63.7	63.1	64.1	60.9	58.7	62.5	59.8	.....	63.0	63.4	62.0
May 9	73.0	60.4	.....	62.1	63.5	......	52.1	......	......	61.6	......	59.8	61.0
May 17	76.2	60.5	62.2	63.5	66.7	53.3	49.2	67.0	62.7	63.8	61.2	62.9	62.7
May 31	91.7	61.1	80.7	62.8	69.2	60.0	49.8	69.7	54.5	68.0	63.3	65.0	65.6
June 1	95.7	61.1	82.0	62.5	72.9	54.0	49.5	69.0	62.5	67.2	62.8	66.8	66.1
June 7	96.4	61.1	82.6	62.6	73.8	54.8	49.7	69.1	65.8	67.5	63.0	69.9	66.3
June 11	93.6	62.0	82.0	62.6	79.0	55.2	49.4	69.3	65.6	67.2	62.0	66.7	68.8
June 16	01.2	62.0	84.2	62.7	76.7	55.9	50.0	69.5	66.4	68.0	64.1	67.9	66.9
July 6	01.4	62.5	84.4	63.1	79.0	58.8	50.0	70.6	68.3	69.6	65.2	68.6	68.5
September 29.	84.3	59.5	67.6	62.7	73.3	60.6	51.0	61.8	61.1	65.7	62.3	66.0	60.8
1925:													
November 11.	64.8	58.8	66.5	68.9	74.4	63.8	50.0	65.6	63.8	59.0	56.7	56.7	61.1
1926:													
July 1	02.7	62.0	80.6	.....	66.3	63.6	50.5	65.6	63.2	63.2	63.6	61.6	61.4

The readings of November 11, 1925, were taken after the sluice-gates had been lowered for 15 days, allowing that time for the water level to fall in the wells. It should be noted that the water level in Holes *A* is high in all three lines. After these observations were taken, the water was artificially removed from all wells. Readings were again taken on July 1, 1926, after the gates had been closed since February 12. The readings were taken at high water, with the dam overtopped. The head-water had not, at any time since the unwatering of the wells, been more than 2 ft. higher than at the time of the observation.

The maximum uplift pressures observed at the Willwood Dam up to the present observations are plotted in Fig. 15. Uplift curves at Lines 1, 2, and 3 have been arbitrarily extended as straight lines from the values observed at the *A*-pipes to the maximum observed reservoir head at the up-stream edge of the dam; also, as straight lines from the values observed at the *D*-pipes to the maximum observed tail-water head at the down-stream edge. The latter is probably an accurate assumption. The former may indicate uplift pressures greater than those actually existing due to the thorough grouting of the foundation.

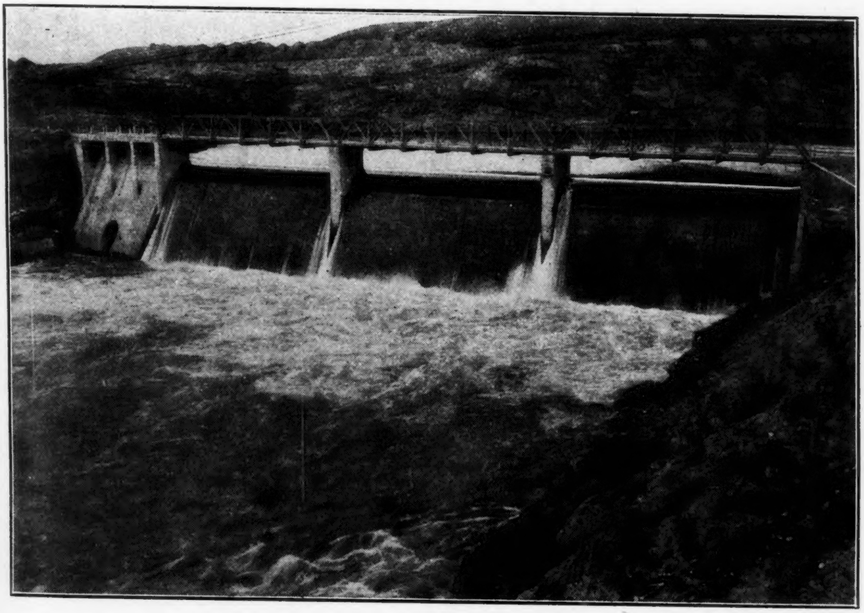

FIG. 12.—GENERAL VIEW OF WILLWOOD DAM, SHOSHONE PROJECT, WYOMING.

FIG. 13.—VIEW SHOWING CHARACTER OF SANDSTONE AT NORTH ABUTMENT, WILLWOOD DAM, SHOSHONE PROJECT, WYOMING.

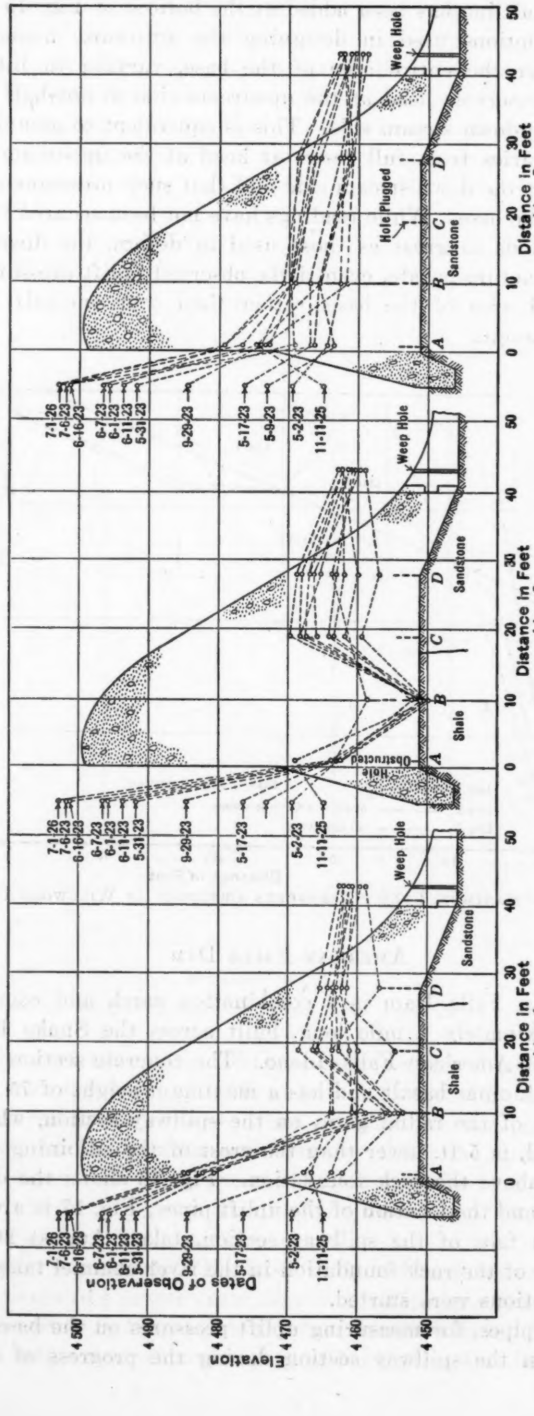

FIG. 14.—UPLIFT PRESSURES OBSERVED AT WILLWOOD DIVERSION DAM.

A full diagonal line has been added at the bottom of Fig. 15 to illustrate the uplift assumptions used in designing the structure, namely, that the pressures act over the entire area of the base, varying in intensity from one-half of full reservoir head at the up-stream side to one-half of full tail-water head at the down-stream side. This is equivalent to assuming that the uplift pressure varies from full reservoir head at the up-stream side to full tail-water head at the down-stream side and that such pressures act over one-half the area of the base. While readings have not been secured for head- and tail-water elevations as great as those used in design, the diagrams clearly show that the structure is safe, even if the observed uplift pressure intensities act over the full area of the base, rather than over one-half the area, as assumed in the design.

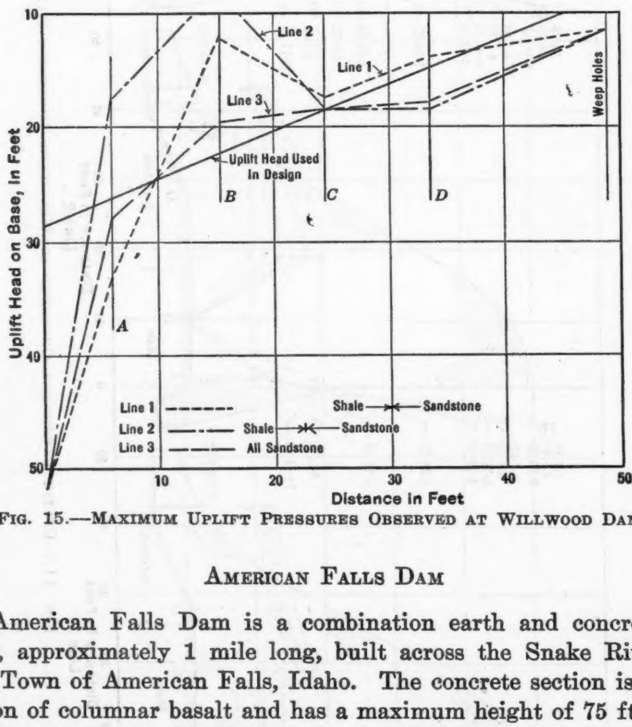

FIG. 15.—MAXIMUM UPLIFT PRESSURES OBSERVED AT WILLWOOD DAM.

AMERICAN FALLS DAM

The American Falls Dam is a combination earth and concrete gravity structure, approximately 1 mile long, built across the Snake River Valley, near the Town of American Falls, Idaho. The concrete section is built on a foundation of columnar basalt and has a maximum height of 75 ft. above the rock. The crest of the radial gates on the spillway section, which occupies the river channel, is 5 ft. lower than the crest of the adjoining concrete sections, or 70 ft. above the rock foundation. Fig. 16 shows the design of the spillway section and the location of the uplift pipes; Fig. 17 is a view showing the down-stream face of the spillway section, taken August 26, 1927; and Fig. 18 is a view of the rock foundation in the river channel taken just before concreting operations were started.

Twelve 3-in. pipes, for measuring uplift pressures on the base of the dam, were installed in the spillway section during the progress of construction.

These were arranged in three lines of four pipes each. One line was located at Station 9 + 00 near the west edge of the river channel; one was placed at Station 13 + 00, near the east edge; and the third was located at Station 11 + 10, or approximately half-way between the other two. Individual pipes were spaced 12 ft. apart in each line and were located as shown in Fig. 16, the up-stream pipe being placed 6 ft. 6 in. from the up-stream face of the dam in each instance.

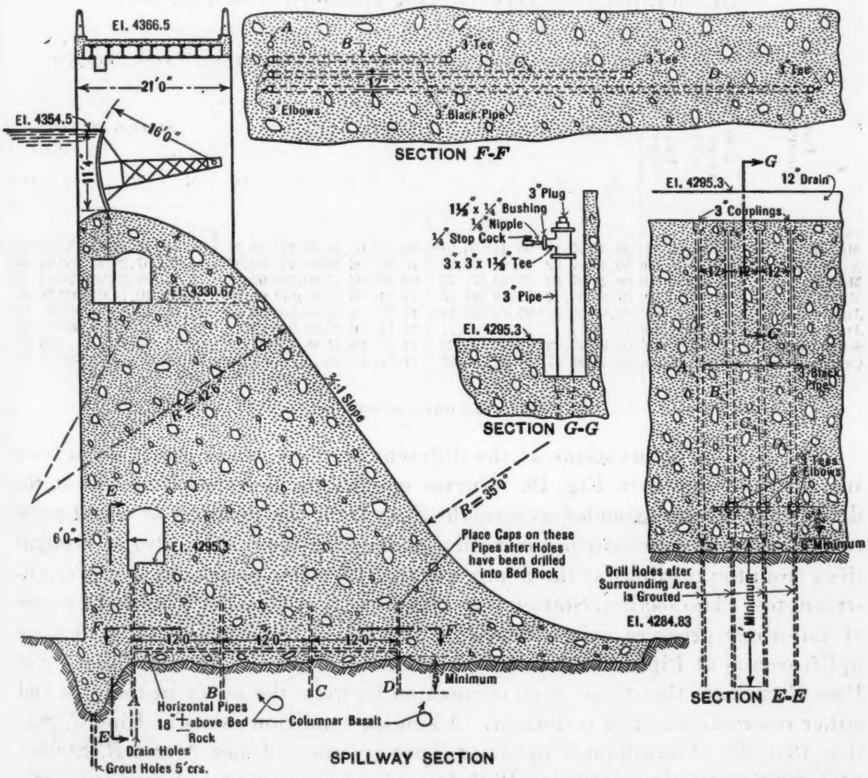

FIG. 16.—SPILLWAY SECTION, AMERICAN FALLS DAM, WITH PIPES FOR UPLIFT PRESSURE.

All the pipes were run to the drainage gallery where observations were made by reading pressures on an altitude gauge, or by sounding in the pipes. The pipes were placed in the concrete when the first lift was poured, leaving the lower ends about 6 in. above the rock surface. After the up-stream part of the foundation was grouted, holes were drilled through the vertical pipes approximately 5 ft. into the rock formation and the tops of the vertical pipes capped, as indicated on Fig. 16.

Observations were started March 25, 1927, and have been taken at intervals of from two weeks to a month since that time. Pipes not flowing were pumped out three days prior to the date of the first measurements. Table 4 gives the results of all measurements made up to and including October 20, 1927, uplift

pressures at the base of the dam being expressed as equivalent water-surface elevations. In Table 4 elevations below the tail-water level show that the water surfaces in the pipes were not being affected by either reservoir or tail-water pressures; or, in other words, that the foundation at these locations was tight on the dates such elevations were measured.

TABLE 4.—MEASURED UPLIFT ON BASE OF AMERICAN FALLS DAM.

(All elevations are between 4289 and 4355 (See Fig. 19)).

Date of observations.	Reservoir water surface elevation.	Tail-water elevation.	Uplift Pressure Expressed as Equivalent Water Surface Elevation.											
			Station 9 + 00.				Station 11 + 10.				Station 13 + 00.			
			A.	B.	C.	D.	A.	B.	C.	D.	A.	B.	C.	D.
1927:														
March 25......	34.20	96.50	98.48	97.14	96.44*	95.94*	06.14	01.48	90.21*	98.91	14.15	06.64	94.55*	89.33*
April 15.......	37.88	96.50	99.48	97.22	97.23	97.23	10.39	02.40	90.11*	99.66	14.65	07.39	94.25*	91.66*
May 2..........	40.40	96.50	00.23	97.22	97.23	97.23	09.89	03.15	90.16*	00.16	16.15	98.39	94.17*	91.76*
May 17.........	44.76	96.50	01.23	98.22	97.73	97.73	12.64	05.65	90.67*	01.16	20.15	10.14	94.10*	96.86
July 1.........	54.46	97.00	98.99	96.51*	95.64*	95.14*	12.39	99.15	93.00*	93.64*	25.15	12.14	95.45*	01.13
July 21........	54.31	97.30	01.23	......	......	......	13.14	00.65	92.94*	......	25.15	11.14	94.80*	02.13
September 19.	49.06	96.50	00.23	97.22	97.23	97.23	12.64	99.15	98.65	97.16	22.15	10.14	......	03.13
October 20....	51.38	96.50	00.48	97.47	97.23	97.23	11.64	00.15	......	97.16	22.15	10.14	93.83*	97.88

* Below tail-water elevations.

The various observations at the different stations where uplift pipes were installed are shown in Fig. 19. Curves connecting the measurements at the different pipes are extended as straight lines from the readings at the *A*-pipes to the reservoir water-surface elevation at the up-stream side; also as straight lines from the readings at the *D*-pipes to the tail-water elevations at the down-stream toe. The section, Station 9 + 00, shows that very little uplift in excess of tail-water pressure exists at any of the pipes. At Station 11 + 10 some uplift occurs at Pipes *A, B,* and *D,* but none at Pipe *C*. The observations at Pipe *C* indicate that there is no connection between the water in the pipe and either reservoir water or tail-water. A similar condition exists at Pipe *C,* Station 13 + 00. Considerable uplift pressure occurs at Pipes *A* and *B,* Station 13 + 00, but relatively little at Pipe *D*.

In Fig. 20 is shown a summary of maximum uplift observations at the American Falls Dam. In designing this dam, it was assumed that uplift pressures would act over one-third the area of the base and that the variation in pressure would be as a straight line running from full reservoir head at the up-stream side of the dam to full tail-water head at the down-stream side. Line 1, Fig. 20, shows the assumption of uplift-pressure variation, while Line 2 shows the equivalent line which would give similar uplift forces if applied to the entire area of the base. Line 3 shows the design assumptions used at the Willwood Dam, namely, that the pressure represented by Line 1 is applied over one-half the area of the base. Line 4 shows the uplift assumptions being used in the design of a high arch dam for an irrigation project in the Northwest, the pressures shown by this line being applied over the full area of the base.

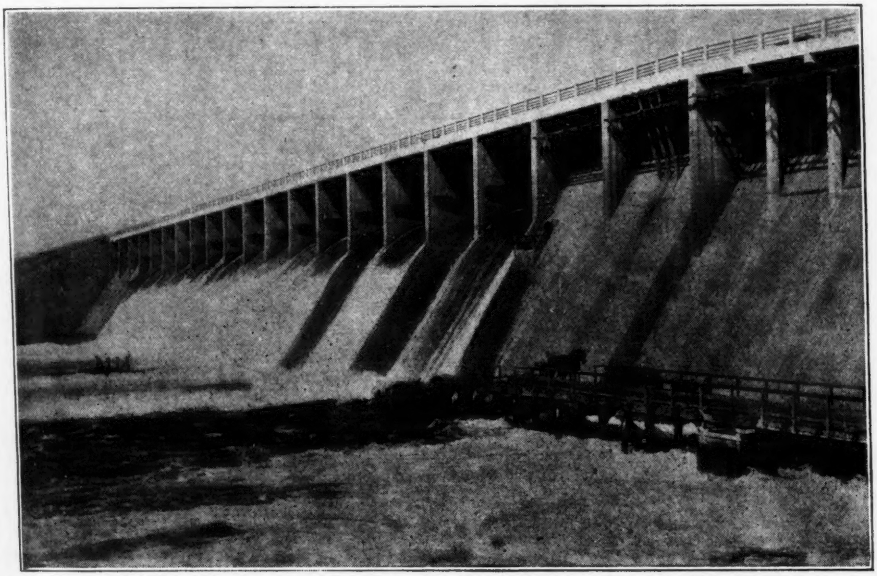

FIG. 17.—GENERAL VIEW OF SPILLWAY SECTION, AMERICAN FALLS DAM, MINIDOKA PROJECT, IDAHO.

FIG. 18.—VIEW OF ROCKY FOUNDATION OF SPILLWAY SECTION, AMERICAN FALLS DAM.

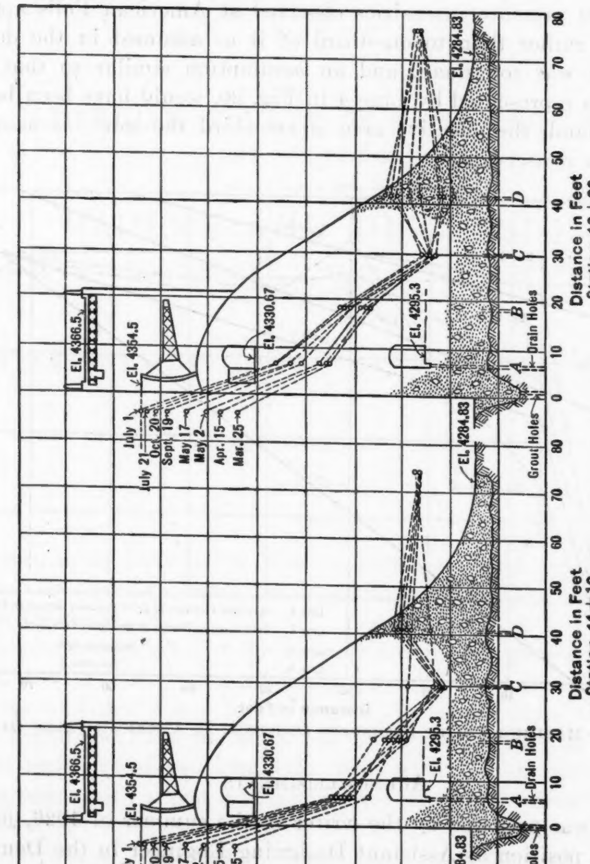

FIG. 19.—UPLIFT PRESSURES OBSERVED AT AMERICAN FALLS DAM.

If the uplift pressure intensities observed at American Falls apply to all the base area, rather than to one-third of it as assumed in the design, the assumed uplift was too small, and an assumption similar to that made at Willwood, or as represented by Line 4 in Fig. 20, would have been better. If, on the other hand, the effective area is one-third the total, as assumed, the design is amply conservative.

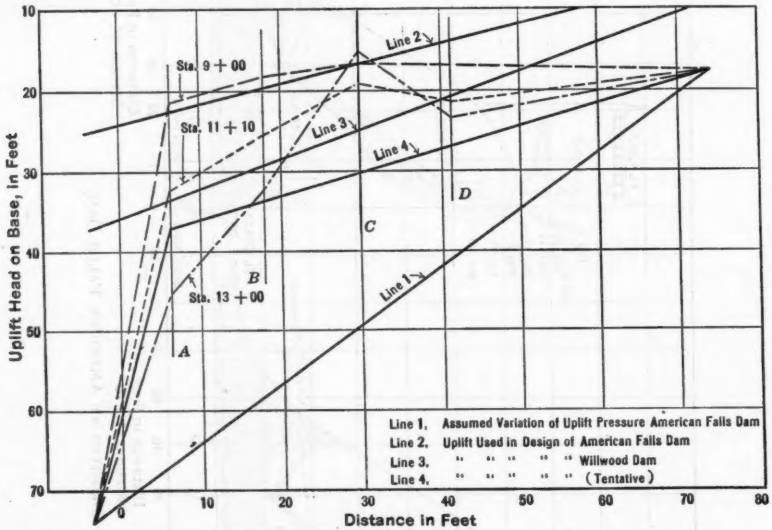

FIG. 20.—MAXIMUM UPLIFT PRESSURES OBSERVED AT AMERICAN FALLS DAM.

ACKNOWLEDGMENTS

This paper was prepared by the writer in the summer of 1926, just before he resigned his position as Assistant Designing Engineer in the Denver Office of the U. S. Bureau of Reclamation. Since that time the manuscript has been revised and enlarged in the Denver Office. Much credit is due R. D. Hubbard, Engineer of that Office, for valuable assistance in compiling and studying the data.

The experimental and research work of the U. S. Bureau of Reclamation is a part of the work carried on under the direction of J. L. Savage, M. Am. Soc. C. E., Chief Designing Engineer, and Ivan E. Houk, M. Am. Soc. C. E., Research Engineer. All engineering and construction work of the Bureau is under the direction of R. F. Walter, M. Am. Soc. C. E., Chief Engineer, with headquarters at Denver, Colo., and all activities of the Bureau are under the general charge of Elwood Mead, M. Am. Soc. C. E., Commissioner, with headquarters at Washington, D. C.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

CONTINUOUS BEAMS OVER THREE SPANS

By I. Oesterblom,* M. Am. Soc. C. E.

Synopsis

The matters of accessibility, natural light, and fresh air combine to make three-span construction more prevalent than any other series of multiple spans for buildings.

In the colder and even in the temperate climates, office buildings, schools, hospitals, hotels, clubs, etc., generally provide for corridors in the center with rooms on either side. In the tropics the same classes of buildings, and also the dwellings, have rooms in the center with verandas on either side. Thus, the problems of two long side spans with a short center span, or two short side spans with a long center span, are constantly offered to the engineer for solution.

For continuous construction, particularly prevalent on account of the increasing use of reinforced concrete, the proper solution of the moment equations is a most tedious task. It has come to be a very frequent custom, therefore, to neglect such important matters as the effect on moment factors of variability in spans, loads, and sections.

This erroneous custom is sometimes very serious in its consequences. It reaches to all divisions of engineering—design, control, and execution; consultants, municipal authorities, and manufacturers of specialties, all are affected. Even engineering committees and technical schools seem to have avoided the problem or condoned the errors.

In textbooks from Continental Europe, the problem is usually covered by the evolution of formulas which may be used in good faith and with good results, although they are generally tedious to apply.† Tables to facilitate the use of the formulas are generally missing, or so elaborate as to be of no use for the most common problems of variable spans in commercial engi-

Note.—Discussion on this paper will be closed in August, 1928.

* Structural Engr., Newark, N. J.

† See, especially, formulas in "Beton Kalender," pub. by Wilhelm Ernst & Sohn, Berlin, Germany, which contains some of the formulas derived by the writer.

neering work. The economic significance of the variables affecting the moment factors is also overlooked.

This paper aims to provide the necessary tables showing moment factors for all the most common conditions, so that the moments may be found with the expenditure of less effort and time. It also shows the significance of the variables and the seriousness of the usual conventional errors. By the use of correct methods, safety and economy of materials are obtained simultaneously.

THE THREE-MOMENT EQUATION

The "three-moment equation" is presented in many different forms, one of which is the following:

$$(M_1 + 2\,M_2)\,\frac{l_{12}}{I_{12}} + (2\,M_2 + M_3)\,\frac{l_{23}}{I_{23}} = -\,\frac{6\,A_{21}}{I_{12}} - \frac{6\,A_{23}}{I_{23}} \quad \ldots\ldots(1)$$

The symbols have the usual significance:

M_1, M_2, etc. = bending moments (at supports).
l_{12}, l_{23}, etc. = spans.
I_{12}, I_{23}, etc. = moments of inertia.
A_{21}, A_{23}, etc. = moment area reaction for simple span conditions.*

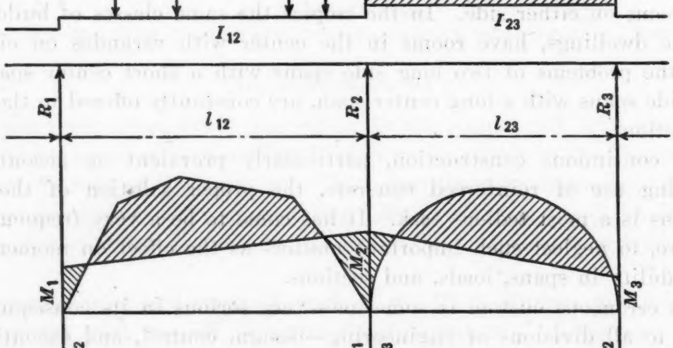

FIG. 1.

The proper meanings of the subscript indices in relation to supports and spans are shown on Fig. 1, according to the following system:

(1) Supports are numbered 1, 2, 3, and 4.
(2) Spans are numbered 12 between Supports 1 and 2; 23 between Supports 2 and 3; etc.
(3) Reactions are numbered as the supports.
(4) Unit loads and moments of inertia are numbered as the spans.

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* That is, the reaction of the area under the moment curve considered as a load, the moment curve being obtained for simple span conditions.

(5) Concentrated loads are numbered as the spans, with subscript numbers for each span to indicate locations, as 121, 122, 123, etc., for loads on Span 12.

(6) The area moment reactions take the first subscript number from the support and the second from the adjacent support on the contributory side. For example, A_{32} is the reaction at Support 3 due to the moment area between Supports 3 and 2.

The form of Equation (1) is "universal" in so far that it takes into account a different section for each span of the beam and any condition of loading. It is not completely universal, however, in so far that it neglects the effects of settlement of supports, or changes due to temperature or similar causes. For purposes of ordinary construction the form is sufficiently comprehensive, as variations in span, section, and load are the only factors that ordinarily affect the stresses.

In the left side of Equation (1) are all the "moment terms"; in the right side are the "load terms". The reasons for these designations are obvious.

Methods of Solution.—For a two-span construction with no end restraints, the end moments, therefore, being equal to zero, the moment at the center support can be found by means of one moment equation only. For each additional span under similar conditions as regards end restraint, one additional equation will be required. The solution of several equations of this type is an elaborate task. Determinants will have to be resorted to for three or more equations. This simplifies the task of solution, but it still requires a great deal of time. For several spans it is customary to use graphic methods, by means of which results can be obtained quickly and accurately. The methods now in use take into account all the usual variables—spans, loads, and moments of inertia.

The graphic method may be the only practical method for five spans or more—even for four spans, it may prove to be the best. For three spans, however, and especially for commercial "repeat work" involving merely minor variations it would be desirable to have a method developed in detail with moment factors available for immediate use. It is the object of this paper to provide these moment factors.

For commercial work, uniform loads or concentrated loads at equal spacing are especially significant. Other cases may be considered as unusual and for them no moment factors need be developed; they should be treated individually. Moments due to equi-distant concentrated loads are so nearly equal to uniform loads that the same moment factors would apply. Consider, therefore, the significance of the area-moment reactions and the need of making the "load terms" more definite.

Clapeyron's Theorem

The moment area for a uniform load is parabolic, and, therefore,

$$2\,A = \frac{2}{3} \times \frac{w\,l^2}{8} \times l = \frac{w\,l^3}{12} \quad \text{.....................(2)}$$

Hence, the three-moment equation in more definite form would be,

$$(M_1 + 2\,M_2)\,\frac{l_{12}}{I_{12}} + (2\,M_2 + M_3)\,\frac{l_{23}}{I_{23}} = -\frac{w_{12}\,l_{12}^{\;3}}{4\,I_{12}} - \frac{w_{23}\,l_{23}^{\;3}}{4\,I_{23}} \quad \text{....(3)}$$

For the specific case in which the moments of inertia are the same for all spans, the values of I disappear by cancellation and:

$$(M_1 + 2\,M_2)\,l_{12} + (2\,M_2 + M_3)\,l_{23} = \frac{w_{12}\,l_{12}^{\,3}}{4} \frac{w_{23}\,l_{23}^{\,3}}{4} \ldots\ldots (4)$$

This is the formula produced by Clapeyron (and almost simultaneously by two other investigators) in the middle of the Nineteenth Century and since known by his name. In this form, but with the idea of introducing amendments later, the equation may be applied to three-span construction. With M_1 and M_4 equal to zero two equations in sequence are needed to find M_2 and M_3. Dropping the terms in which M_1 and M_4 are factors,

$$2\,M_2\,(l_{12} + l_{23}) + M_3\,l_{23} = -\frac{1}{4}\,w_{12}\,l_{12}^{\,3} - \frac{1}{4}\,w_{23}\,l_{23}^{\,3} \ldots\ldots\ldots (5)$$

$$M_2\,l_{23} + 2\,M_3\,(l_{23} + l_{34}) = -\frac{1}{4}\,w_{23}\,l_{23}^{\,3} - \frac{1}{4}\,w_{34}\,l_{34}^{\,3} \ldots\ldots\ldots (6)$$

Support Moments.—For further operations a symbol, K, for an oft-occurring expression is introduced as follows:

$$K = 4\,(l_{12}\,l_{23} + l_{12}\,l_{34} + l_{23}\,l_{34}) + 3\,l_{23}^{\,2} \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Solving for $M_3\,l_{23}$ in Equation (5) and introducing the values of M_3 in Equation (6):

$$M_3\,l_{23} = -\frac{1}{4}\,w_{12}\,l_{12}^{\,3} - \frac{1}{4}\,w_{23}\,l_{23}^{\,3} - 2\,M_2\,(l_{12} + l_{23}) \ldots\ldots\ldots (8)$$

$$-K\,M_2 = \frac{1}{2}\,w_{12}\,l_{12}^{\,3}\,(l_{23} + l_{34}) + \frac{1}{2}\,w_{23}\,l_{23}^{\,3}\left(\frac{1}{2}\,l_{23} + l_{34}\right) - \frac{1}{4}\,w_{34}\,l_{34}^{\,3}\,l_{23} . (9)$$

Equation (9) now will give all the support moments, M_2, and after these are found all the support moments, M_3.*

First, considering Span 12 only loaded with w_{23} (w_{34}, therefore, being equal to zero),

$$-K\,M_2 = \frac{1}{2}\,w_{12}\,l_{12}^{\,3}\,(l_{23} + l_{34})$$

and,

$$M_2 = -\frac{w_{12}\,l_{12}^{\,3}}{2\,K}\,(l_{23} + l_{34}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

For the second span loaded, and with w_{12} and w_{34} equal to zero, in like manner,

$$-K\,M_2 = \frac{1}{2}\,w_{23}\,l_{23}^{\,3}\left(\frac{1}{2}\,l_{23} + l_{34}\right)$$

and,

$$M_2 = -\frac{w_{23}\,l_{23}^{\,3}}{4\,K}\,(l_{23} + 2\,l_{34}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Finally, with Span 34 only loaded, and with w_{12} and w_{23} equal to zero,

$$-K\,M_2 = -\frac{1}{4}\,w_{34}\,l_{34}^{\,3}\,l_{23}$$

* The term, "support moment," refers to the moment at a support as distinguished from "span moment," within the span itself.

and,

$$M_2 = \frac{w_{34}\, l_{34}^{\,3}}{4\, K} \cdot l_{23} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Thus, have been found the moment effects for the second support due to uniform loads on the first, second, and third spans, respectively. The expressions for M_2 being found, Equation (8) may be used, together with Equations (10), (11), and (12), to find similar expressions for M_3.

Setting w_{12}, w_{23}, and w_{34} equal to zero in like manner, the equations for Span 12 loaded will be,

$$K\, M_3\, l_{23} = -\frac{1}{4}\, K\, w_{12}\, l_{12}^{\,3} + w_{12}\, l_{12}^{\,3}\, (l_{12} + l_{23})\, (l_{23} + l_{34}) = \frac{1}{4}\, w_{12}\, l_{12}^{\,3}\, l_{23}^{\,2}$$

and,

$$M_3 = + \frac{w_{12}\, l_{12}^{\,3}}{4\, K}\, l_{23} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Similarly, for Span 23 loaded:

$$K\, M_3\, l_{23} = -\frac{1}{4}\, K\, w_{23}\, l_{23}^{\,3} + \frac{1}{2}\, w_{23}\, l_{23}^{\,3}\, (l_{12} + l_{23})\, (l_{23} + 2\, l_{34})$$

$$= -\frac{1}{4}\, w_{23}\, l_{23}^{\,3}\, (2\, l_{12}\, l_{23} + l_{23}^{\,2})$$

and,

$$M_3 = -\frac{w_{23}\, l_{23}^{\,3}}{4\, K}\, (2\, l_{12} + l_{23}) \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

Finally, for Span 34 loaded:

$$K\, M_3\, l_{23} = -2\, M_2\, (l_{12} + l_{23}) = -\frac{1}{2}\, w_{34}\, l_{34}^{\,3}\, (l_{12} + l_{23})\, l_{23}$$

and,

$$M_3 = -\frac{w_{34}\, l_{34}^{\,3}}{2\, K}\, (l_{12} + l_{23}) \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Span Moments.—From the expressions for the support moments one may proceed to find equations for the span moments through the medium of expressions for the reactions. The relations between support moments and reactions for any one span are given by the equation,

$$R_L\, l_{12} = M_2 - M_1 + \frac{1}{2}\, w_{12}\, l_{12}^{\,2} \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

In like manner the relations between span moments and the reactions will be,

$$M_x = M_1 + R_L\, x - \frac{1}{2}\, w_{12}\, x^2 \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

R_L indicates the reaction at the left end of the span and M_x, the span moment at a distance, x, from the left support. $M_{x\,\max.}$ is expressed by M_{12}, M_{23}, and M_{34}, respectively. Differentiating M_x in regard to x, and setting the derivative equal to zero, gives for $M_{x}\max.$:

$$x = \frac{R_L}{w_{12}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

and,

$$M_x \text{max.} = M_1 + \frac{R_L^2}{2\, w_{12}} \dots\dots\dots\dots\dots\dots\dots (19)$$

Solving for R_L in Equation (16) and using that value in Equation (19):

$$M_{12} = M_1 + \frac{1}{2\, w_{12}} \left(\frac{M_2}{l_{12}} - \frac{M_1}{l_{12}} + \frac{w_{12}\, l_{12}}{2}\right)^2 \dots\dots\dots\dots (20)$$

Equations (18) and (19) are useful and important; but Equation (20) is the one to be used for the specific purpose of finding maximum span moments. All three equations obviously are good for any span with proper modifications of the indices for M, w, and l. Therefore, the following equations hold for Spans 23 and 34:

$$M_{23} = M_2 + \frac{1}{2\, w_{23}} \left(\frac{M_3}{l_{23}} - \frac{M_2}{l_{23}} + \frac{w_{23}\, l_{23}}{2}\right)^2 \dots\dots\dots\dots (21)$$

$$M_{34} = M_3 + \frac{1}{2\, w_{34}} \left(\frac{M_4}{l_{34}} - \frac{M_3}{l_{34}} + \frac{w_{34}\, l_{34}}{2}\right)^2 \dots\dots\dots\dots (22)$$

It should be noted that for the particular cases under investigation all terms containing M_1 and M_4 disappear, as these two moments are always zero.

Moment Effects from Adjacent Spans

As may be noted, Equations (20), (21), and (22) will give expressions for the maximum bending moments in the loaded spans, but obviously will not give any results for adjacent spans. This is on account of the presence of the zero load element, w, as a divisor in the second term of the equations, which makes the expressions indefinite. To find the meaning of this indefinite feature the three-moment diagrams may be considered in sequence, as shown in Fig. 2.

As previously, the supports are indicated by the numerals, 1, 2, 3, and 4; the spans by the numbers, 12, 23, and 34; and the conditions of loading by the letters, A, B, and C. In Fig. 2, A shows a diagram for Span 12 loaded; B, for Span 23; and C, for Span 34. The quantities, $M_{f_{12}}$, $M_{f_{23}}$, and $M_{f_{34}}$, would be the moments at points of maximum effect for the loaded spans if they were freely supported and not continuous; and M_{12}, M_{23}, and M_{34}, the actual maximum moments for the loaded spans.

It is obvious from Fig. 2 that there can be no maximum span moments in the interior of spans adjacent to the loaded spans. This, therefore, is the interpretation of the indefinite feature of Equations (20), (21), and (22).

Combination of Moment Effects.—For purposes of analysis the individual diagrams are instructive. As the ultimate aim of this paper is practical, however, the results from the combination of the three are more interesting. It is not desired to know the maximum effect in the unloaded spans, but rather the effect where the combined moments will be a maximum.

In individual cases it is easy to establish the points of maximum effect for loaded spans due to the loads on these spans. Equations (16) and (18) will give these with little effort after the negative moments have been found. After the addition of secondary effects from adjacent spans the points of maxi-

mum moment will not be quite the same. The combined moments at the two points would be substantially the same, however, as the change of curvature between the points is insignificant.

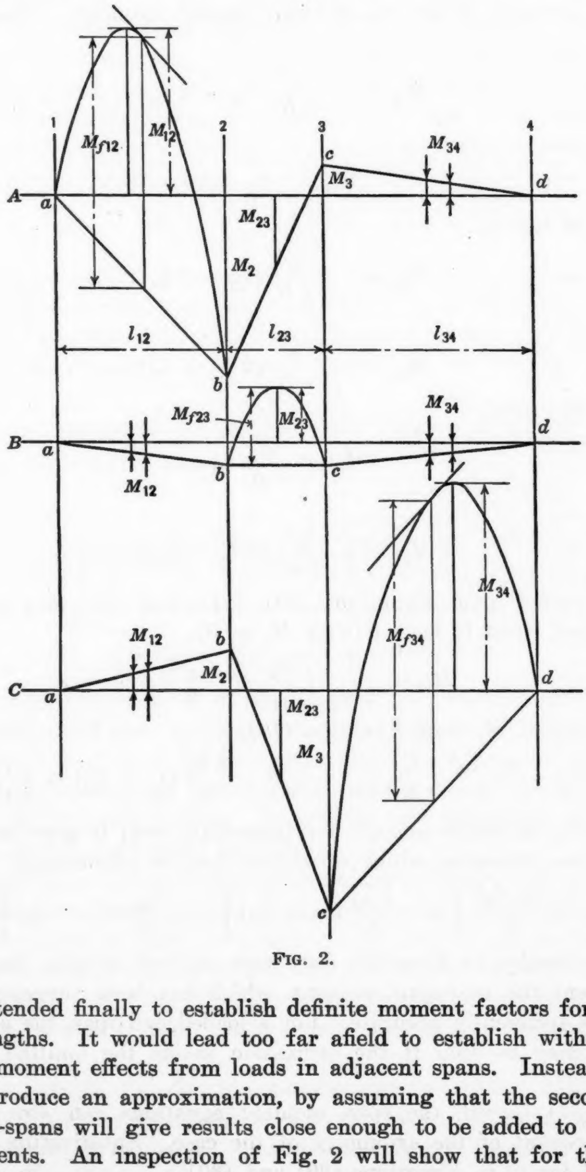

FIG. 2.

It is intended finally to establish definite moment factors for given ratios of span lengths. It would lead too far afield to establish without error the secondary moment effects from loads in adjacent spans. Instead it is necessary to introduce an approximation, by assuming that the secondary effects at the mid-spans will give results close enough to be added to the two principal moments. An inspection of Fig. 2 will show that for the totals this is substantially correct. On all the spans the lines representing the two unloaded effects are compensating, and the resultant of the two is almost horizontal. In the outside spans the secondary effects are generally small, while

in the center spans the mid-points in most cases of reasonable design are truly points of maximum combined effect. All these considerations combine to make the proposed approximation almost equal to definite accuracy.

On this basis the secondary mid-span moment effects can now be written directly as averages of the two adjacent support moments. Then, with Span 12 loaded,

$$M_{23} = -\frac{l_{12}^3}{8\,K}(l_{23} + 2\,l_{34}) \quad \text{(23)}$$

and,

$$M_{34} = \frac{l_{12}^3}{8\,K}\,l_{23} \quad \text{(24)}$$

With Span 23 loaded,

$$M_{12} = -\frac{l_{23}^3}{4\,K}(l_{23} + 2\,l_{34}) \quad \text{(25)}$$

and,

$$M_{34} = -\frac{l_{23}^3}{4\,K}(2\,l_{12} + l_{23}) \quad \text{(26)}$$

With Span 34 loaded,

$$M_{12} = \frac{l_{34}^3}{8\,K}\,l_{23} \quad \text{(27)}$$

and,

$$M_{23} = -\frac{l_{34}^3}{8\,K}(2\,l_{12} + l_{23}) \quad \text{(28)}$$

For the loaded spans Equations (20), (21), and (22) may now be used. Assuming, first, Span 12 loaded (with $M_1 = 0$),

$$M_{12} = \frac{1}{2\,w_{12}}\left(\frac{M_2}{l_{12}} + \frac{w_{12}\,l_{12}}{2}\right)^2$$

and with values of M_2 from Equation (10),

$$M_{12} = \frac{w_{12}\,l_{12}^2}{8}\left\{\frac{l_{12}^2}{K^2}(l_{23} + l_{34})^2 - \frac{2\,l_{12}}{K}(l_{23} + l_{34}) + 1\right\} \quad \text{(29)}$$

This expression is simple enough for immediate use; it may be written in a different form, however, which sometimes has its advantages,

$$M_{12} = \frac{w_{12}\,l_{12}^2}{8}\left\{1 - \frac{2\,l_{12}}{K}(l_{23} + l_{34})\left[1 - \frac{l_{12}}{2\,K}(l_{23} + l_{34})\right]\right\} \quad \text{(30)}$$

If the square bracket in Equation (30) were omitted entirely, the remainder would represent the mid-span moment, which has been suggested by some authorities as sufficiently accurate. For a loaded end span the error is considerable, as may be seen if the expression inside the omitted bracket be evaluated.

For Span 34 loaded, the span moment equations can now be written directly on account of the symmetry of the case. Substituting Indices 12 for 34 and 34 for 12 in Equations (29) and (30),

$$M_{34} = \frac{w_{34}\,l_{34}^2}{8}\left\{\frac{l_{34}^2}{K^2}(l_{23} + l_{12})^2 - \frac{2\,l_{34}}{K}(l_{23} + l_{12}) + 1\right\} \quad \text{(31)}$$

and,

$$M_{34} = \frac{w_{34}\, l_{34}^2}{8} \left\{ 1 - \frac{2\, l_{34}}{K} (l_{23} + l_{34}) \left[1 - \frac{l_{34}}{2\, K} (l_{23} + l_{34}) \right] \right\} \ldots\ldots (32)$$

For moment effects in the center span due to loading in that span, again it is possible to write an approximation with a considerable accuracy and thereby eliminate long calculations, which would hardly add anything of practical significance. The maximum moment effect will be found so close to mid-span and the disturbing negative moment line for all reasonable span conditions will be so nearly horizontal that a mid-span expression for M_{23} is fully justified. On this basis,

$$M_{23} = \frac{w_{23}\, l_{23}^2}{8} \left\{ 1 - \frac{2\, l_{23}}{K} (l_{12} + l_{23} + l_{34}) \right\} \ldots\ldots\ldots\ldots (33)$$

To ascertain directly and immediately useful moment factors for all the most frequently occurring span and load conditions in three-span construction all the necessary formulas have now been found. Table 1 gives a summary of those expressions required.

TABLE 1.—SCHEDULE OF EQUATIONS FOR VARIOUS MOMENTS AND SPANS.

Load on.	M_{12}.	M_2.	M_{23}.	M_3.	M_{34}.
Span 12................	Equation (30)	Equation (10)	Equation (23)	Equation (13)	Equation (24)
Span 23................	Equation (25)	Equation (11)	Equation (33)	Fquation (14)	Equation (26)
Span 34................	Equation (27)	Equation (12)	Equation (28)	Equation (15)	Equation (32)

CONDITIONS OF SYMMETRY

Clearly it is impracticable to make a table of moment factors for all possible and reasonable span combinations. For increments of 6 in. and with a range from 8 ft. 0 in. to 30 ft. 0 in., which would cover all common cases, there would be no less than 2 025 combinations, for each of which there would have to be 15 moment factors. Neither is this necessary, as the moment factors are merely matters of span ratios.

This analysis will be limited to conditions of perfect symmetry or slight asymmetrical variations, as these cases are the only ones of frequent practical occurrence. First, let $l_{12} = l_{34} = l$ and $l_{23} = xl$, in which, x is the span ratio. Then moment factors may be established for all span ratios between 0.20 and 2.50 in increments of 0.10. Finally, it will be pointed out how these moment factors may be used to ascertain correctly moments for cases of asymmetric span relations.

For this purpose the equations enumerated in Table 1 will be used, with the span designations as noted and with the loads for the loaded spans equal to unity in proper sequence. The moment factor formulas (Table 2) are merely simplifications as outlined on the basis of the equations noted in Table 1. The table of "Moment Factors" (Table 3) is a further development for given span ratios.

TABLE 2.—MOMENT FACTOR FORMULAS FOR THREE-SPAN CONSTRUCTION.
(Two outside spans alike and equal to unity; middle span equal to x.)
$(K = 8\,x + 3\,x^2 + 4)$

Loading.	M_{12}.	M_2.	M_{23}.	M_3.	M_{34}.
Span 12 loaded.	$\frac{1}{8} - \frac{x+1}{4\,K}\left(1 - \frac{x+1}{2\,K}\right)$	$-\frac{x+1}{2\,K}$	$-\frac{x+2}{8\,K}$	$+\frac{x}{4\,K}$	$+\frac{x}{8\,K}$
Span 23 loaded.	$-\frac{2\,x^3 + x^4}{8\,K}$	$-\frac{2\,x^3 + x^4}{4\,K}$	$\frac{x^2}{8} - \frac{2\,x^3 + x^4}{4\,K}$	$-\frac{2\,x^3 + x^4}{4\,K}$	$-\frac{2\,x^3 + x^4}{8\,K}$
Span 34 loaded.	$+\frac{x}{8\,K}$	$+\frac{x}{4\,K}$	$-\frac{x+2}{8\,K}$	$-\frac{x+1}{2\,K}$	$\frac{1}{8} - \frac{x+1}{4\,K}\left(1 - \frac{x+1}{2\,K}\right)$

Dead and Live Load Effects.—It has come to be a practice among American designers to combine dead and live loads in computing moments. This is a very unfortunate practice, because the live load effects vary greatly, whereas the dead load effects do not. To endeavor to produce total load moment factors, comprehensive enough to cover every possible ratio of dead to live load, is merely to accept a crude method, when a correct method is equally simple. In Table 3, therefore, no attempt has been made at moment-factor summation, which the writer considers incorrect in principle, inaccurate as regards results, and leading to waste in materials of construction. In addition, both dead and live loads may vary from span to span, which makes the separation of moment factors necessary. Finally, there may be variation in moments of inertia, for which case, as will be seen, separated factors are essential.

MOMENT FACTORS

In Table 3 are given the coefficients, ϕ, of the moments for any given set of dimensions and conditions of loading. To find any moment, first find the ratio, x, of the center span to the end span, then multiply the moment factors, ϕ, for spans and load conditions, by $w\,l^2$. Both dead and live loads for all spans must be used in this computation and the results added. Always use the outside span in the formula as all moment factors have been computed on the basis of the outside span being equal to unity.

A comparative review of the moment factors in Table 3, particularly in comparison with conventional moment factors for equal span construction, is very instructive. It is especially noteworthy that negative moments occur in the center spans whenever these are short comparatively; also, in the end spans, when they are short. Indeed, the center spans are very severely punished by the end span loads, and the span ratio has to reach 0.60 before the danger of negative moments is avoided. This point is especially significant in view of the universal custom of computing moments as positive through an erroneous use of the conventional moment factors for equal spans.

Application of Factors—Symmetrical Spans.—An example will show the practical working of the formulas and tables.

TABLE 3.—MOMENT FACTORS, ϕ, FOR MAXIMUM BENDING MOMENTS IN THREE-SPAN CONSTRUCTION.

FORMULA, $M = \phi\, w\, l^2$

(Symmetrical span arrangement; x = ratio of middle to side spans; all moment factors are computed on basis of outside spans.)

Values of x.	Values of K.	MAXIMUM BENDING MOMENTS FOR:								
		Outside Spans.			Middle Spans.			Points of Support.		
		First or third span loaded.	Second span loaded.	Third or first span loaded.	First or third span loaded.	Second span loaded.	Third or first span loaded.	First or third span loaded.	Second span loaded.	Third or first span loaded.
0.20	5.72	0.078	−0.000	0.004	−0.048	0.004	−0.048	−0.105	−0.001	0.009
0.30	6.67	0.081	−0.001	0.006	−0.043	0.009	−0.043	−0.098	−0.002	0.011
0.40	7.68	0.084	−0.003	0.007	−0.039	0.015	−0.039	−0.091	−0.005	0.013
0.50	8.75	0.086	−0.005	0.007	−0.036	0.022	−0.036	−0.086	−0.009	0.014
0.60	9.88	0.088	−0.007	0.008	−0.033	0.031	−0.033	−0.081	−0.014	0.015
0.70	11.07	0.090	−0.011	0.008	−0.031	0.040	−0.031	−0.077	−0.021	0.016
0.80	12.32	0.091	−0.015	0.008	−0.028	0.051	−0.028	−0.073	−0.029	0.016
0.90	13.93	0.093	−0.019	0.008	−0.026	0.063	−0.026	−0.068	−0.038	0.016
1.00	15.00	0.094	−0.025	0.008	−0.025	0.075	−0.025	−0.067	−0.050	0.017
1.10	16.43	0.095	−0.031	0.008	−0.024	0.088	−0.024	−0.064	−0.063	0.017
1.20	17.92	0.096	−0.039	0.008	−0.022	0.103	−0.022	−0.061	−0.077	0.017
1.30	19.47	0.097	−0.047	0.008	−0.021	0.118	−0.021	−0.059	−0.093	0.017
1.40	21.08	0.098	−0.055	0.008	−0.020	0.135	−0.020	−0.057	−0.111	0.017
1.50	22.75	0.099	−0.065	0.008	−0.019	0.151	−0.019	−0.055	−0.130	0.017
1.60	24.48	0.100	−0.075	0.008	−0.018	0.170	−0.018	−0.053	−0.151	0.016
1.70	26.27	0.101	−0.087	0.008	−0.018	0.188	−0.018	−0.051	−0.173	0.016
1.80	28.12	0.101	−0.099	0.008	−0.017	0.208	−0.017	−0.050	−0.197	0.016
1.90	30.03	0.102	−0.111	0.008	−0.016	0.229	−0.016	−0.048	−0.223	0.016
2.00	32.00	0.103	−0.125	0.008	−0.016	0.250	−0.016	−0.047	−0.250	0.016
2.10	34.03	0.103	−0.140	0.008	−0.015	0.272	−0.015	−0.046	−0.279	0.015
2.20	36.12	0.104	−0.155	0.008	−0.015	0.295	−0.015	−0.044	−0.310	0.015
2.30	38.27	0.104	−0.171	0.008	−0.014	0.318	−0.014	−0.043	−0.342	0.015
2.40	40.48	0.105	−0.188	0.007	−0.014	0.344	−0.014	−0.042	−0.375	0.015
2.50	42.75	0.105	−0.205	0.007	−0.013	0.370	−0.013	−0.041	−0.411	0.015

Example 1.—A 9-ft. corridor span and two outside spans, 22 ft. 6 in. each; dead load, 67 lb. per sq. ft.; live load, 60 lb. per sq. ft for outside, and 80 lb. per sq. ft. for middle, spans; and span ratio, $x = 0.40$. From Table 3 directly:

Dead Load Bending Moments:

First and Third Spans (equal):

First span load..... $+ 0.084 \times 67 \times 22.5^2$

Second " " ..... $- 0.003 \times 67 \times 22.5^2$

Third " " ..... $+ 0.007 \times 67 \times 22.5^2$

Total effect .... $+ 0.088 \times 67 \times 22.5^2 = + 2\,980$ ft-lb.*

Middle Span:

First span load..... $- 0.039 \times 67 \times 22.5^2$

Second " " ..... $+ 0.015 \times 67 \times 22.5^2$

Third " " ..... $- 0.039 \times 67 \times 22.5^2$

Total effect .... $- 0.063 \times 67 \times 22.5^2 = - 2\,130$ ft-lb.

Middle Supports:

First span load..... $- 0.091 \times 67 \times 22.5^2$

Second " " ..... $- 0.005 \times 67 \times 22.5^2$

Third " " ..... $+ 0.013 \times 67 \times 22.5^2$

Total effect .... $- 0.083 \times 67 \times 22.5^2 = - 2\,800$ ft-lb.

* The writer prefers the notation, "lb. ft." to distinguish units of moment from "ft-lb.," as units of energy.

Live Load Bending Moments:

First and Third Spans (equal):

First span load.....	$+ 0.084 \times 60 \times 22.5^2$	$=$	$+ 2\,550$
Second " "	$- 0.003 \times 80 \times 22.5^2$	$=$	$- 120$
Third " "	$+ 0.007 \times 60 \times 22.5^2$	$=$	$+ 210$
Maximum effect			$+ 2\,760$ ft-lb.

Middle Span:

First span load.....	$- 0.039 \times 60 \times 22.5^2$	$=$	$- 1\,180$
Second " "	$+ 0.015 \times 80 \times 22.5^2$	$=$	$+ 610$
Third " "	$- 0.039 \times 60 \times 22.5^2$	$=$	$- 1\,180$
Maximum effect			$- 2\,360$ ft-lb.

Middle Supports:

First span load.....	$- 0.091 \times 60 \times 22.5^2$	$=$	$- 2\,770$
Second " "	$- 0.005 \times 80 \times 22.5^2$	$=$	$- 200$
Third " "	$+ 0.013 \times 60 \times 22.5^2$	$=$	$+ 395$
Maximum effect			$- 2\,970$ ft-lb.

Total Bending Moments:

First and third spans (equal)	$+ 2\,980 + 2\,760$	$=$	$+ 5\,740$ ft-lb.
Middle span	$- 2\,130 - 2\,360$	$=$	$- 4\,490$ ft-lb.
Middle supports	$- 2\,800 - 2\,970$	$=$	$- 5\,770$ ft-lb.

The example is instructive in many respects. It shows how the moment factors may be summarized, when the load is equal for all spans and constant; and how they may be used separately for variable loads or loads of different magnitude. It shows that the load effects from the outside spans predominate in the middle span, so that the negative moments overbalance the positive moments. It shows how in the first span the moment from the second span load must be omitted to obtain the maximum effect and how in the middle span for the same reason its own load must be omitted. It shows, finally, that even with the outside span live loads omitted, the center span load will not be sufficient to cause a positive moment effect in the middle span as against the dead load effect from the outside spans. About most of these very important matters, conventional methods of design give no information.

Different Moments of Inertia

This leads to a consideration of three-span construction having a different moment of inertia in each span. For this purpose take Equation (3) and multiply by I_{12}; then,

$$(M_1 + 2\,M_2)\,l_{12} + (2\,M_2 + M_3)\,l_{23}\,\frac{I_{12}}{I_{23}} = -\frac{w_{12}\,l_{12}^{\,3}}{4} - \frac{w_{23}\,l_{23}^{\,3}}{4} \times \frac{I_{12}}{I_{23}} . \quad (34)$$

or in a more useful form, letting $q = \dfrac{I_{12}}{I_{23}}$,

$$(M_1 + 2\,M_2)\,l_{12} + (2\,M_2 + M_3)\,q\,l_{23} = -\frac{w_{12}\,l_{12}^{\,3}}{4} - q\,\frac{w_{23}\,l_{23}^{\,3}}{4} \ldots (35)$$

Expressed in words, Equation (35) means that if the moment of inertia varies from span to span, the moment factors already found may still be used, provided certain adjustments of spans and loads be introduced; that is:

(1) One span should be selected as the principal span to remain without change and with the original loads applied.

(2) The other two spans should be transformed (lengthened or shortened) in the ratio of the moments of inertia; the slenderer beams should be increased, and the heavier beams decreased, in length.

(3) The loads of the same two spans should be divided by the square of the same figure with which the spans were multiplied; or, in other words, for an increased span there should be decreased load and *vice versa*.

Application.—For purposes of demonstration and comparison take Example 1 as before, but with the moment of inertia for the middle span only a fraction of that for the outside span.

Example 2.—Same span conditions as in Example 1. Dead load, 67 lb. per sq. ft. for outside spans and 60 lb. for middle span; live loads, 60 and 80 lb., respectively; moments of inertia ratio (outside to middle spans), 4:1; middle transformed span, therefore, $4 \times 9 = 36$ ft.; span ratio, $\frac{36}{22.5} = 1.60$; modified dead load, $\frac{60}{16} = 3.75$ lb. per sq. ft.; and modified live loads, $\frac{80}{16} = 5$ lb. per sq. ft. With the original information, and with spans and loads modified, the moment factors can now be applied directly.

Dead Load Bending Moments:

First and Third Spans (equal):

First span load...	+ 0.100	× 67	× 22.5²	=	+ 3 390
Second " " ...	— 0.075	× 3.75	× 22.5²	=	— 140
Third " " ...	+ 0.008	× 67	× 22.5²	=	+ 270
Total effect					+ 3 520 ft-lb.*

Middle Span:

First span load...	— 0.018	× 67	× 22.5²	=	— 610
Second " " ...	+ 0.170	× 3.75	× 22.5²	=	+ 320
Third " " ...	— 0.018	× 67	× 22.5²	=	— 610
Total effect					— 900 ft-lb.

Middle Supports:

First span load...	— 0.053	× 67	× 22.5²	=	— 1 800
Second " " ...	— 0.151	× 3.75	× 22.5²	=	— 290
Third " " ...	+ 0.016	× 67	× 22.5²	=	+ 540
Total effect					— 1 550 ft-lb.

Live Load Bending Moments:

First and Third Spans (equal):

First span load...	+ 0.100	× 60	× 22.5²	=	+ 3 040
Second " " ...	— 0.075	× 5	× 22.5²	=	— 190
Third " " ...	+ 0.008	× 60	× 22.5²	=	+ 240
Maximum effect					+ 3 280 ft-lb.

Live Load Bending Moments (*Continued*):

Middle Span:

First span load...	— 0.018 × 60	× 22.5²	=	—	550
Second " " ...	+ 0.170 × 5	× 22.5²	=	+	430
Third " " ...	— 0.018 × 60	× 22.5²	=	—	550
Maximum effect				—	1 100 ft-lb.

Middle Supports:

First span load...	— 0.053 × 60	× 22.5²	=	—	1 610
Second " " ...	— 0.151 × 5	× 22.5²	=	—	380
Third " " ...	+ 0.016 × 60	× 22.5²	=	+	485
Maximum effect				—	1 990 ft-lb.

Total Bending Moments:

First and third span (equal)....	+ 3 520 + 3 280	=	+ 6 800
Middle span	— 910 — 1 100	=	— 2 010
Middle supports	— 1 550 — 1 990	=	— 3 540

Asymmetrical Span Arrangement

Thus far, only symmetrical cases have been examined. It would extend the discussion too far, and without particular value, to cover the full range of asymmetrical arrangements. The universal case may be solved without much difficulty by applying the formulas scheduled and introducing at the same time—if need be—span and load modifications as already described for variable moments of inertia.

There are numerous cases in which the span arrangement is almost symmetrical, the outside spans differing in length by a few feet only. In a moderate climate a school building, for example, may have a 10-ft. corridor in the center and 20 and 22-ft. rooms on the sides; or, in warmer climates, a 20-ft. room in the center with 12 and 14-ft. verandahs on the sides.

A survey of the moment factors in Table 3, particularly for the supplementary load effects from adjacent spans, will make it plain that such minor variations from true symmetry will not cancel the usefulness of the tabular figures. Each end span, with its inner support, may be computed quite accurately as if the opposite side were the same and, then, the middle span, assuming the end spans to be equal to an average of the two.

Assume, for instance, spans of 20 ft. 0 in., 9 ft. 0 in., and 22 ft. 0 in. The spans ratios are then 0.45 and 0.41, respectively. For these differences the variations in the load effects are visible to a small extent in the third decimal only. A simple example using a larger asymmetry would readily show further approximation.

Conventional Errors

In usual designing practice the solution by the graphical method or the three-moment theorem is not very common; instead, conventional moment factors are ordinarily applied. These factors were never intended to apply except for equal spans. How dangerously incorrect the use of these factors

are may be shown by continuing the foregoing examples in accordance with the conventional method as follows:

$$\text{Outside spans} \ldots\ldots\ldots\ldots \frac{127 \times 22.5^2}{10} = + 6\,430 \text{ ft-lb.}$$

$$\text{Middle spans} \ldots\ldots\ldots\ldots \frac{140 \times 9^2}{12} = + \quad 940 \text{ ft-lb.}$$

$$\text{Middle supports} \ldots\ldots\ldots\ldots \frac{-133 \times 15.75^2}{12} = - 2\,750 \text{ ft-lb.}$$

A comparison of these results with those previously obtained is enlightening. A summary of the three methods is shown in Table 4.

TABLE 4.—Comparison of Moments, in Foot-Pounds, by Various Methods.

Location.	By Three-Moment Theorem:		By conventional moment factors.
	Equal values of *I*.	Unequal values of *I*.	
Outside spans	+5 740	+6 800	+6 430
Middle spans	−4 490	−2 010	+ 940
Middle supports	−5 770	−3 540	−2 750

Practical Consequences

Obviously, for the example chosen—and it represents a very common case—the moments for both the middle span and its support are most erroneously calculated. This is true not only with the rough conventional moment factors, but also when the three-moment theorem is applied without allowance for the change in cross-section.

The middle span moment by the conventional method is positive, whereas it should be decidedly negative and increase by a parabolic curve until the still larger support moment is reached. By the conventional method the support and outside span moments are also incorrect, but in this example not dangerously so. In regard to the use of the three-moment theorem it should be noted that modification for the change of section is most necessary, as the differences in results are very large—for the middle span and support moments, 55% and 49%, respectively. In this case, it is true the error is on the safe side, but with other span proportions it would be the reverse.

At any rate a question of economy or strength has been raised: Who would advocate using carelessly large quantities of materials when they are not needed, or leaving them out when they are required for strength? The question might also be asked: What chance, for instance, has a slender slab of concrete, proportioned for a moment of 940 ft-lb., to resist a negative moment of 3 540 ft-lb.?

Careless Designs.—The ignorance or lack of care in this matter would have brought disastrous results in reinforced concrete construction, had it not been for compensating features. When the two outer spans are long and the middle spans short, or *vice versa*—the cases where conventional methods are

most in error—the heavier spans with their supports generally act as independent units and as such are strong enough to stand of themselves. The slenderer spans also act independently, and are able somehow to hang on to the supports even in spite of numerous cracks, which are not as large as they might be (on account of generous load compensations) and are efficiently concealed by laitance, dust, floor coverings, partitions, base-boards, and the like.

Revision of Current Practice Essential.—This general subject has been taboo both to university and to municipal authorities. Consulting engineers likewise seem to be unable to spare the time to design correctly. The large reinforcement bar distributors, "pioneers and consultants in reinforced concrete," are possibly the worst offenders. Their various "tin pan systems", most excellent for long-span construction, are lagging behind in details suitable for a correct computation of continuity.

Perhaps even the Joint Committee on Concrete and Reinforced Concrete is somewhat to blame. It would have been consistent and quite in order for this Committee to omit all reference to moment factors, which are not truly subject to regulation by anything akin to a code; but to write several pages about moments for equal spans and then to state* merely that "continuous beams with unequal spans * * * shall be designed for actual moments", is surely inconsistent and only invites others to "escape from an unpleasant reality".

One effect on bureaucratic minds has been to make the conventional moment factors universal for almost all spans and conditions. Indeed a competent designer is sometimes forced to do what he knows to be wrong, because some one in authority has spoken with the appearance of great understanding. Possibly, and conceivably, this paper may bring some of these hazy or incorrect ideas to a proper focus.

* *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, p. 1191.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS AND DISCUSSIONS

THE DESIGN OF AIRPLANE WING-BEAMS

By Joseph S. Newell,* Jun. Am. Soc. C. E.

Synopsis

This paper presents the method developed by the United States Army Air Corps for the analysis and design of the beams used in airplane wings. These beams, which in the conventional biplane structure are generally continuous over three or more supports, are subjected to combined axial and transverse loads due to the fact that they act as the chord members of the lift and internal drag trusses at the same time that they carry the air loads. Although the air loads are actually applied to the wing-beams by the ribs as a series of concentrated loads, it is customary to assume them to act as if they were uniformly distributed and to design accordingly. The first part of this paper presents, in the form developed by the writer, the formulas that are now used for obtaining the stresses in wing-beams under various types of loadings, emphasis being placed on those for the uniformly distributed and the concentrated transverse loads acting in conjunction with an axial compression.

Equations are given for single spans as well as continuous beams. Tables of functions in radians, are included in the Appendix to reduce, to a minimum, the labor of applying the formulas. (See Tables 8 and 9.) A graphical method is also given, which is more general than the analytical in that it provides for the effects of a variation in moment of inertia within a given span. The method, however, is limited in its application to beams continuous over three or, under certain conditions, four supports. The effect of variations in the ratio of lateral to axial load on the location of points of inflection in continuous or restrained beams is outlined, and methods are given by which some of the difficulties encountered in the application of the

Note.—Written discussion on this paper will be closed in **August, 1928.**

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formulas may be avoided. In short, an effort is made in Part I to present a precise method for the determination of stresses in members under combined loadings.

Part II is devoted to a description of the method developed by the Forest Products Laboratory, at Madison, Wis., for computing the allowable stresses on wooden beams of various shapes when subjected to transverse loads alone or in combination with axial loads. This method, which is partly theoretical, but mostly empirical, emphasizes the effect of the shape of such beams on their modulus of rupture and elastic limit stresses, an effect which has been found to be of the utmost importance in wooden beams. Little or no information is available, however, regarding this shape effect in steel or duralumin.

Parts I and II, therefore, completely cover the analysis and design of wooden beams subjected to combined loadings, but do not treat the design of metal beams.

Part III presents and discusses the results of several tests made on pin-ended struts both with and without lateral loads. These tests were conducted at McCook Field, Dayton, Ohio, for the purpose of establishing the reliability of the methods developed in Parts I and II, and some very interesting data were obtained. Sufficient tests have been made to prove the dependability of the precise formulas for computing the imposed stresses, but the data on the reliability of the method for obtaining the allowable stresses are meager and do not permit forming more than a general conclusion. This method is, however, the most rational one available, and it is now being used by both the Army and Navy Air Services in the design of military airplanes. Its use is recommended by the Air Regulations Division of the U. S. Department of Commerce for all commercial airplanes which apply for license by that Department.

The test data on struts without lateral loads are especially interesting in that they demonstrate that the Euler load may be reached quite consistently on struts having slenderness ratios of 60, 70, and 80, if precautions are taken to eliminate all eccentricities due to variations in the material. The methods of eliminating such eccentricities in the laboratory and their apparent effects on the behavior of short columns are briefly discussed.

Historical and Bibliographical

During the World War the rapid development of the airplane necessitated a vast amount of research in many fields, but especially in those connected with the strength of materials. Not only was it necessary to develop new structural materials of high strength and light weight and to investigate the strength properties of materials already in common use when employed in new ways, but new, or at least more precise, methods of analysis were required. One of the foremost problems of the latter type was to develop a means for determining the stresses in wing-beams subjected to combined axial and transverse loads.

In 1915, Messrs. H. Booth and H. Bolas developed a method of analysis based on the differential equation of the elastic curve.* In 1916, Mr. Arthur Berry, a Fellow of King's College, Cambridge, England, made several simplifications to this method and greatly facilitated its application to actual airplane beams by including a set of functions for use with the three-moment equation.† Developments and modifications of various phases of the problem were made by several others, among the more important being those of Messrs. W. L. Cowley and H. Levy.‡ In Germany, H. Müller-Breslau§ developed a similar method.

While the "Berry method" was developed only for the case of a uniformly distributed transverse load in combination with axial compression, or tension, Müller-Breslau, in "Technische Berichte,"§ covered the cases of a uniformly distributed transverse load and of several concentrated loads in a span, and also outlined the problem of the stability of beams as the axial load approaches the Euler load. In his "Graphische Statik"§ the treatment of the subject is much more complete and equations are developed for the general case of any loading that can be represented as a function of X, the distance from the origin.

The United States Naval Air Service adopted the "Berry method" when America entered into the World War, whereas the Army Air Service used various approximate methods; and it was not until 1922 that the Army began to investigate the more precise methods of analysis. At that time a study was made of both the Berry and Müller-Breslau methods, and it was seen that the theoretical basis for both methods was the same, the differences being in the choice of the origin and in nomenclature. Of the two, the Müller-Breslau equations appeared to be less laborious and the Army Air Service undertook to simplify their application still further and to present them in a form readily adapted to use in the routine design of airplane structures. The result was the presentation of the equations in a form very similar to that given in this paper in reports of the U. S. Army Air Service.‖ More complete reports were issued in 1924.¶

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* "Some Contributions to the Theory of Engineering Structures, with Special Reference to the Problem of the Airplane," by H. Booth and H. Bolas, pub. by Air Dept., British Admiralty, 1915.

† "The Calculation of Stresses in Aeroplane Wing Spars," by Arthur Berry, *Transactions, No. 1*, Royal Aeronautical Soc., Lond., 1919.

‡ In papers issued by the British Advisory Committee for Aeronautics and, later, published in *Proceedings*, Royal Aeronautical Soc., London.

§ Pub. in 1918 in "Technische Berichte der Flugzeugmeisterei der Fliegertruppen," in Charlottenburg, and, more recently, 1925, in Band II, Abt. II, of his "Graphische Statik der Baukonstruktionen," Alfred Kroner, Leipzig, Germany.

‖ Serial Repts. Nos. 2212 and 2213, Eng. Div., U. S. Army Air Corps, as preprints to "Airplane Design," Chapter II, June, 1923.

¶ "The Investigation of Structural Members under Combined Axial and Transverse Loads," McCook Field Serial Rept. No. 2400, and, later, *Air Service Information Circular, Aviation, No. 493*, by J. S. Newell, Jun. Am. Soc. C. E.; "The Investigation of the Movements of Points of Inflection on Beams under Various Combinations of Load," 1924, McCook Field Serial Report No. 2324, *Air Service Information Circular, Aviation, No. 471*, by Lieut. H. A. Sutton and J. S. Newell, Jun. Am. Soc. C. E. Other writings touching on this subject are: "Applied Elasticity," Chapter 6, by J. Prescott, Longmans Green & Co., 1924; "Aeronautics in Theory and Experiment," Chapter 8, by Cowley and Levy, Longmans Green & Co., Lond., 1920; "Aeroplane Structures," Chapter 10, by Pippard and Pritchard, Longmans Green & Co., Lond., 1919; and "Theory of Airplane Structural Members Subjected to Combined Axial and Transverse Loads," by J. E. Younger, Univ. of California Publications in Engineering, No. 8, Vol. 2, Berkeley, Calif., 1926.

Part I also presents the "Precise Method" now used by the U. S. Army and Navy Air Services, which is based on the same general theory as the methods given in earlier texts, but which, with the use of Table 9, has been conceded, by many aeronautical engineers who are familiar with other methods, to be less laborious. It suffices for the computation of the stresses encountered in the types of beams and the loadings that ordinarily occur in airplane design.

Part I.—Stresses in Wing-Beams

Action of a Beam Under Combined Axial and Transverse Load.—Due to the side load alone, the beam, Fig. 1, would deflect a distance, y', at a distance, x, from the left support, and the bending moment at that point would be,

$$M = \frac{w\,x^2}{2} - \frac{w\,L\,x}{2}$$

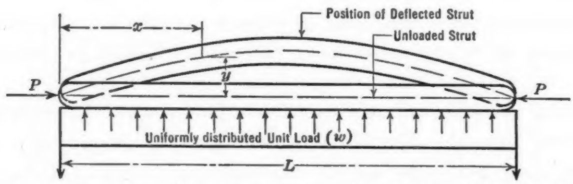

Fig. 1.—Beam with One Axial and a Uniform Transverse Load.

If a compressive load, P, be applied as shown, the moment at x would be increased by $-Py'$, since the load, P, would act at a distance, y', from the axis of the deflected member. This increase in moment would cause a greater deflection at x which, in turn, would result in a further increase in the moment. If the load, P, were not too great, these increments to the moment and deflection would become smaller and smaller until eventually, the beam would reach a state of equilibrium. If, however, the load, P, were sufficiently large, the increments of deflection would be successively greater and greater until the beam failed. It is apparent, then, that it should be made possible to represent increments of deflection or bending moment by a mathematical series of some kind which, if the axial compression were not too great, would converge, so that the limit of the series could be taken to represent conditions when the member reached a state of equilibrium.

If P were tension instead of compression, the deflection, y', due to the side load alone would be reduced instead of increased; and, as P was increased, the beam would tend to straighten and the moment at any point would be reduced. The failure, when it occurred, would be a tension failure, and there would be no tendency toward elastic instability or buckling as would be the case with a compressive axial load.

It is apparent, then, that an axial compressive load, which increases the bending moment at every point, is of far greater importance in the design of members under combined loads than an axial tension, which tends to decrease the secondary bending. For this reason the formulas developed

herein have been confined almost exclusively to the case of a compressive load, although some attention has been given to axial tension.

If the beam is continuous over two or more supports, the bending moments will be increased, throughout, by the application of an axial compressive load. The ordinary three-moment equation, which would be used on a continuous beam in conjunction with any of the approximate methods, makes no provision for this change in moment over the supports and so vitiates the effect of any series or other device used in such formulas to provide for the axial load. The methods developed in this paper, however, provide for the axial load both in the three-moment equation and in the formulas for the moments in the spans by the use of mathematical series. It so happens that the series used with axial compression are identical with those of the trigonometric functions, sines, cosines, and tangents. The use of special tables of sines, cosines, and tangents of ratios,* or radians, will greatly facilitate the application of these formulas. For axial tension loads the series involved are those of hyperbolic sines and cosines, which may be found in almost any set of mathematical tables in a form that is readily adaptable to use in these equations.

Basic Assumptions.—The basic assumptions from which the formulas are developed are as follows:

(1) Plane cross-sections before bending remain plane and normal to the longitudinal fibers after bending.
(2) The intensity of stress is proportional to the strain throughout the member and the ratios of stress to strain, the moduli of elasticity, are the same in tension and compression.
(3) Every longitudinal fiber is free to extend or contract under stress as if separate from the other fibers.
(4) The member is straight and homogeneous and of uniform cross-section between supports.
(5) The axial load is applied in such a way as to produce no bending in the member due to eccentricities, that is, it is applied so as to pass through the centroid of each cross-section of the undeflected member.

Nomenclature.—With the exception of one or two abbreviated forms, the nomenclature used is standard in literature on mechanics and is self-explanatory with the aid of the diagrams.

Conventions for Signs.—Forces are considered positive when acting upward; shear, when the algebraic sum of all forces to the left of the section is positive; bending moments, when they tend to cause compression in the upper fibers of a beam; slope, when the line rises from left to right; and deflection, when the deflected position of the point is above the original. These conventions conform to the direction of the load and deflection of an airplane wing structure and great care must be exercised with the signs when modifying the formulas for other conventions.

Derivation of the Formulas.—The derivation of the formulas for the most common type of loading, a uniformly distributed lateral load in conjunction

* "Fünfstellige Tafeln der Kreis- und Hyperbolfunktionen, sowie der Funktionen e^x und e^{-x} mit den natürlichen Zahlen als Argument," Keiichi Hayashi, Berlin und Leipzig, 1921, W. de Gruyter & Co.

with an axial compression, are given in detail, while the equations for other types are presented in a manner to show the differences so that they may be used as an aid in the development of expressions for any type of loading. For brevity, computations involving only simple algebra or arithmetic are omitted.

Case I.—Strut Under Axial Compression and Uniformly Distributed Transverse Load

Fig. 2 shows a member supported at two points and subjected to a uniformly distributed transverse load, an axial compression, and restraining moments applied at the points of support.

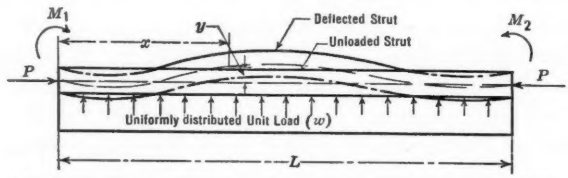

FIG. 2.—EFFECT OF RESTRAINING MOMENTS AT SUPPORTS.

The expression for the moment at any point is,

$$M = M_1 + \left(\frac{M_2 - M_1}{L}\right) x - \frac{w\,L\,x}{2} + \frac{w\,x^2}{2} - P\,y \ldots\ldots\ldots (1)$$

By making the usual assumptions of the beam theory,

$$M = E\,I \frac{d^2 y}{d\,x^2}$$

Whence,

$$E\,I \frac{d^2 y}{d\,x^2} + P\,y = M_1 + \left(\frac{M_2 - M_1}{L}\right) x - \frac{w\,L\,x}{2} + \frac{w\,x^2}{2}$$

If differentiated twice with respect to x, this becomes,

$$\frac{d^2 M}{d\,x^2} + \frac{P}{E\,I} M = w$$

or, writing $\frac{1}{j^2}$ for $\frac{P}{E\,I}$, j being $\sqrt{\frac{E\,I}{P}}$,

$$\frac{d^2 M}{d\,x^2} + \frac{1}{j^2} M = w$$

The solution for this differential equation is,

$$M = C_1 \sin \frac{x}{j} + C_2 \cos \frac{x}{j} + w\,j^2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

C_1 and C_2 being the constants of integration.

When $x = 0$, $M = M_1$, and when $x = L$, $M = M_2$, hence,

$$C_1 = \frac{M_2 - w\,j^2}{\sin \frac{L}{j}} - \frac{M_1 - w\,j^2}{\tan \frac{L}{j}} = \frac{M_2 - w\,j^2 - (M_1 - w\,j^2) \cos \frac{L}{j}}{\sin \frac{L}{j}}$$

and,

$$C_2 = M_1 - w\,j^2$$

For brevity:

$$D_1 = M_1 - w\,j^2$$

and,

$$D_2 = M_2 - w\,j^2$$

The moment at any point on the span is,

$$M = \left(\frac{D_2 - D_1 \cos \frac{L}{j}}{\sin \frac{L}{j}}\right) \sin \frac{x}{j} + D_1 \cos \frac{x}{j} + w\,j^2 \qquad (3)$$

To find the location of the section of maximum moment, differentiate Equation (2), equate the first derivative to zero, and solve; whence,

$$\frac{d\,M}{d\,x} = 0 = \frac{C_1}{j} \cos \frac{x}{j} - \frac{C_2}{j} \sin \frac{x}{j}$$

and,

$$\tan \frac{x}{j} = \frac{C_1}{C_2} = \frac{D_2 - D_1 \cos \frac{L}{j}}{D_1 \sin \frac{L}{j}} \qquad (4)$$

From the value of $\frac{x}{j}$, determined from Equation (4) by the use of Tables 8 and 9, in the Appendix, the distance x, to the section of maximum moment, is readily obtained. The value of x, obtained in this way, must lie between zero and L. Otherwise, either M_1 or M_2 is the maximum on the member.

The maximum moment may be found by substituting the value from Equation (4) in Equation (3) and simplifying; whence,

$$M_{\text{max.}} = \frac{D_1}{\cos \frac{x}{j}} + w\,j^2 \qquad (5)$$

It will be noted that,

$$\frac{D_1}{\cos \frac{x}{j}} = D_1 \sec \frac{x}{j}$$

which is equal to

$$D_1 \sqrt{\tan^2 \frac{x}{j} + 1}$$

Hence, if the exact location of the section of maximum moment is not desired, the value of $M_{\text{max.}}$ may be found without determining $\frac{x}{j}$. Care must be taken, however, with the sign used for the secant.

The deflection at any point is found by substituting the value of M from Equation (2) in Equation (1) and solving for y,

$$y = \frac{1}{P}\left[M_1 + \frac{M_2 - M_1}{L}x - \frac{w\,L\,x}{2} + \frac{w\,x^2}{2} - \frac{D_2 - D_1\cos\frac{L}{j}}{\sin\frac{L}{j}}\sin\frac{x}{j} - D_1\cos\frac{x}{j} - w\,j^2\right] \quad \text{(6)}$$

The first derivative of Equation (6) gives the slope of the tangent to the elastic curve at any point:

$$i = \frac{1}{P}\left[\frac{M_2 - M_1}{L} - \frac{w\,L}{2} + w\,x - \frac{C_1}{j}\cos\frac{x}{j} + \frac{C_2}{j}\sin\frac{x}{j}\right] \quad \text{(7)}$$

Equation of Three Moments.—In two contiguous spans, shown in Fig. 3, the slope of the tangent to the elastic curve at the center support will be the same for both spans, the member being continuous over this support.

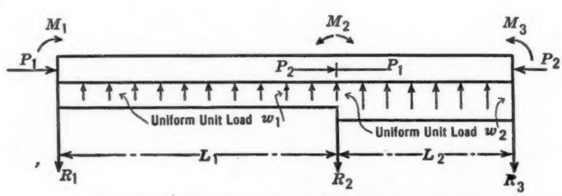

FIG. 3.—TWO SPANS ON THREE SUPPORTS.

At R_2, $x_1 = L_1$ for the left-hand, and $x_2 = 0$ for the right-hand, span. Using subscripts to differentiate between the symbols for the respective spans and substituting these values in the expressions for slope at R_2,

$$i_1 = \frac{M_2 - M_1}{L_1\,P_1} - \frac{w_1\,L_1}{2\,P_1} + \frac{w_1\,L_1}{P_1} - \frac{C_1\cos\frac{L_1}{j_1}}{j_1\,P_1} + \frac{C_1\sin\frac{L_1}{j_1}}{j_1\,P_1} \quad \text{(8)}$$

in which,

$$C_1 = \frac{M_2 - w_1\,j_1^{\,2} - (M_1 - w_1\,j_1^{\,2})\cos\frac{L_1}{j_1}}{\sin\frac{L_1}{j_1}}$$

and,

$$C_2 = M_1 - w_1\,j_1^{\,2}$$

$$i_2 = \frac{M_3 - M_2}{L_2\,P_2} - \frac{w_2\,L_2}{2\,P_2} - \frac{C'_1}{j_2\,P_2} \quad \text{(9)}$$

in which,

$$C'_1 = \frac{M_3 - w_2\,j_2^{\,2} - (M_2 - w_2\,j_2^{\,2})\cos\frac{L_2}{j_2}}{\sin\frac{L_2}{j_2}}$$

However $i_1 = i_2$ at the center support. Substituting the values for C_1, C_1' and C_2, in Equations (8) and (9), combining the terms, and simplifying, the following result is obtained:

$$\frac{M_1 L_1}{I_1}\left[\frac{\frac{L_1}{j_1}\operatorname{cosec}\frac{L_1}{j_1}-1}{\left(\frac{L_1}{j_1}\right)^2}\right]+\frac{M_3 L_2}{I_2}\left[\frac{\frac{L_2}{j_2}\operatorname{cosec}\frac{L_2}{j_2}-1}{\left(\frac{L_2}{j_2}\right)^2}\right]$$

$$+\,M_2\left[\frac{L_1}{I_1}\left(\frac{1-\frac{L_1}{j_1}\cot\frac{L_1}{j_1}}{\left(\frac{L_1}{j_1}\right)^2}\right)\right]+M_2\left[\frac{L_2}{I_2}\left(\frac{1-\frac{L_2}{j_2}\cot\frac{L_2}{j_2}}{\left(\frac{L_2}{j_2}\right)^2}\right)\right]$$

$$=\frac{w_1 L_1^3}{I_1}\left[\frac{\tan\frac{L_1}{2\,j_1}-\frac{L_1}{2\,j_1}}{\left(\frac{L_1}{j_1}\right)^3}\right]+\frac{w_2 L_2^3}{I_2}\left[\frac{\tan\frac{L_2}{2\,j_2}-\frac{L_2}{2\,j_2}}{\left(\frac{L_2}{j_2}\right)^3}\right]\ \ldots\ldots\text{(10)}$$

Multiplying Equation (10) by 6 it becomes,

$$\frac{M_1 L_1 \alpha_1}{I_1}+2\,M_2\left\{\frac{L_1}{I_1}\beta_1+\frac{L_2}{I_2}\beta_2\right\}+\frac{M_3 L_2 \alpha_2}{I_2}$$

$$=\frac{w_1 L_1^3}{4\,I_1}\gamma_1+\frac{w_2 L_2^3}{4\,I_2}\gamma_2\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(11)}$$

in which,

$$\alpha=6\left[\frac{\frac{L}{j}\operatorname{cosec}\frac{L}{j}-1}{\left(\frac{L}{j}\right)^2}\right]$$

$$\beta=3\left[\frac{1-\frac{L}{j}\cot\frac{L}{j}}{\left(\frac{L}{j}\right)^2}\right]$$

$$\gamma=3\left[\frac{\tan\frac{L}{2\,j}-\frac{L}{2\,j}}{\left(\frac{L}{2\,j}\right)^3}\right]$$

The sines, cosines, and tangents of $\frac{L}{j}$, and also of α, β, and γ, in terms of $\frac{L}{j}$, will be found in the special tables previously recommended.*

* "Fünfstellige Tafeln der Kreis- und Hyperbolfunktionen, sowie der Funktionen e^x und e^{-x} mit den natürlichen Zahlen als Argument," Keiichi Hayashi, Berlin und Leipzig, 1921, W. de Gruyter & Co.

It often happens that moments are introduced at the points of support of continuous beams because of fittings that are not concentric. Equation (11) may be altered to provide for this condition as follows:

$$\frac{M_1\,L_1\,\alpha_1}{I_1} + 2\,M_{-2}\left(\frac{L_1}{I_1}\,\beta_1\right) + 2\,M_{+2}\left(\frac{L_2}{I_1}\,\beta_2\right) + \frac{M_3\,L_2\,\alpha_2}{I_2}$$

$$= \frac{w_1\,L_1^{\,3}\,\gamma_1}{4\,I_1} + \frac{w_2\,L_2^{\,3}\,\gamma_2}{4\,I_2} \quad \text{.....................(12)}$$

In Equation (12), M_{-2} and M_{+2} are the moments an infinitesimal distance to the left and right of the point of support, respectively. Equation (12) contains an additional unknown which necessitates another equation for a solution. This is obtained from the relation between M_{-2} and M_{+2}, M_{+2} being equal to M_{-2} plus or minus the eccentric moment, M_e. Care must be taken with the sign of M_e. It should be considered positive if it increases the moment from M_{-2} to M_{+2} as one goes from left to right over the point of support.

When the wing truss of an airplane deflects, the panel points do not necessarily lie on a straight line, although the divergence is generally not great. Equation (11) may be modified to provide for differences in the elevation of supports by adding the following terms to the right-hand side of the equation, $\frac{6\,E\,(y_1 - y_2)}{L_1}$ and $\frac{6\,E\,(y_3 - y_2)}{L_2}$, y_1, y_2, and y_3 being the deflections of the respective panel points.

If a strut is, subjected to an axial compression and restraining moments at each end, but no lateral load, the value of w in Equations (4) and (5) is 0 and:

$$\tan\frac{x}{j} = \frac{M_2 - M_1\cos\frac{L}{j}}{M_1\sin\frac{L}{l}} \quad \text{...................(13)}$$

$$M_{\text{max.}} = \frac{M_1}{\cos\frac{x}{j}} \quad \text{........................(14)}$$

For the condition of a strut with a uniform side load, but no restraining moments, the section of maximum moment occurs at mid-span, and it has a value of,

$$M_{\text{max.}} = w\,j^2\left(1 - \sec\frac{L}{j}\right) \quad \text{..................(15)}$$

If the algebraic series is substituted for the secant term, Equation (15) takes the form,

$$M = \frac{M_0}{1 - \frac{5\,P\,L^2}{48\,E\,I}}$$

which corresponds to Johnson's approximate formula that provides for axial

loads, if $\frac{1}{10}$ is substituted for $\frac{5}{48}$, and with Perry's formula if $\frac{1}{\pi^2}$ is substituted for $\frac{5}{48}$.

Case II.—Axially Loaded Strut with Concentrated Transverse Load

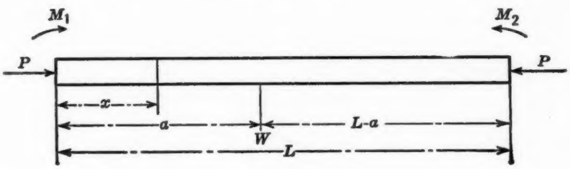

FIG. 4.

Fig. 4 shows a span subjected to a concentrated lateral load, an axial compression, and moments applied at each point of support. As it is impossible to write a single equation for the moment at any point in the span for this type of loading, one equation will be written for the segment to the left of the load and another for that to the right.

The expression for the moment at any point between the left support and the load is,

$$M_L = M_1 + \left(\frac{M_2 - M_1}{L}\right) x - \frac{W (L - a) x}{L} - P y \ldots\ldots\ldots (16)$$

Between the load and the right support,

$$M_R = M_1 + \left(\frac{M_2 - M_1}{L}\right) x - \frac{W (L - a) x}{L} + W (x - a) - P y \ldots (17)$$

The second derivative of Equations (16) and (17) is,

$$\frac{d^2 M}{d x^2} = - P \frac{d^2 y}{d x^2}$$

or,

$$\frac{d^2 M}{d x^2} - \frac{P}{E I} M = 0$$

The solution of this equation when applied to the segment to the left of the load is,

$$M_L = C_1 \sin \frac{x}{j} + C_2 \cos \frac{x}{j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

and for the segment to the right,

$$M_R = C_3 \sin \frac{x}{j} + C_4 \cos \frac{x}{j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

in which, C_1, C_2, C_3, and C_4 are the constants of integration.

From Equations (16), (17), (18), and (19) the deflection is found to be,

$$P y_L = M_1 + \left(\frac{M_2 - M_1}{L}\right) x - \frac{W (L - a) x}{L} - C_1 \sin \frac{x}{j} - C_2 \cos \frac{x}{j} \ldots (20)$$

and,

$$P\, y_R = M_1 + \frac{M_2 - M_1}{L}\, x - \frac{W\,(L - a)\, x}{L} + W\,(x - a)$$

$$- C_3 \sin \frac{x}{j} - C_4 \cos \frac{x}{j} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

These equations, when differentiated, give the slope, i, which is,

$$P\, i_L = \frac{M_2 - M_1}{L} - \frac{W\,(L - a)}{L} - \frac{C_1}{j} \cos \frac{x}{j} + \frac{C_2}{j} \sin \frac{x}{j} \ldots\ldots (22)$$

$$P\, i_R = \frac{M_2 - M_1}{L} - \frac{W\,(L - a)}{L} + W - \frac{C_3}{j} \cos \frac{x}{j} + \frac{C_4}{j} \sin \frac{x}{j} \ldots. (23)$$

From the conditions of the structure, when $x = 0$, $M = M_1$; when $x = a$, $y_L = y_R$ and $i_L = i_R$; and when $x = L$, $M = M_2$. These conditions are sufficient to determine the four constants, which are,

$$C_1 = \frac{M_2 - M_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} + W j \sin \frac{a}{j} \left(\cot \frac{L}{j} - \cot \frac{a}{j} \right)$$

$$C_2 = M_1$$

$$C_3 = \frac{M_2 - M_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} + W j \sin \frac{a}{j} \cot \frac{L}{j}$$

and,

$$C_4 = M_1 - W j \sin \frac{a}{j}$$

Differentiating Equation (18) to find the sections of maximum moment,

$$\frac{d\, M_L}{d\, x} = \frac{C_1}{j} \cos \frac{x}{j} - \frac{C_2}{j} \sin \frac{x}{j} = 0$$

and,

$$\tan \frac{x}{j} = \frac{C_1}{C_2}$$

Dividing Equation (18) by $\cos \frac{x}{j}$,

$$\frac{M_L}{\cos \frac{x}{j}} = C_1 \tan \frac{x}{j} + C_2$$

Therefore,

$$\frac{M_{\text{max.}}}{\cos \frac{x}{j}} = C_1 \left(\frac{C_1}{C_2} \right) + C_2$$

or,

$$M_{L\,(\max.)} = \frac{C_1^2 + C_2^2}{C_2} \cos \frac{x}{j} \qquad (24)$$

and,

$$M_{R\,(\max.)} = \frac{C_3^2 + C_4^2}{C_4} \cos \frac{x}{j} \qquad (25)$$

It will be noted that, for a single concentrated load, the section of maximum moment may come either to the left or right of the load, depending on its position and the magnitude of the moments at the supports. It may be necessary, therefore, to compute the values of $\tan \frac{x}{j}$ for both segments in order to ascertain in which the section of maximum moment is located. If $x < a$ for the left-hand segment, use the formula for that segment to obtain the maximum moment. However, if $x > a$ when computed from the values of C_1 and C_2, the section of $M_{\text{max.}}$ lies to the right of the load and its location and magnitude should be computed from C_3 and C_4.

It is conceivable that the shape of the moment curves to the left and right of the load will be such that the point of zero slope for each curve will lie on the opposite side of the load from the curve itself. As the equation for the location of the section of maximum moment is, in reality, simply a means for determining the point where the slope of the moment curve is zero, it will be found that for such a condition the value of x, obtained from C_1 and C_2, will be greater than that of a, and that for C_3 and C_4, it will be less. This indicates that the maximum moment will be at the section where the load is applied and may be determined by substituting a for x; or that the concentrated load does not produce a maximum in the span, in which case either M_1 or M_2 is the maximum.

Case II*a*.—Axially Loaded Strut with a Series of Concentrated Transverse Loads

From Equation (18) and the values for C_1 and C_2,

$$M_L = \frac{M_2 - M_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} \sin \frac{x}{j} + W j \sin \frac{a}{j} \left(\cot \frac{L}{j} - \cot \frac{a}{j} \right) + M_1 \cos \frac{x}{j}$$

For several loads to the right of Section x, the first and last terms of this expression remain the same, but the second term takes the form,

$$j \sin \frac{x}{j} \sum \left[W \sin \frac{a}{j} \left(\cot \frac{L}{j} - \cot \frac{a}{j} \right) \right]$$

which may be simplified to,

$$\frac{-j \sin \frac{x}{j}}{\sin \frac{L}{j}} \sum \left[W \sin \frac{L - a}{j} \right]$$

so that M_L becomes,

$$M_L = \frac{M_2 - M_1 \cos \dfrac{L}{j}}{\sin \dfrac{L}{j}} \sin \frac{x}{j} + M_1 \cos \frac{x}{j}$$

$$- \frac{j \sin \dfrac{x}{j}}{\sin \dfrac{L}{j}} \sum \left(W \sin \frac{L - a}{j} \right) \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

By a similar process, M_R may be written,

$$M_R = \frac{M_2 - M_1 \cos \dfrac{L}{j}}{\sin \dfrac{L}{j}} \sin \frac{x}{j} + M_1 \cos \frac{x}{j}$$

$$- \frac{j \sin \dfrac{L - x}{j}}{\sin \dfrac{L}{j}} \sum \left(W \sin \frac{a}{j} \right) \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The moment at any point on the span due to the series of concentrated loads, therefore, may be found from,

$$M = \frac{M_2 - M_1 \cos \dfrac{L}{j}}{\sin \dfrac{L}{j}} \sin \frac{x}{j} + M_1 \cos \frac{x}{j} - \frac{j \sin \dfrac{x}{j}}{\sin \dfrac{L}{j}} \sum \left(W \sin \frac{L - a}{j} \right)$$

$$- \frac{j \sin \dfrac{L - x}{j}}{\sin \dfrac{L}{j}} \sum \left(W \sin \frac{a}{j} \right) \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which, the $\sum \left(W \sin \dfrac{L - a}{j} \right)$-terms apply to loads having $a > x$ and the $\sum \left(W \sin \dfrac{a}{j} \right)$-terms, to the loads where $a < x$.

If the same procedure be followed for a continuous beam having concentrated side loads as was used in the case for the uniformly distributed side load, the following equation of three moments results,

$$\frac{M_1 \, L_1 \, \alpha_1}{I_1} + 2 \, M_2 \left\{ \frac{L_1 \, \beta_1}{I_1} + \frac{L_2 \, \beta_2}{I_2} \right\} + \frac{M_3 \, L_2 \, \alpha_2}{I_2}$$

$$= \sum \frac{6 \, W_1 \, j_1^2}{I_1} \left[\frac{\sin \dfrac{a_1}{j_1}}{\sin \dfrac{L_1}{j_1}} - \frac{a_1}{L_1} \right]$$

$$+ \sum \frac{6 \, W_2 \, j_2^2}{I_2} \left[\frac{\sin \dfrac{L_2 - a_2}{j_2}}{\sin \dfrac{L_2}{j_2}} - \frac{L_2 - a_2}{L_2} \right] \ldots\ldots\ldots\ldots (29)$$

In this Equation (29) α and β have the same values as in Case I and may be found in Table 8 in the Appendix. It should be noted that the left-hand side of Equation (29) is identical to that of Equation (11), such differences as there are being on the right-hand side in connection with the terms for the load. Equation (29), therefore, may be treated in exactly the same way as Equation (12) to allow for the effect of an eccentric moment at one of the supports or for the deflection of the supports.

If a combined uniform and concentrated loading were applied to a continuous beam the three-moment equation could be arranged to provide for this condition by including, on the right-hand side, terms sufficient to account for each load, leaving the left-hand side of the equation unaltered.

Formulas similar to those obtained in Case I may be obtained for pin-ended struts, or for struts in which one end is pinned and one end restrained, by substituting, $M_1 = M_2 = 0$, or $M_1 = 0$ in the equations.

For a pin-ended column, with a concentrated lateral load in the middle of the span, the moment at any point is,

$$M = C_1 \sin \frac{x}{j} \qquad (30)$$

in which,

$$C_1 = \frac{-W\,j}{2 \cos \frac{L}{2\,j}}.$$

The maximum moment is at mid-span and is,

$$M_{\text{max.}} = -\frac{W\,j}{2} \tan \frac{L}{2\,j} \qquad (31)$$

The deflection at any point in the span is,

$$y = \frac{1}{P}\left[\frac{W\,j \sin \frac{x}{j}}{2 \cos \frac{L}{2\,j}} - \frac{W\,x}{2}\right] \qquad (32)$$

which becomes, at mid-span,

$$y = \frac{w\,j}{2\,P}\left[\tan \frac{L}{2\,j} - \frac{L}{2\,j}\right] \qquad (33)$$

Case III.—Axially Loaded Strut with Uniformly Varying Lateral Load

Fig. 5 shows a strut subjected to an axial compression, a lateral load varying uniformly from zero at one end to W at the other, and restraining moments at each point of support. The derivation of the equations for this type of loading is practically identical in every step with that for a uniform load.

The moment at any point in the span is,

$$M = M_1 + \frac{M_2 - M_1}{L}\,x - \frac{W\,L\,x}{6} + \frac{W\,x^3}{6\,L} - P\,y \qquad (34)$$

and the second derivative becomes,

$$\frac{d^2 M}{d x^2} + \frac{P}{E I} M = \frac{W x}{L}$$

for which the solution is,

$$M = C_1 \sin \frac{x}{j} + C_2 \cos \frac{x}{j} + \frac{W}{L} x j^2 \quad \text{.............. (35)}$$

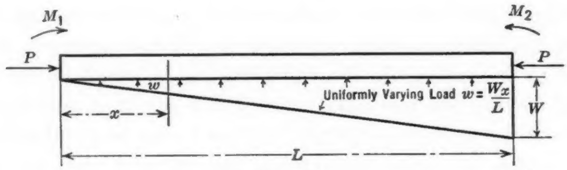

FIG. 5.

When $x = 0$, $M = M_1$, and when $x = L$, $M = M_2$, from which it will be found that,

$$C_1 = \frac{M_2 - W j^2 - M_1 \cos \frac{L}{j}}{\sin \frac{L}{j}}$$

and,

$$C_2 = M_1$$

Equation (35) does not offer a simple expression for finding the location of the section of maximum moment because its first derivative, when equated to zero, gives,

$$C_2 \sin \frac{x}{j} - C_1 \cos \frac{x}{j} = \frac{W j^3}{L}$$

which may be converted into,

$$\tan \frac{x}{j} = \frac{C_1}{C_2} - \frac{W j^3 (C_1^2 + C_2^2)}{C_1 C_2 W j^3 \mp C_2^2 \sqrt{L^2 (C_1^2 + C_2^2) - (W j^3)^2}} \quad \text{..... (36)}$$

It is to be noted that two values of x will be found from Equation (36), depending on the sign used for the radical term in the denominator. One value of x will probably not lie on the span and, therefore, may be neglected.

The magnitude of the maximum moment is found by substituting the value of x from Equation (36) in Equation (35) and solving.

The deflection at any point may be found from,

$$y = \frac{1}{P} \left(M_1 + \frac{M_2 - M_1}{L} x - \frac{W L x}{6} + \frac{W x^3}{6 L} - C_1 \sin \frac{x}{j} - C_2 \cos \frac{x}{j} - \frac{W x j^2}{L} \right) \quad \text{..................... (37)}$$

and the slope at any point from,

$$i = \frac{1}{P} \left(\frac{M_2 - M_1}{L} - \frac{W L}{6} + \frac{W x^2}{2 L} - \frac{C_1}{j} \cos \frac{x}{j} + \frac{C_2}{j} \sin \frac{x}{j} - \frac{W j^2}{L} \right) \quad \text{.. (38)}$$

The three-moment equation is obtained by the same procedure as for the uniformly distributed load,

$$\frac{M_1\,L_1\,\alpha_1}{I_1} + 2\,M_2\left(\frac{L_1}{I_1}\,\beta_1 + \frac{L_2}{I_2}\,\beta_2\right) + \frac{M_3\,L_2\,\alpha_2}{I_2}$$

$$= \frac{W_1\,L_1\,j_1^{\,2}}{I_1}\,[2\,(\beta_1 - 1)] + \frac{W_2\,L_2\,j_2^{\,2}}{I_2}\,(\alpha_2 - 1)\ldots\ldots\ldots\ldots(39)$$

If the load varies from W at the left support to zero at the right in each span, the left-hand side of Equation (39) remains the same, but the right-hand side is changed to,

$$\frac{W_1\,L_1\,j_1^{\,2}}{I_1}\,(\alpha_1 - 1) + \frac{W_2\,L_2\,j_2^{\,2}}{I_2}\,[2\,(\beta_2 - 1)]\ldots\ldots\ldots\ldots(40)$$

Case IV.—General Case

If in Equation (28), $w\,da$ is substituted for the concentrated load, which was assumed to act at a distance, a, from the left point of support, a general expression can be written for any point, x, and for any type of transverse load acting in conjunction with an axial compression.

Such an expression is,

$$M = \frac{M_2 - M_1 \cos\dfrac{L}{j}}{\sin\dfrac{L}{j}}\sin\frac{x}{j} + M_1\cos\frac{x}{j} - \frac{j\sin\dfrac{x}{j}}{\sin\dfrac{L}{j}}\int w\sin\frac{L-a}{j}\,d\,a$$

$$- \frac{j\sin\dfrac{L-x}{j}}{\sin\dfrac{L}{j}}\int w\sin\frac{a}{j}\,d\,a\ldots\ldots\ldots\ldots\ldots\ldots(41)$$

The $\int w\sin\dfrac{L-a}{j}\,d\,a$-term provides for a load to the right of the section being investigated; that is, between x and L; while the term, $\int w\sin\dfrac{a}{j}\,d\,a$ provides for the loads to the left, or between 0 and x.

In a like manner, if $\int w\,d\,a$ is substituted for W in Equation (29), a general form is obtained for the three-moment equation:

$$\frac{M_1\,L_1\,\alpha_1}{I_1} + \frac{2\,M_{-2}\,L_1\,\beta_1}{I_1} + \frac{2\,M_{+2}\,L_2\,\beta_2}{I_2} + \frac{M_3\,L_2\,\alpha_2}{I_2} = \frac{6\,E\,(y_1 - y_2)}{L_1}$$

$$+ \frac{6\,E\,(y_3 - y_2)}{L_1} + \frac{6\,j_1^{\,2}}{I_1}\int_0^{L_1}\left(\frac{\sin\dfrac{a_1}{j_1}}{\sin\dfrac{L_1}{j_1}} - \frac{a_1}{j_1}\right)w_1\,d\,a$$

$$+ \frac{6\,j_2^{\,2}}{I_2}\int_0^{L_2}\left(\frac{\sin\dfrac{L_2 - a_2}{j_2}}{\sin\dfrac{L_2}{j_2}} - \frac{L_2 - a_2}{L_2}\right)w_2\,d\,a\ldots\ldots\ldots\ldots(42)$$

The three-moment equations developed for Cases I, II, and III, may be obtained from Equation (42) by integrating between the proper limits and simplifying the resultant expressions.

Case V.—Beam Subjected to Uniformly Distributed Lateral Load and Axial Tension

All the foregoing formulas have been derived for cases where the axial load caused compression in the member. The same procedure may be followed when the axial load causes tension except that $-P$ should be substituted for P. The equation for moment at any point in the span is, then,

$$M = M_1 + \frac{M_2 - M_1}{L} x - \frac{w L x}{2} + \frac{w x^2}{2} + P y$$

and its second derivative becomes,

$$\frac{d^2 M}{d x^2} - \frac{P}{E I} M = w$$

The solution of this differential equation is,

$$M = C_1 \sinh \frac{x}{j} + C_2 \cosh \frac{x}{j} - w j^2$$

in which,

$$j = \sqrt{\frac{E I}{P}}$$

$$C_1 = \frac{M_2 + w j^2 - (M_1 + w j^2) \cosh \frac{L}{j}}{\sinh \frac{L}{j}}$$

and,

$$C_2 = M_1 + w j^2$$

It will be noted that the circular functions occurring in the former cases have been replaced by hyperbolic functions and that some of the signs have been changed. These changes are brought about by the change in sign of the second term in the differential equation and emphasize the fact that the utmost care must be taken with the signs when deriving formulas for other types of loading.

The location of the section of maximum moment is found from,

$$\tanh \frac{x}{j} = \frac{D_1 \cosh \frac{L}{j} - D_2}{D_1 \sinh \frac{L}{j}}$$

and,

$$M_{\text{max.}} = \frac{D_1}{\cosh \frac{x}{j}} - w j^2$$

in which,

$$D_1 = M_1 + w\,j^2$$

and,

$$D_2 = M_2 + w\,j^2$$

Expressions may be derived for the deflection and slope as they were in Case I and a three-moment equation may also be obtained which is identical with that for Case I, except that α_h, β_h, and γ_h have the following values:

$$\alpha_h = \frac{6\left(1 - \frac{L}{j}\,\mathrm{cosech}\,\frac{L}{j}\right)}{\left(\frac{L}{j}\right)^2}$$

$$\beta_h = \frac{3\left(\frac{L}{j}\coth\frac{L}{j} - 1\right)}{\left(\frac{L}{j}\right)^2}$$

and,

$$\gamma_h = \frac{3\left(\frac{L}{2\,j} - \tanh\frac{L}{2\,j}\right)}{\left(\frac{L}{2\,j}\right)^3}$$

Values of α_h, β_h, and γ_h are given in Table 9 in the Appendix. Values of the hyperbolic functions are not included since they may be found in almost any set of mathematical tables.

The case sometimes arises in which a beam is subjected to compression in one span and tension in the next, and the three-moment equation may be altered to provide for this condition by using the coefficients based on the circular functions in all terms relating to the span under compression and the coefficients based on the hyperbolic functions for those relating to the span in tension. The moments and deflections within the spans are then found by the use of the formulas for axial compression and tension, as the case may be. While this condition seldom occurs in practice, it may be found in the wing-beam of an airplane, due to the action of the drag truss stresses, which may oppose and overcome those from the lift truss in one bay, but not in the next.

Graphical Methods.—It will be noted that all the analytical expressions for combined axial and transverse loads developed thus far have been based on the assumption that the moment of inertia of the beam was constant throughout each bay. The assumption that I varies, leads to a differential equation that is so cumbersome as to appear incapable of solution, at least in a form satisfactory for practical use. As a matter of fact, the equation has not been solved, except for one or two special cases, one of which involves the use of Bessel's functions. For this reason precise formulas that are applicable, to tapered beams such as those used in many of the smaller airplanes, have not yet been derived. In some cases, satisfactory results may be obtained by using, in the formulas based on a constant, I, a mean value of

the moment of inertia in each bay. Such a method is inherently approximate and is not to be recommended for the design of small wing-spars in which the margin of safety is likely to be low.

A graphical method has been devised by Miss Barbara Gough.* It is applicable to continuous beams with three supports, the bending moments at two of which are known, for any type of transverse load and any variation in cross-section. The axial load is assumed to be compressive.

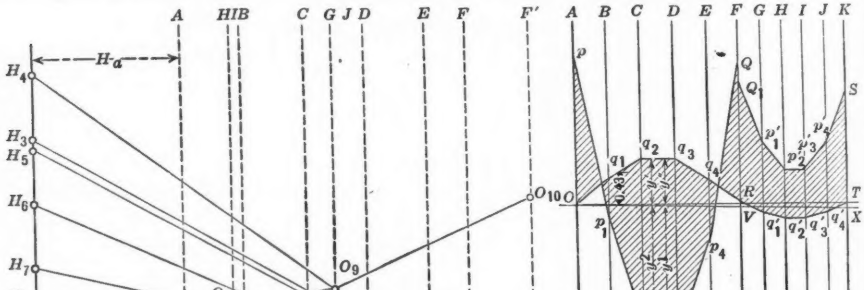

FIG. 6.—GRAPHICAL SOLUTION APPLIED TO A SMALL WING SPAR.

This method, while slightly approximate and subject to the errors of any graphical method, is simple in its application and gives quite accurate results. It has been found to be capable of extension to cover the case of a continuous beam with four supports if the structure and loading are symmetrical so that the moments at the two central supports are identical. As this is the condition existing in almost all single-bay, tapered-wing airplanes, the graphical method has a considerable field of application.

Fig. 6 shows a complete solution of the method as applied to the wing-spar of a small airplane.

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Procedure for Span 1-2 (Fig. 6 (c)).—First divide the span into n equal sections, five or six will generally suffice from the standpoint of accuracy. Then choose the ratio of reduction in horizontal scale, λ, necessary to keep the solution on the sheet of paper to be used. With these two values decided upon, compute the polar distances, H_A, H_B, H_C, H_D, H_E and H_F (Fig. 6(a)) from the formula,

$$H = \frac{E\,I\,n}{P\,\lambda\,L}$$

in which,

E = the modulus of elasticity;

I = the moment of inertia of the member at the section being investigated;

n = the number of sections (five in Fig. 6(c));

λ = the scale of reduction (40 in Fig. 6(c)); and,

L = the actual, full-sized length of the span.

The computations for these values will be found in Table 1.

Attention is called to the fact that the polar distances vary inversely with λ and that if the scale of reduction is small the polar distances will be so large that the intersection of the rays will not fall on the paper. These polar distances should be plotted from a common base line, drawn at right angles to the OX-axis of the beam, and the lines AA, BB, etc., drawn in (see Fig. 6(a)).

The values of y' for Span 1-2 (Fig. 6(c)), are next computed. For the type of loading used y' is found from,

$$y' = \frac{M_1}{P} + \frac{V\,x}{P} + \frac{w\,x^2}{2\,P} + \frac{K\,w\,x^3}{6\,L\,P}$$

in which, V represents the shear just to the right of Support 1. M_1 is the bending moment at Support 1 and must be a known quantity, such as that from a cantilever wing tip. It amounts to 13 800 in-lb. in this example. The last term, $\frac{K\,w\,x^3}{6\,L\,P}$, provides for the triangular segment in the loading curve representing the increment in load between two consecutive sections. V, however, depends on the moment at the second support, which is one of the things to be found and so is unknown. It is necessary, therefore, to assume a value for V, the most logical being that found by the use of the ordinary three-moment equation. In the illustrative example ("Procedure for Span 1-2"), the value used for V was obtained from an approximate analysis of the beam, but the exact value chosen has little influence, because the method of construction eliminates the effects of small differences in V. In computing the values of y', x is the actual, full-scale length of the member, not the length on the reduced scale diagram (Fig. 6).

The values of y' are then plotted as ordinates from the horizontal axis OX, giving the points, p, p_1, p_2, etc. The next step is to assume a slope for, and draw in, the line, Oq_1, and the string, H_0O_0, parallel to it. The location of the point, H_0, is chosen arbitrarily to bring the completed polar diagram on

TABLE 1.—COMPUTATION OF POLAR DISTANCES, H, AND VALUES OF THE DEFLECTIONS, y'.

Section.	SPAN 1-2: $P_1 = 6\,750$; $n = 5$; $\lambda = 40$; $L = 99.5$						SPAN 2-3: $P_2 = 8\,750$; $n = 5$; $\lambda = 40$; $L = 67.0$					
	A.	*B.*	*C.*	*D.*	*E.*	*F.*	*F'.*	*G.*	*H.*	*I.*	*J.*	*K.*
I	7.8	10.9	14.1	17.2	20.4	22.5	22.5	13.5	9.0	9.0	13.5	22.5
$\frac{E}{P}$	237	237	237	237	237	237	183	183	183	183	183	183
$\frac{EI}{P}$	1 837	2 586	3 334	4 064	4 832	5 338	4 118	2 471	1 647	1 647	2 471	4 118
$\frac{n}{L}$	0.00125	0.00125	0.00125	0.00125	0.00125	0.00125	0.00187	0.00187	0.00187	0.00187	0.00187	0.00187
H	2.31	3.25	4.19	5.13	6.07	6.70	7.66	4.61	3.07	3.07	4.61	7.66
$\frac{M_1}{P_1}$	+2.04	+ 2.04	+ 2.04	+ 2.04	+ 2.04	+ 2.04						
$\frac{V}{P}$	−0.159	− 0.159	− 0.159	− 0.159	− 0.159	− 0.159						
x	0	19.9	39.8	59.7	79.6	99.5						
$\frac{Vx}{P}$	0	− 3.16	− 6.32	− 9.49	−12.66	−15.82						
$\frac{W}{2P}$	0.00143	0.00143	0.00143	0.00143	0.00143	0.00143						
x^2	0	396	1 584	3 564	6 336	9 900						
$\frac{Wx^2}{2P}$	0	+ 0.57	+ 2.27	+ 5.10	+ 9.06	+14.16						
$\frac{Kw}{6LP}$	0	0.00000194	0.00000194	0.00000194	0.00000194	0.00000194						
x^3	0	7 880	63 045	212 776	504 358	985 075						
$\frac{Kwx^3}{6LP}$	0	+ 0.02	+ 0.12	+ 0.41	+ 0.98	+ 1.91						
y'	+2.04	− 0.53	− 1.89	− 1.94	− 0.58	+ 2.29	0	0.86	1.25	1.25	0.86	0

NOTE.—Values of y' for Span 2-3 (Fig. 6 (c)) are computed for $y' = \frac{M}{P_2}$, in which, M is the moment due to the transverse load only, assuming the span to be a simply supported beam. The assumed value of $V = -1\,075$ lb. E for spruce is taken as 1 600 000 lb. per sq. in. (10% moisture). In Span 2-3: $M_G = M_J = 7\,538$; $y'_G = y'_J = 0.86$; $M_H = M_I = 10\,940$; and $y'_H = y'_I = 1.25$. $RQ_1 = \frac{P_1}{P_2}$. $RQ = \frac{6\,750}{8\,750} \times 2.25 = 1.74$.

the paper. Ray H_0 O_0 is continued until it intersects the axis, *BB,* at O_1. The distance, p_1 q_1, is now scaled from the string polygon and laid off from H_0 along the base line to locate H_1. H_1 O_1 is then drawn in and continued to O_2 on the axis, *CC*. q_1 q_2 is next drawn parallel to this line and the distance, p_2 q_2, scaled and laid off from H_1 to locate H_2. This procedure is continued until the point, *R* (Fig. 6(*c*)), is reached on the line through the intermediate support.

R Q (Fig. 6(*c*)), is a measure of the bending moment at this point and, as continuity is required in the two spans, it is necessary to multiply the ordinate by a factor such that the value obtained from the product of *R Q* and the axial load in Span 1-2 will be the same as the product, $R\ Q_1$, times the axial load in Span 2-3. This factor is obviously $\frac{P_1}{P_2}$, so that, $R\ Q_1 = \frac{P_1}{P_2} R\ Q$.

It may be shown that the ordinates, y'', to the curve, Oq_1 q_2 q_3 q_4 *R* (Fig. 6(*c*)), measured from the axis *OR*, are equal to the deflections in Span 1-2. Hence, if all three supports lie on the same straight line, all the deflections at *O, R,* and *T,* must be zero and a straight line through *O* establishes the point, *T*.

The problem now resolves itself into constructing the curve, $R\, q_1'$ q_2' q_3' q_4' *T,* so that it will satisfy the conditions in Span 2-3 and close on the point, *T,* with zero deflection.

Procedure for Span 2-3.—Divide Span 2-3 (Fig. 6(*c*)), into *n* equal sections, not necessarily the same number as was used in Span 1-2, and compute the polar distances H_F, H_G, H_H, H_I, H_J, and H_K in the same way as in Span 1-2. Draw in the lines, *F' F'*, *G G*, etc., through the poles (Fig. 6 (*a*)). The values of *n*, *I*, *P*, and *L* for Span 2–3 should, of course, be used.

If the bending moment at the third support is zero, as in the case of a pin-joint, the ordinate, *ST,* must be zero, because this ordinate, when multiplied by P_2, gives the value of the bending moment. For such a condition the line, Q_1 *S* (Fig. 6(*c*)), would become Q_1 *T,* and the curve, Q_1 p_1' p_2' p_3' p_4' *S,* would close on *T*. However, in the illustrative example the beam and loading are symmetrical about the mid-point of Span 2-3, so the $M_2 = M_3$, hence, the ordinate, *T S,* = $R\ Q_1$. The curve, Q_1 p_1' p_2' p_3' p_4' *S* (Fig. 6(*c*)), is constructed by plotting the ordinates, y', from the line Q_1 *S.* The values of y' are obtained from $y' = \frac{M}{P_2}$, in which, *M* is the moment at the section due to the transverse load only, assuming the moments at Supports 2 and 3 to be zero.

The curve, Q_1 q_1' q_2' q_3' q_4' *T* (Fig. 6(*c*)), is then constructed. It is to be noted (Fig. 6(*a*)), that there are two computed polar distances for *F,* one based on the axial load in Span 1-2, the other on that in Span 2-3. The slope of the line, Rq_1', is obtained by laying off one-half the ordinate, *RQ,* from H_4 to obtain H_5. Line H_5 O_5 is drawn and continued to O_6 on the axis, *F' F'* (Fig. 6(*a*)). One-half of RQ_1 is then laid off from H_5 to locate H_6 and the line H_6 O_6, is drawn in. Rq_1' is next drawn parallel to H_6 O_6. H_7 is

determined as in the case of Span 1-2 by making $H_6\,H_7$ equal to the ordinate, $p_1\,q_1'$, and the slope of $q_1'\,q_2'$ is the same as the ray, $H_7\,O_7$, O_7 being located at the point of intersection of the ray, $H_6\,O_6$, with the axis, GG. The construction is continued in this way and $q_4'\,T$ should close on T. It is obvious that this will not occur unless the correct slope has been assumed for the line, Oq_1. By measuring the error of closure and plotting it against the assumed slope, as is done in Fig. 6(*b*), the trend of the error of closure may be found by drawing a curve through the plotted points. This curve will indicate the slope necessary for zero error of closure so that the third trial should come very near to closing and the fourth close exactly if the drafting is done carefully in each case.

In some cases the $H\,H$-distances are laid off above the last point, in others, below it. They are to be drawn in the same direction as the pq intercept at the section; that is, above when q is above p, and below when q is below p.

If the loading is not uniformly varying in the span, the equation for Py' must obviously be altered to give the moment at any point for the type of loading used. If the points of support do not lie on a straight line this can be provided for by having the $q_1\,q_2\,q_3$-curve intersect the lines at the supports at a distance above, or below, O, R, and T, equal to the known deflection of these points.

When the diagram has been completed and gives a satisfactory closure on T, the bending moments at any point may be found by scaling the vertical distance between the $p_1\,p_2\,p_3$ and $q_1\,q_2\,q_3$-curves (Fig. 6(*c*)) and multiplying by the axial load at the point. If the deflection at any point is wanted, the vertical intercept between the $q_1\,q_2\,q_3$-curves and the line, $O\,R\,T$, gives the actual value.

Fig. 6 gives all the data pertaining to the rear upper beam of a small airplane and, with Table 1, gives all the work required for the complete determination of the bending moments and deflections of this beam, except that the diagrams obtained from the first trials are not shown. The cross-hatched area represents the area between the curves used in deriving the bending moments at any point, the depth of this area, in inches, times the axial load, in pounds, giving the moment in inch-pounds.

This method gives the bending moments at all points on the span, which is a very desirable feature when designing a tapered beam, since the critical section may be at a point other than the section of maximum bending moment if the rate of change of bending moment and taper are not the same.

Critical Points Encountered in Using the Formulas.—Each of the precise formulas is difficult to solve for certain values of $\frac{L}{j}$. In some cases the formula will assume an indeterminate form, such as $\frac{\infty}{\infty}$ or $\frac{0}{0}$; in others, the functions involved will vary so rapidly that reliable results cannot be obtained with ordinary straight-line interpolation. For instance, in the expression for bending moment, given in Equation (3), the coefficient of $\sin\frac{x}{j}$ becomes

infinite when $\sin \frac{L}{j}$ is zero. As $\sin \frac{L}{j} = 0$ when $\frac{L}{j} = 0, \pi, 2\pi$, etc., it is apparent that these points are critical when applying the formulas. It is possible, however, to compute the moment for values of $\frac{L}{j}$ just above and just below the critical point, plot the results, and obtain the desired value for the moment from a curve drawn through the points.

Considering Equation (5), the maximum moment in a span becomes indeterminate when $D_1 = 0$, that is,

$$M_{\text{max.}} = \frac{0}{0} + w\,j^2$$

since $\tan \frac{x}{j}$ as found from $\dfrac{D_2 - D_1 \cos \frac{L}{j}}{D_1 \sin \frac{L}{j}}$ is infinite; whence $\frac{x}{j} = \frac{\pi}{2}$ and $\cos \frac{x}{j} = 0$. It is, however, possible to avoid this complication by observing that $\frac{x}{j} = \frac{\pi}{2}$ and using the expression for the moment at any point in the span, which becomes,

$$M = \frac{D_2}{\sin \frac{L}{j}} + w\,j^2$$

For points near $D_1 = 0$ the tangent varies rapidly, and it is impossible to obtain the values of $\frac{x}{j}$ and $\cos \frac{x}{j}$ with sufficient precision by the ordinary method of interpolation. The simplest method of attack in such a case is to interchange D_1 and D_2 in the formulas for $M_{\text{max.}}$. x will then be measured from the right-hand support; that is, the interchange of D_1 and D_2 is equivalent to turning the beam end for end. Unless $\frac{L}{j}$ is very close to π in value, this method will alter the value of $\frac{x}{j}$ sufficiently to permit a solution without recourse to any special formulas.

A second method depends on the fact that,

$$\frac{1}{\cos A} = \sec A = \sqrt{\tan^2 A + 1}$$

Applying this relationship to the expression for maximum moment in the span and substituting for $\tan \frac{x}{j}$ its value in terms of D_1 and D_2,

$$M_{\text{max.}} = \frac{1}{\sin \frac{L}{j}} \sqrt{D_1^2 + D_2^2 - 2\,D_1\,D_1 \cos \frac{L}{j}} + w\,j^2$$

By using this formula it is possible to find the maximum moment without solving for $\frac{x}{j}$, but the location of the moment is, of course, not obtained.

A third method, which is slightly approximate, depends on the fact that when $\frac{x}{j}$ is near $\frac{\pi}{2}$, sin A is nearly equal to 1, so that $\frac{1}{\cos A}$ may be equated to tan A. Then,

$$M_{\text{max.}} = D_1 \tan \frac{x}{j} + w\, j^2 = \frac{D_2 - D_1 \cos \frac{L}{j}}{\sin \frac{L}{j}} + w\, j^2$$

This last method involves an error of less than 1% in the first term of the equation when $\frac{x}{j}$ is between 1.45 and 1.70.

The three-moment equation also has critical points and for certain values of $\frac{L}{j}$ appears to give infinite values. M_2 appears to become infinite in the formula,

$$M_2 = \frac{\frac{w_1\, L_1^{\,3}\, \gamma_1}{4\, I_1} + \frac{w_2\, L_2^{\,3}\, \gamma_2}{4\, I_2} - \frac{M_1\, L_1\, \alpha_1}{I_1} - \frac{M_3\, L_2\, \alpha_2}{I_2}}{2\left\{\frac{L_1}{I_1}\, \beta_1 + \frac{L_2}{I_2}\, \beta_2\right\}}$$

when $\frac{L}{j} = \pi$ since α, β, and γ are each infinite. As a matter of fact this expression really becomes $\frac{\infty}{\infty}$ and may be evaluated in the ordinary manner for solving indeterminate forms by differentiation. It is apparent from this equation, however, that M_2 becomes infinite when the denominator is zero. Substituting the trigonometric functions for β, the denominator becomes zero when,

$$\frac{1 - \frac{L_1}{j_1} \cot \frac{L_1}{j_1}}{\frac{L_2}{j_2} \cot \frac{L_2}{j_2} - 1} = \frac{P_1\, L_1}{P_2\, L_2}$$

If the loading is symmetrical so that $P_1 = P_2$, $L_1 = L_2$, etc., the moment at the center support will become infinite when

$$\frac{1 - \frac{L}{j} \cot \frac{L}{j}}{\frac{L}{j} \cot \frac{L}{j} - 1} = 1$$

or,

$$1 - \frac{L}{j} \cot \frac{L}{j} = \frac{L}{j} \cot \frac{L}{j} - 1$$

Whence,

$$\frac{L}{j}\cot\frac{L}{j} = 1$$

and,

$$\frac{L}{j} = \tan\frac{L}{j}$$

This condition is fulfilled when $\frac{L}{j}$ is approximately 4.49, as tan 4.49 = 4.45.

The moment at the center support will not be infinite when $\frac{L}{j} = 0$, although tan 0 = 0, since 0 cot 0 is not equal to unity.

When the beam is continuous over more than two spans the problem of solving for the moments at the support for $\frac{L}{j}$ in one span equal to π becomes more involved. No difficulty is encountered in the use of any of these formulas if $\frac{L}{j}$ is kept less than π, and it is recommended that the sizes of beams to be used in airplane wings be so chosen that this condition will be satisfied in every span.

Movement of Points of Inflection.—As a part of the work of developing the precise formulas, a study was made as to the effects produced by changes of both axial and lateral load on the points of inflection. Fig. 7 shows graphically the changes in the magnitude and location of the points of inflection for a continuous beam subjected to a uniformly distributed lateral load and varying axial compressive loads. These curves are given for the span, $L = 155$, of the beam shown in the diagram; and for values, w, the uniformly distributed load, equal 5 and 25, respectively. The load is considered as acting along the neutral axis of the beam and applied at the outer points of support. Curves are given for each value of w for various axial loads, ranging from zero to 13 612 lb. Table 2 gives the values from which Fig. 7 was plotted. This table covers two different loading conditions, Cases A and B, the difference being in the ratio of side load to the value of M_1, the moment due to the cantilever tip load.

A glance at either Table 2 or Fig. 7 shows that the location of the points of inflection varies with the magnitude of the axial load, the variation being quite appreciable. Another important point brought out by Table 2 is that multiplying all the loads on a beam, subjected to combined loads, by a given amount will increase the bending moments at any point on the beam by more than that amount. For instance, a beam having an axial compression of 1 000 lb., a uniform lateral load of 5 lb. per in., and a restraining moment, M_1, of 10 000 in-lb., has a moment, M_2, of 10 206 in-lb. and a maximum span moment of — 5 122 in-lb. When each of the loads is multiplied by 5 so that $P = 5\,000$ lb., $w = 25$ lb. per in., and $M_1 = 50\,000$ in-lb., the value of M_2 becomes 54 877 in-lb., instead of 51 030 in-lb., while M_{1-2} is — 28 672 in-lb. instead of — 25 610 in-lb. The distance between points of inflection also

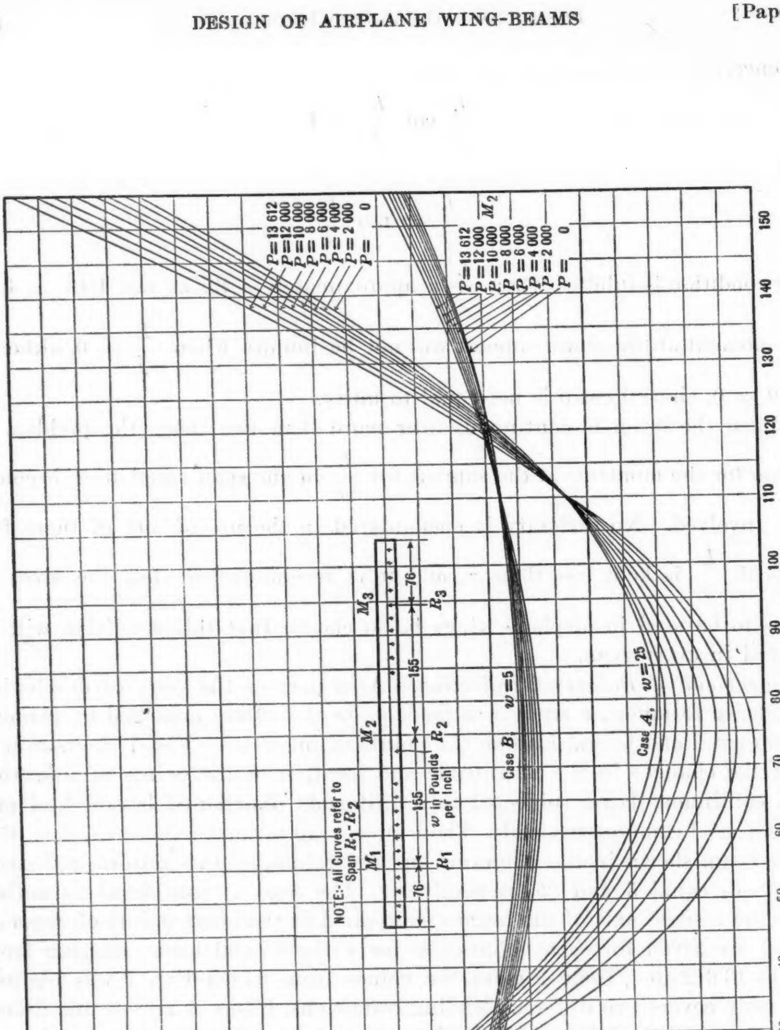

FIG. 7.—MOMENT CURVES.

increases from 89.468 in., to 90.063 in. Further study shows that had the axial load remained at 1 000 lb., while all the other loads were multiplied by 5, the moments in the span and at the support would have increased five times and the distance between points of inflection would not have changed. In other words, the points of inflection remain unchanged as long as the axial load remains constant and the ratio of lateral load to restraining moments is the same.

TABLE 2.—Maximum Moments and Location of Points of Inflection on a Continuous Beam.

Axial load, P.	Moment, M_2.	Maximum moment in Span M_{1-2}.	Distance from R_1.	Distance from R_1 to Points of Inflection.		Distance between points of inflection.
				Outer.	Inner.	
Case A: M_1 = 50 000 In-Lb.; w = 25 Lb. per In.						
0	50 078	−25 036	77.480	32.720	122.240	89.520
1 000	51 028	−25 606	77.221	32.489	121.953	89.464
2 000	51 832	−26 345	77.022	32.190	121.854	89.664
3 000	52 782	−27 085	76.783	31.879	121.687	89.808
4 000	53 797	−27 854	76.523	31.557	121.489	89.932
5 000	54 877	−28 672	76.243	31.211	121.274	90.063
6 000	56 026	−29 574	75.955	30.837	121.073	90.236
7 000	57 261	−30 518	75.643	30.450	120.886	90.386
8 000	58 593	−31 623	75.326	29.994	120.658	90.664
9 000	60 021	−32 697	74.961	29.553	120.369	90.816
10 000	61 576	−33 916	74.583	29.062	120.104	91.042
11 000	63 265	−35 254	74.178	28.530	119.826	91.296
12 000	65 118	−36 750	73.748	27.950	119.546	91.596
13 612	68 498	−39 442	72.980	26.938	119.022	92.084
Case B: M_1 = 10 000 In-Lb.; w = 5 Lb. per In.						
0	10 016	− 5 008	77.480	32.723	122.237	89.514
1 000	10 206	− 5 122	77.220	32.486	121.954	89.468
2 000	10 366	− 5 270	77.024	32.188	121.860	89.672
3 000	10 556	− 5 417	76.783	31.875	121.691	89.816
4 000 to 11 000	*	*	†	†	†	†
12 000	13 023	− 7 356	73.750	27.939	119.561	91.622
13 612	13 700	− 7 882	72.978	26.948	119.008	92.060

* Values are one-fifth those for Case A.

† Values are the same as for Case A.

It is because this movement of the points of inflection is appreciable under the action of large axial loads that the various approximate formulas for the analysis of continuous beams subjected to combined loadings are unsatisfactory for airplane design. Unless a three-moment equation providing for the effects of the axial load is used, the location of the points of zero moment in the spans will be incorrect and any of the formulas, which depend on the length of the "pin-ended" span between points of inflection, will give erroneous values.

For "roughing out" a design such methods are satisfactory, but they should not be relied on for the final design of airplane wing-beams. Since the moment of inertia enters into all the formulas that provide for an axial

load, it is sometimes desirable to use one of the approximate methods for the preliminary design before the final analysis by the more precise method is feasible but an experienced designer can generally approximate the required section so closely that the use of approximate analytical methods is unnecessary.

General Remarks on the Use of the Formulas.—It is recommended that at least four significant figures be used in all computations involving the foregoing formulas. In preliminary investigations, or for the purpose of obtaining a rough check, three figures will often suffice but, as the final result often depends on small differences between large quantities, three significant figures may give misleading results.

In discussions of members subjected to axial loads, failure is generally understood to mean "elastic instability" or "buckling". Usually, before such a failure occurs in practice the member actually fails by rupture of the fibers due to excessive unit stresses, and statements regarding the criteria for failure must be read with these facts in mind. Such criteria implicitly assume the material to have a constant, finite modulus of elasticity, but an infinite proportional limit and ultimate allowable stress. While no engineering material has such properties these formulas and the resulting criteria for failure are very useful in determining the loads under which failure of a structure is likely to occur.

PART II.—Computation of Allowable Stresses in Wooden Beams Under Combined Axial and Transverse Loads

Introduction.—If a short wooden strut of square cross-section is subjected to an axial load it will fail when the stress over any section reaches the crushing strength of the material. Assuming this to be first quality spruce with a 15% moisture content, that is, the material and conditions for which airplane beams are designed, the crushing strength of the strut will be about 5 000 lb. per sq. in. Now remove the axial load and subject the member to pure flexure of some kind sufficient to break it, and from this test compute the modulus of rupture by the usual formula, $f = \frac{M\,y}{I}$. An average value for this modulus will be found to be about 9 400 lb. per sq. in.

It appears, then, that for pure axial loads failure occurs when the intensity of stress on the fibers of the spruce is about 5 000 lb. per sq. in., whereas under a lateral load failure does not take place until a stress of 9 400 lb. per sq. in. has been reached by the extreme fiber. A question that immediately suggests itself is, "At what stress intensity will failure occur on a member subjected to both an axial and a lateral load?" For a short rectangular member this stress will be between 5 000 and 9 400 lb. per sq. in., depending on the ratio of stress due to bending to the total stress. For members of greater length and for other shapes the problem is complicated by the variation in stress intensity that will cause failure as a simple column and, further, by the fact that the modulus of rupture varies with the shape of the member.

The method of predicting the stress intensity at which failure will occur is partly theoretical and partly empirical. By its use a value is obtained which may be called the allowable stress or the stress at maximum load. It is, however, no more an actual stress than the modulus of rupture, since both are obtained by extending to conditions beyond the elastic limit of the material, formulas that are developed for conditions below this limit. The value might well be called an "apparent" allowable stress since it is an apparent stress just as the modulus of rupture is the apparent maximum fiber stress in a simple beam at failure. The method was developed at the Forest Products Laboratory by J. A. Newlin, M. Am. Soc. C. E., and Mr. G. W. Trayer as a continuation of the work on "Form Factors" performed by Mr. R. L. Hankinson, of the Materials Section, McCook Field, Dayton, Ohio.*

The Effect of the Shape of a Member on Its Modulus of Rupture.—Before discussing the computation of the allowable stresses under combined load a description of the method of obtaining the modulus of rupture for beams of various cross-sectional shapes is necessary. The material upon which the description is based is airplane spruce having the following properties:

Moisture content	15%
Modulus of elasticity	1 300 000 lb. per sq. in.
Modulus of rupture	9 400 lb. per sq. in.
Stress at elastic limit in bending	6 200 lb. per sq. in.
Maximum compression parallel to the grain	5 000 lb. per sq. in.
Compression stress at elastic limit	4 000 lb. per sq. in.

These values are based on the results of a great number of tests on standard specimens 28 in. long and 2 by 2 in. in cross-section. It was found that specimens of other shapes have different properties. For instance, a circular stick having a cross-sectional area of 4 sq. in. will, when loaded as a simple beam, support practically the same load as the 2 by 2-in. rectangular specimen having its sides vertical, assuming, of course, that the pieces are of the same quality. Moreover, a 2 by 2-in. rectangular specimen tested with one of its diagonals vertical will support the same or a slightly greater load than the one with its sides vertical. Yet the section modulus, $\frac{I}{y}$, for the square section with its sides vertical is about 18% greater than that of the circle of the same area and about 41% greater than the square having one diagonal vertical.

If these three sections carry the same loads over the same length of span the bending moments at the time of rupture must be the same, so that the only remaining variable is the apparent fiber stress at failure, the modulus of rupture value. The results of a number of tests show that this value does vary for different sections, being about 115% of the standard value in the case of a circular section and about 140% for the square section with one diagonal vertical.

* It is fully described in Reports No. 181 and 188, National Advisory Committee for Aeronautics, Washington, D. C.

A great many observers have noted that the fiber stress at elastic limit in bending, as computed by the ordinary beam theory, is materially greater than the elastic limit stress parallel to the grain, and many theories have been advanced to explain it. One of the best known is that the fiber stresses and strains are not proportional to the distances from the neutral axis even within the elastic limit. However, if it is assumed that the beam theory holds and that the stresses within the elastic limit are proportional to the distances of the fibers from the neutral axis, the difference becomes one of greater actual fiber stress in the beam in pure flexure rather than in a strut under compression parallel to the grain. The ability of the beam to take this increase in stress may be accounted for if the minute wood fibers, when subjected to compression along their length, are considered to act as Euler columns more or less bound together. As long as all the fibers are stressed in the same degree, as in the case of the ideal column, they can offer little or no support to each other, with the result that when one of the fibers reaches its limit it yields slightly and throws its load on to its neighbors so that they all yield. In this way the failure of the member may be accounted for as a progressive failure of its fibers, each of which is carrying the same load as its neighbor, so that none can help the others. In the case of a beam, however, all fibers are not equally stressed. Consequently, the less stressed fibers lend support to those carrying the maximum loads, or to those that have been stressed beyond their elastic limits, so that the elastic limit curve for the beam as a whole is not reached until what appears to be a very high stress has been developed in the extreme fibers.

This effect can best be illustrated, perhaps, by a consideration of a square beam tested with one diagonal vertical. The most stressed fiber may then be visualized as a single fiber extending the length of the upper corner of the beam, which is the corner carrying the maximum stress if the load acts vertically downward. If, then, it is assumed that there are two fibers in the layer next below the corner, each will be subjected to a stress less than that in the extreme fiber and, if the extreme fiber passes its elastic limit and yields slightly, these adjacent fibers will immediately absorb the slight increment of load. Similarly, when these two fibers reach their elastic limits the three or four in the next layer below them will absorb the extra stresses due to their yielding, and this action will continue through successive layers so that several increments may be added before the beam as a whole will appear to have passed its elastic limit. This same conception of the absorption of stress from a fiber that has yielded may be extended to one that has failed, and it is immediately apparent that the number of fibers adjacent to and supporting the critical one is of the utmost importance. This, of course, depends on the shape of the cross-section of the member, or on its "form", from which a "form factor" may be defined as a coefficient that evaluates the support given to the most stressed fiber by all the other fibers in a beam. Assuming a form factor of unity for the standard 2-in. square section tested with its sides vertical and horizontal (that is, one in which the fibers in the layer adjacent to the most stressed are subjected to very

nearly the same load as the most stressed), it may be shown that the factor for a circular section of the same area is 1.15, while for the square section having its diagonals vertical and horizontal, it is 1.41.

It is obvious then that for an I-section, the factor cannot be as great as 1.0, since the cut-out portion reduces the number of lightly stressed fibers that would otherwise offer support to the more highly stressed fibers in the flanges. By evaluating the supporting action which the fibers in the various parts of the cross-section exert, it should be possible to forecast the elastic limit stress and the stress at rupture for various types of sections. A formula that does evaluate this effect, giving "form factors" for the stress at elastic limit, is:

$$F_e = 0.65 + 0.35 \left[0.293 \left(\frac{\alpha}{57.3} - \sin \alpha \cos \alpha \right) \frac{b - b'}{b} + \frac{b'}{b} \right] \ldots (43)$$

and, with a slight change of constants, Equation (44) is obtained to give the modulus of rupture factor:

$$F_u = 0.50 + 0.50 \left[0.293 \left(\frac{\alpha}{57.3} - \sin \alpha \cos \alpha \right) \frac{b - b'}{b} + \frac{b'}{b} \right] \ldots (44)$$

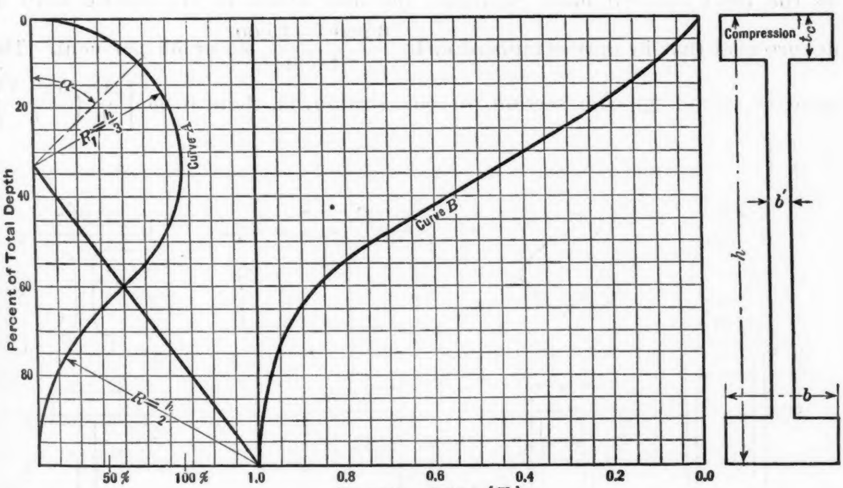

FIG. 8.

Equations (43) and (44) are empirical and are derived as follows: Assuming R_1 in Fig. 8 to be 1.0, the total area between Curve A and the vertical axis, that is, the total supporting effect of the fibers, is:

$$A = \frac{1}{2} \left[\frac{143.13}{57.3} + \frac{3}{2} \times 2 - \frac{53.13}{57.3} \times \left(\frac{3}{2} \right)^2 \right]$$

The area of the portion above the dashed line which represents the flange to depth ratio is:

$$A' = \frac{1}{2} \left[\frac{\alpha}{57.3} - \sin \alpha \cos \alpha \right]$$

The supporting ratio, K, shown on Curve B is:

$$\frac{A'}{A} = 0.293 \left(\frac{\alpha}{57.3} - \sin \alpha \cos \alpha\right)$$

These formulas represent the conditions when the depth of the compression flange is not more than 60% of the total depth of the beam. Within this limit α is the angle the versed-sine of which is $\frac{t_c}{h}$.

If the width of the flange of an I-beam, or box-beam, is b and the width of the web, b', the supporting ability of the compression chord is $\frac{A'}{A}\left(\frac{b-b'}{b}\right)$ times the supporting ability of a rectangular section of the breadth, b. The supporting ability of the web alone is $\frac{b'}{b}$ times that of a rectangle of the breadth, b, so the total becomes $\frac{A'}{A}\left(\frac{b-b'}{b}\right) + \frac{b'}{b}$. From the strength properties of spruce it has already been noted that the total support given to the most stressed fibers increases the fiber stress at the elastic limit in flexure over that in pure compression by $\frac{6\,200 - 4\,000}{4\,000}$, or 55 per cent. The increase for an I-beam section, or box section, is, then, $0.55\left[\frac{A'}{A}\left(\frac{b-b'}{b}\right) + \frac{b'}{b}\right]$.

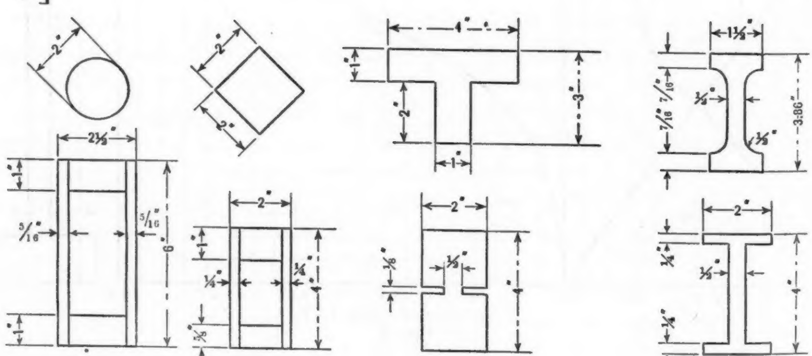

FIG. 9.

The ratio of the elastic limit stress in bending to the elastic limit of the material in direct compression will be 1 plus this quantity and the form factor will be this ratio divided by 1.55. Consequently, when the values of A and A' are substituted, the form factor for I-beam sections and box sections takes the form of Equation (43). Fig. 9 shows several sections to which this formula has been applied and Table 3 gives a comparison between the computed and actual form factors. It will be noted that the agreement is excellent, even between the more extreme sections.

TABLE 3.

Type.	Form factor modulus of rupture.	
Circular section	Test	1.15
	Formula	....
Square section with sides at 45°	Test	1.41
	Formula	....
I-beam section	Test	0.70
	Formula	0.70
T-beam section	Test	0.78
	Formula	0.80
Box section with equal chords	Test	0.69
	Formula	0.69
Box section with unequal chords	Test	0.71
	Formula	0.74
I-beam section with very thick chords	Test	0.89
	Formula	0.89
I-beam section with very thin chords	Test	0.64
	Formula	0.64

Thus far, the assumption has been made that the stresses do not exceed the elastic limit of the material. When this limit is passed there is practically no theoretical justification for applying a formula similar to Equation (43) to determine the effect of the form of a section on its properties. It has been found, however,* that if 0.50 and 0.50 be substituted for 0.65 and 0.35, Equation (43) will give a factor for the modulus of rupture that is in excellent accord with test results. (See Equation (44)).

TABLE 4.—DATA TO SUPPLEMENT FIG. 10.

Type.	$\frac{I}{c}$.	F.	$\frac{FI}{c}$.	Maximum load, in pounds.
(a)	7.58	....	....	3 200
(b)	8.33	....	....	3 178
(c)	4.73	....	....	2 280
(d)	5.33	....	....	2 285
(e)	4.15	0.86	3.57	3 897
(f)	4.73	0.74	3.50	3 867
(g)	9.88	0.68	6.71	......
(h)	10.40	0.65	6.76	......

Neither of these formulas is directly applicable to beams the top and bottom surfaces of which are not at right angles to the vertical axis, a case that often occurs in wing-beams on account of their being beveled to conform to the wing contour. They do apply, however, to the properties of an equivalent section that is symmetrical about the vertical axis, and the height of which is the mean height of the original section, and the width and flange areas of which equal those of the original section. Fig. 10 shows four typical wing-beams with their equivalent sections and the values in Table 4 compare the loads carried by the original and equivalent sections made from spruce having the same strength properties as the original beams. The results are in excellent agreement.

* National Advisory Committee for Aeronautics, Rept. 181, "Form Factors of Beams Subjected to Lateral Loads Only," by J. A. Newlin and G. W. Trayer, p. 4.

By use of these "equivalent" sections the form factor for practically any type of wooden beam likely to be used in an airplane wing may be obtained and the modulus of rupture computed, thus determining the allowable stress for a beam subjected to pure flexure.

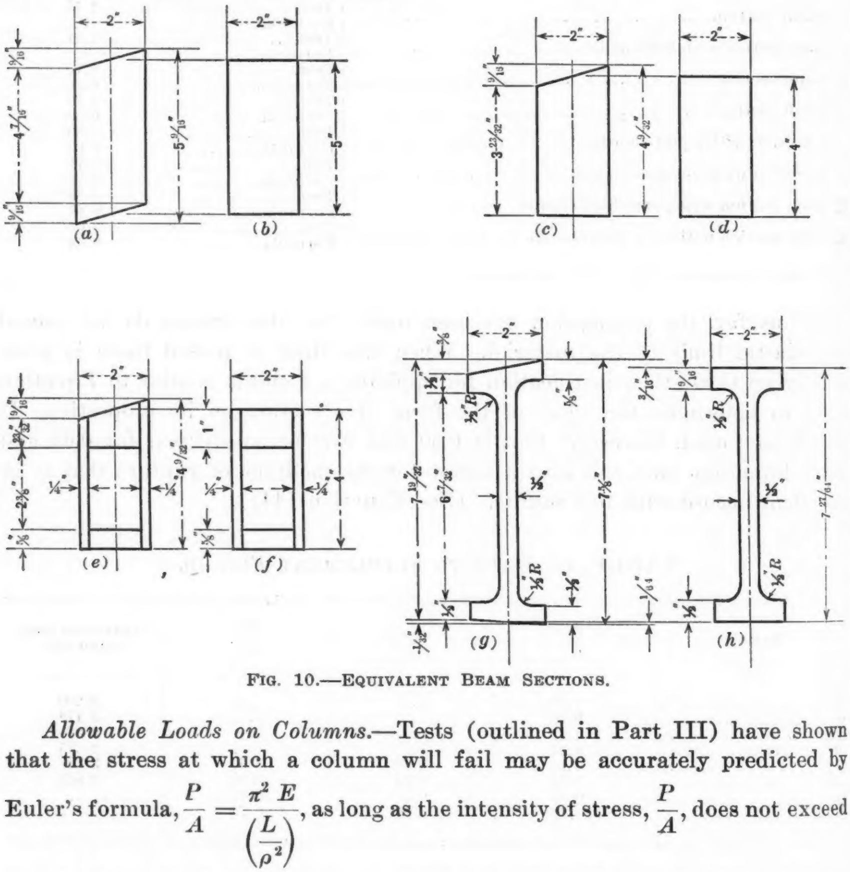

FIG. 10.—EQUIVALENT BEAM SECTIONS.

Allowable Loads on Columns.—Tests (outlined in Part III) have shown that the stress at which a column will fail may be accurately predicted by Euler's formula, $\frac{P}{A} = \frac{\pi^2 E}{\left(\frac{L}{\rho^2}\right)}$, as long as the intensity of stress, $\frac{P}{A}$, does not exceed the elastic limit of the specimen, and as long as the load is applied in such a way as to eliminate eccentricities of the material, workmanship, etc. When the stress intensity exceeds this limit, the value of $\frac{P}{A}$ at maximum load may be computed from:

$$\frac{P}{A} = F - (F - f)\left[\frac{\frac{L}{\rho}}{\sqrt{\frac{\pi^2 E}{f}}}\right]^{\frac{2f}{F-f}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

in which, F is the maximum compressive strength of the material parallel to the grain, and f, the elastic limit stress in compression. The value of the

modulus of elasticity, E, to be used in Equation (45) is that obtained for stresses below the elastic limit.

Equation (45) is based on the fact that after the elastic limit is passed there is, apparently, a gradual change in the stiffness of a column, which is equivalent to a reduction in the modulus of elasticity. The Forest Products Laboratory obtained remarkably close agreement with test results when it was assumed that the specimens remained straight until the maximum load was reached, and that the modulus of elasticity was at all times constant on a cross-section and was equal to the unit stress at any instant divided by the unit strain at that instant. In other words, the modulus of elasticity beyond the elastic limit is to be determined from the slope of the secant line drawn from the origin to intersect the stress-strain curve for the material at the value of $\frac{P}{A}$ being considered. Assuming the stress-strain curve to be a parabola between f and F, Equation (45) may be developed in a manner similar to that used to derive Johnson's parabolic formula.

The assumption just made as to the variation in E does not agree with the conception of the modulus of elasticity as the slope of the tangent to the stress-strain curve for stresses beyond the elastic limit, but it does appear to agree with test results when a column is carefully centered to eliminate eccentricities.

Equation (45) will give a parabolic curve having its vertex at the maximum compressive strength of the material when $\frac{L}{\rho} = 0$ and with a point of tangency to the Euler curve at a value of $\frac{P}{A}$ equal to the elastic limit of the material. For spruce, having its elastic limit in compression at 80% of the ultimate compressive strength, this formula gives an eighth-power parabola. For other woods and other conditions, the power of the curve would be changed and it is recommended by the Forest Products Laboratory that a fourth-power curve be used. This gives somewhat higher allowable stresses than Johnson's second-power parabola, but is more conservative than the theoretical eighth-power curve and appears to be in good accord with the results of tests. It is the writer's opinion, however, that for airplane work, where the factors of safety are low, even the fourth-power curve gives results that are too high, and, therefore, the use of Johnson's parabola is recommended for short columns in conjunction with Euler's formula for long ones. The point of tangency of the Johnson curve with the Euler curve being at the place where the unit stress on a cross-section is one-half the ultimate compressive strength, the value of $\frac{L}{\rho}$, which distinguishes the long from the short column, may be found from,

$$\frac{L}{\rho} = \sqrt{\frac{\pi^2 E}{f'}}$$

in which, f' is one-half the compressive strength of the material.

There are, then, two expressions for computing the allowable stress in a column of any length, one being Euler's formula, which is applicable as long as the fiber stress is less than one-half the crushing strength, the other being Johnson's formula which applies when the fiber stress is greater.

Stress in Members under Combined Loadings.—In determining the allowable stresses in members under combined bending and compression it is necessary to use these formulas, each in its proper sphere, and, in addition, the form-factor formulas must be utilized to provide for the change in modulus of rupture with change in shape of the section.

Fig. 11 contains curves based on these formulas and, by its use, the apparent allowable stress for any type of beam ordinarily encountered in airplane work may be readily computed for various combinations of axial and bending stresses.

The right half of the diagram contains curves that are practically self-explanatory. They are used to determine the modulus of rupture and elastic-limit-in-bending stresses for beams of various shapes. They are, in reality, the curves obtained by multiplying the form factors for beams of various shapes from Equations (43) and (44) by the average value of the modulus of rupture of 9 400 lb. per sq. in., and the elastic-limit-in-bending value of 6 200 lb. per sq. in. The curves are plotted for various ratios of $\frac{t_c}{h}$ and $\frac{b'}{b}$ and give the desired modulus of rupture and elastic limit values in pounds per square inch instead of in terms of the form factor.

The curves in the left half of the diagram are concerned with the axial loads and are obtained in the following manner. The $\frac{L}{\rho}$-curves for the condition of zero bending, that is, $\frac{f_b}{f_t} = 0$, have values equal to those determined by Euler's or Johnson's formulas for the given value of $\frac{L}{\rho}$. Assuming that the axial load is kept constant and that the ratio of $\frac{f_b}{f_t}$ is increased to 0.2, which is the same as assuming that the strut deflects under load, it is readily seen that the maximum stress, f_t, will be $\frac{1}{1 - 0.2} = 1.25$ times that developed under the critical load from Johnson's or Euler's formulas. Similarly, when $\frac{f_b}{f_t} = 0.5$, f_t is twice the stress under the critical load. The group of $\frac{L}{\rho}$-curves given in Fig. 11 were obtained by computing the critical stresses for each value of $\frac{L}{\rho}$ and multiplying each of these stresses by the proper factor to provide for the different ratios of $\frac{f_b}{f_t}$.

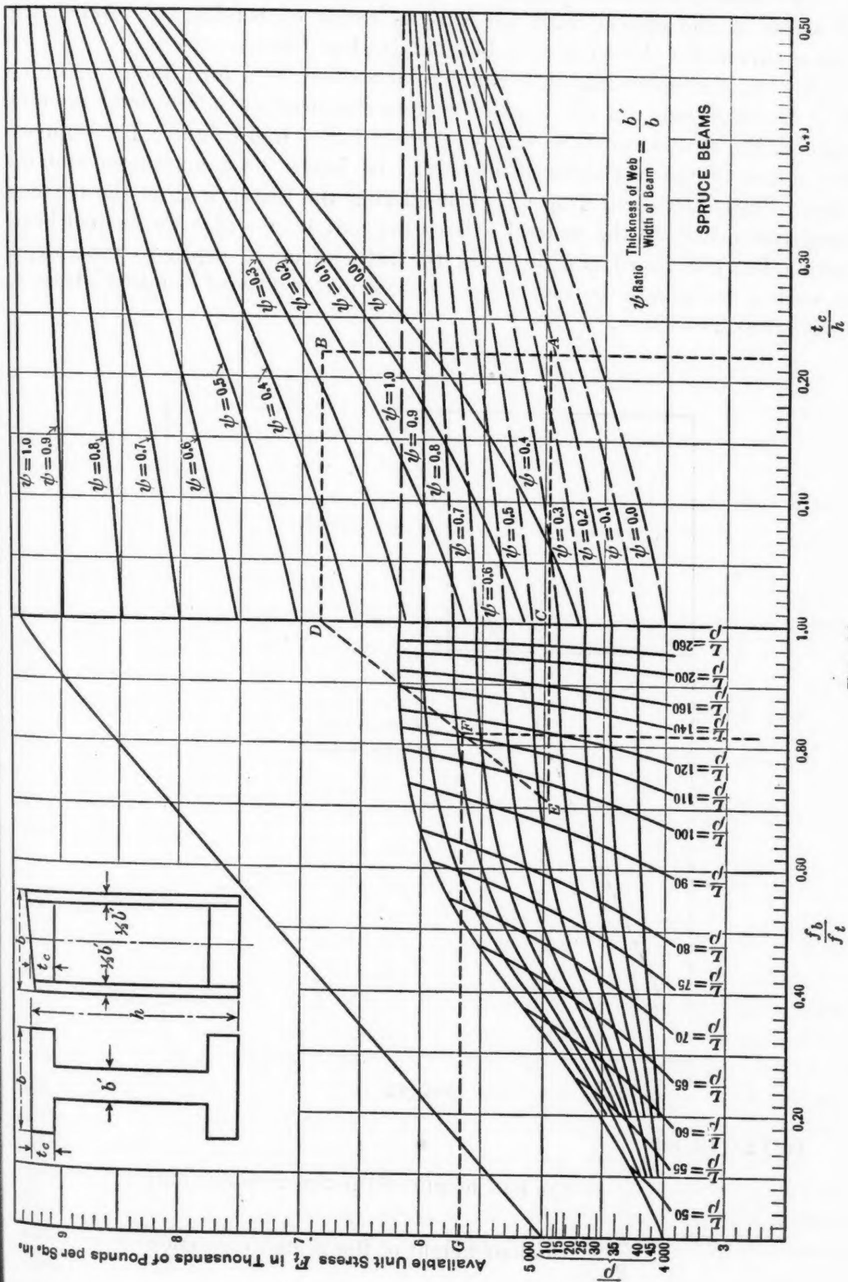
SPRUCE BEAMS
ψ Ratio Thickness of Web / Width of Beam = b′/b
Available Unit Stress F_t in Thousands of Pounds per Sq. In.
t_c/h
f_b/f_t
L/ρ

FIG. 11.

The other group, running approximately horizontal, indicates the intensity of stress at the elastic limit for various ratios of bending to total stress. These curves are plotted from values obtained as follows.

Assume a member having a rectangular section, or a form factor of unity. If it be subjected to a slight axial compression while a lateral load is being applied, the neutral surface will be slightly below mid-height. The supporting action of the less stressed fibers will no longer be a maximum and the elastic limit stress will drop off. Considering the distance from the extreme compressive fiber to the neutral axis as the half height of a theoretical beam having but one chord and no webs, the ratio of chord depth to total depth of such a beam may be determined for various ratios of bending stress to total stress as follows.

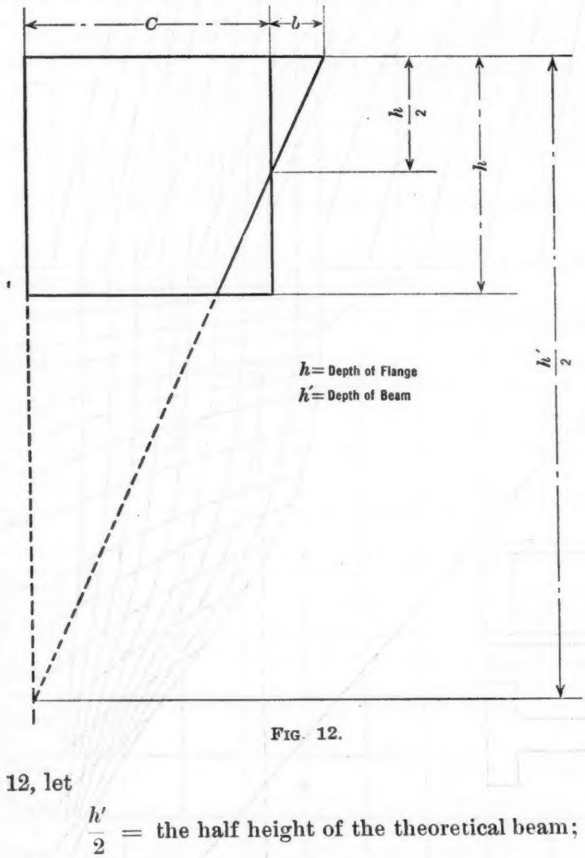

FIG. 12.

In Fig. 12, let

$\frac{h'}{2}$ = the half height of the theoretical beam;

$\frac{h}{2}$ = the half height of the beam in question;

b = the total bending stress; and,

c = the direct compressive stress.

Then, from similar triangles,

$$\frac{b}{\frac{h}{2}} = \frac{b + c}{\frac{h'}{2}}$$

or,

$$\frac{h}{h'} = \frac{b}{b + c}$$

In other words, the ratio of the flange depth to the total depth of the theoretical beam is the same as the ratio of the bending stress to the total stress. Since the theoretical beam has webs of zero thickness the form factor, Equation (43) becomes,

$$F_e = 0.65 + 0.35 \left[0.293 \left(\frac{\alpha}{57.3} - \sin \alpha \cos \alpha\right)\right]$$

The elastic limit stress for the combined load is then determined by taking the product of the elastic limit stress in ordinary bending, 6 200 lb. per sq. in., times the form factor of the theoretical beam for the particular ratio of bending stress to total stress. In the case of pure axial load, F_e becomes 0.65 which is the ratio between the elastic limit in compression to that in simple bending.

If the member being considered has a form factor, as, for instance, a box-beam, the elastic limit stress in direct compression would remain unchanged, but that in bending would be lowered. To take a specific case assume a form factor of 0.90. The elastic limit in bending would then be $0.90 \times 6\,200 = 5\,580$ lb. per sq. in., and the constants in the formula given, would be changed. The first would become $\frac{4\,000}{5\,580} = 0.717$, and the form factor equation for stress at the elastic limit, which would be applied to the theoretical beam subjected to combined loads, would be,

$$F_e = 0.717 + 0.283 \left[0.293 \left(\frac{\alpha}{57.3} - \sin \alpha \cos \alpha\right)\right]$$

The elastic limit curves shown in Fig. 11 were drawn by the use of equations derived in this way.

It is now possible to consider the maximum load condition for various ratios of direct to bending stresses. For the sake of simplicity consider a member in the Euler column class. Obviously, the Euler load is the maximum that can be obtained for a zero ratio of bending unit stress to total unit stress; and the stress must be less than the elastic limit stress in compression parallel to the grain since this is always less than one-half the ultimate for airplane spruce. Assume that the Euler stress for such a member is 2 000 lb. per sq. in. If the column deflected a little it would still carry the Euler load, but a bending stress would be introduced. Deflection would increase until the elastic limit was reached and the total stress, due to the axial compressive load and the bending stresses arising from the deflection, would

follow the $\frac{L}{\rho}$ = 80-curve in Fig. 11 until it intersected the elastic limit curve determined by the shape of the member. This intersection represents the stress in an axially loaded Euler column when deflected to the elastic limit. The stress at maximum load under eccentric or other combined loading would always be somewhat greater than this elastic limit stress; hence this intersection serves as a starting point in the determination of the stress at maximum load. It has been found that the stress at maximum load will be intermediate between this point and the modulus of rupture and will lie on a straight line connecting these two points, at least within the limits of precision of the ordinary test.

The method of using the diagram, Fig. 11, to determine the allowable stress under combined loadings is, then, as follows: Compute the ratios, $\frac{t_c}{h}$ and $\frac{b'}{b}$, for the member in question, and with them use the curves on the right side of the diagram to determine the modulus of rupture and the elastic-limit-in-bending, Points B and A on the diagram. Project these values horizontally to the center of Fig. 11 (Points D and C) and continue the line, $A\,C$, parallel to the dotted elastic limit curves. Compute the slenderness ratio of the member and draw a line parallel to the $\frac{L}{\rho}$-curves as shown by the dashed line, $E\,F$, to intersect the prolongation of the line, $A\,C$, at F. Connect F and D with a straight line, shown as a dashed line in the diagram. The stress at maximum load, that is, the allowable stress as used in the design, will then be found on this dashed line, generally at some point between D and E, depending on the ratio of bending stress to total stress. For a value, $\frac{f_b}{f_t}$, of 0.82, the line, $E\,D$, indicates an allowable stress of 5 700 lb. per sq. in.; for 0.95, 6 500 lb. per sq. in., etc.

Fig. 11 was constructed by A. S. Niles, Jr., Assoc. M. Am. Soc. C. E., from data furnished by the Forest Products Laboratory. It is now used for the computation of the allowable stresses in wing-beams and all other spruce members subjected to combined loading on all airplanes built for the Army Air Corps, for the Bureau of Aeronautics of the U. S. Navy, and for commercial airplanes designed to meet the airworthiness requirements of the U. S. Department of Commerce. The right-hand portion alone may be used to determine the modulus of rupture for I-sections and box-sections subjected to simple flexure.

In computing the values of $\frac{L}{\rho}$ for use in airplane wing-beam design, it is customary to determine the distance between points of inflection from a modified form of Equations (16) and (17), Part I. A monographic chart has been drawn to reduce the labor of determining this distance and it may be found in the Handbook of Instructions for Airplane Designers, published

FIG. 13.—TESTING MACHINE FOR SMALL STRUTS.

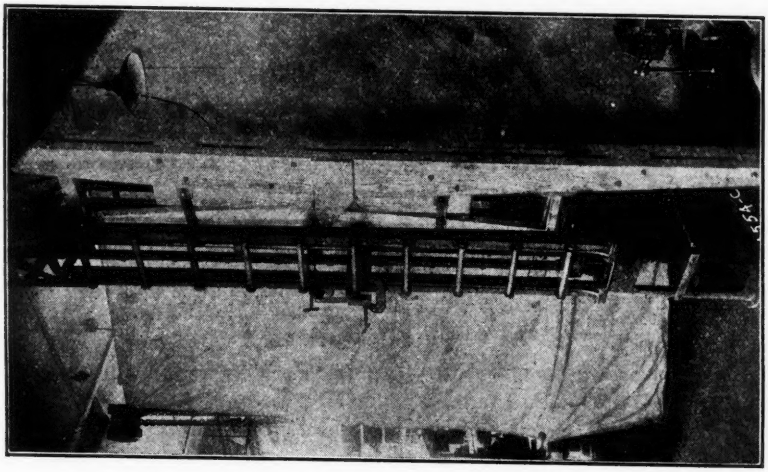

FIG. 14.—VERTICAL JIG FOR TESTING STRUTS.

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by the Army Air Corps for the guidance of designers of military airplanes, or in the equivalent publication of the U. S. Department of Commerce.

In computing ρ neglect the short filler blocks used at the points of attachment of fittings. For beams that are tapered but slightly in a bay, use an average value of ρ.

For beams that have a large taper, or that vary greatly from the conditions encountered in conventional design, Fig. 12 should not be assumed to give satisfactory results, but the strength of such members should be determined, or checked, by test.

Part III.—Pin-Ended Struts

Results of Tests.—For a complete verification of the formulas derived in Part I it would be necessary to make a large number of tests on specimens subjected to each of the different loading conditions; but, as the basis of the formulas for all conditions is the same, it was concluded after some thought that a series of tests vindicating one or two of the formulas would demonstrate the reliability of the remainder. It was also concluded that, since a check on a theory was to be made, it would be justifiable to arrange the tests to simulate theoretical conditions as much as possible, making allowance later for any variation between practical and theoretical conditions.

A series of tests was accordingly made at McCook Field on small spruce struts loaded as columns between knife-edges and subjected to a concentrated load at mid-span, the lateral load being omitted in several cases and the struts tested as simple columns. A second series of tests was run later, on spruce beams of approximately 2 by 6-in. cross-section, comparable to the beams in an airplane of the observation type, in order to determine the reliability of the formulas developed in Part I for computing the design stresses when combined with the method given in Part II for obtaining the allowable stresses. The results of both series of tests which, although limited in number, gave very satisfactory results, are given in Part III. Figs. 13 and 14 show two types of testing machines used.

Tests on Small Struts.—Fig. 13 gives a clear idea of the method of loading and of obtaining the deflections of the small struts. Each strut was supported between knife-edges in such a way that it could deflect freely up or down, but would be restrained against lateral deflection by the bearing of its flat ends against the plates to which the knife-edges were fastened. The span was taken as the distance between knife-edges. The concentrated side load was obtained by hanging a weight of 50 or 100 lb. to a yoke placed at the center of the span. The axial load was applied by turning the wheel shown at the far end of the testing machine. This motion actuated a screw and pulled the two "heads" together, putting the specimen into compression. The amount of this load was measured by a dynamometer which was calibrated frequently in a standard Olsen testing machine. Deflections of the specimen were read at mid-span to 0.001 in. from a Wissler dial deflectometer.

As the method of deriving the precise formulas is similar to that used in the development of the Euler column formula, it was thought that the equations would prove satisfactory for long struts, but might be unreliable for

short, stiff specimens. For this reason the tests were made on struts having slenderness ratios of 50, 60, 70, and 80. The dimensions and properties of the specimens are given in the Table 5.

TABLE 5.—PROPERTIES OF SPECIMEN STRUTS.

Series.	Slenderness ratio, $\frac{L}{\rho}$.	Height and width, in inches.	Span length between knife-edges, in inches.	Area of cross-section, in square inches.	Moment of inertia, in inches.	NUMBER OF TESTS.	
						Combined loading.	Axial load only.
80	80	1.280	29.50	1.638	0.2237	33	2
70	70	1.125	22.77	1.270	0.1344	7	4
60	60	1.028	17.82	1.059	0.0932	11	2
50	50	0.755	10.90	0.571	0.0271	20	2

Curves comparing the observed and computed deflections of fourteen specimens are given in Figs. 15 to 18. The specimens are numbered 80-6, 60-5, etc., the figure before the dash referring to the series to which the strut belongs, that after the dash giving the number of the specimen in that series. The modulus of elasticity is, with one exception, that obtained by a cross-bending test on the specimen. The computed deflections for all specimens were obtained by the use of Equation (33) of Part I.

Results of Preliminary Tests.—The first series of struts tested was that in which the slenderness ratio was 80. Fig. 15 shows the deflection of Strut 80-6 with no side load, as well as the actual and computed deflections under a side load of 50 and 100 lb. The curves in this diagram are good examples of those obtained from a number of specimens. They cannot be said to show a completely satisfactory agreement between observed and computed values, either for the Euler load or for the deflections. The maximum observed load of 4 125 lb. was obtained by running up the axial load to the limit that the strut would carry and releasing it before failure occurred. The side loads were then applied and the results given in the other curves obtained.

The value of E which, when substituted in Euler's formula, would give a Euler load equal to the maximum observed value, was then computed. Fig. 16 gives a comparison of actual and computed deflections when the latter values were obtained from this modified value of E. It will be noted that the observed and computed values check almost exactly.

It was recognized that this apparent modification of the modulus of elasticity was caused by eccentricities due either to the method of loading or to lack of uniformity of the material. It was also seen from the second set of curves for Specimen 80-6 that the formulas would give correct curves of deflection if the effect of eccentricities could be cancelled. A number of specimens were tested, therefore, as simple columns, care being exercised to apply the load exactly at the center of the end cross-section, and the modulus of elasticity was computed from Euler's formula using the maximum observed load as the Euler load. These values of E were compared

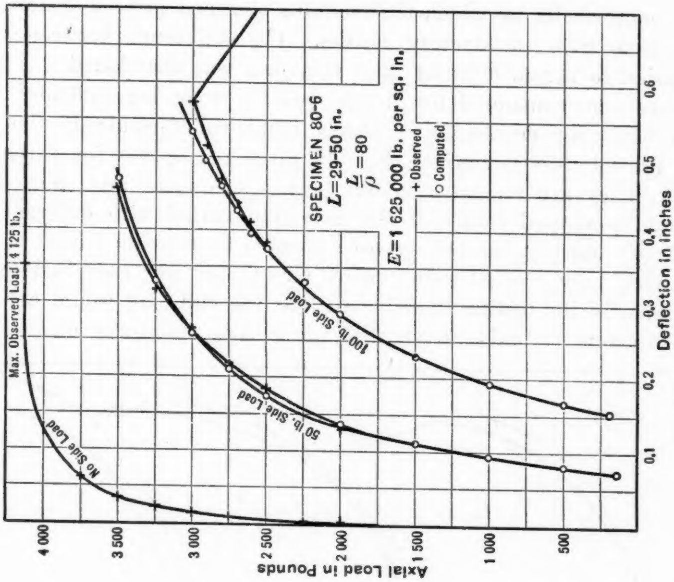

FIG. 16.—LATERALLY LOADED STRUTS.

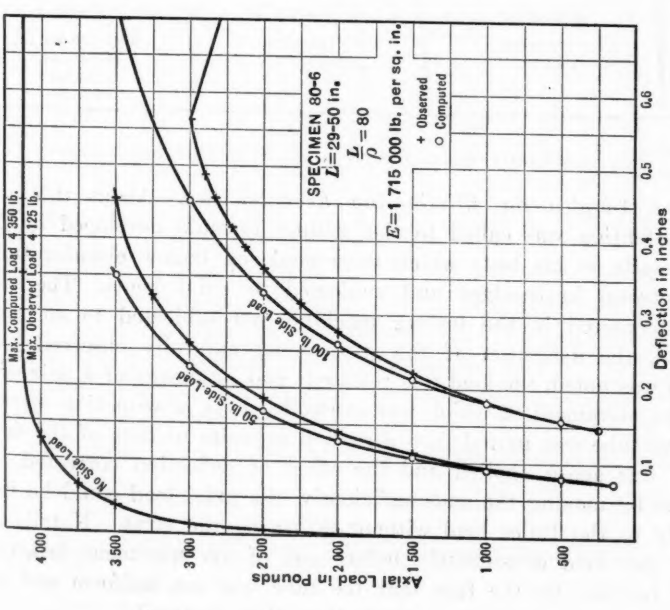

FIG. 15.—DEFLECTION OF STRUTS.

with those obtained by the cross-bending method in the hope that the difference between them would be sufficiently constant so that a coefficient could be derived empirically by which the modulus obtained in cross-bending could be modified to give satisfactory results. The difference between the two moduli varied so much, however, that this idea was abandoned.

The tests were continued for the purpose of amassing sufficient data to furnish a basis for developing a purely empirical formula if the precise equations proved unsatisfactory. Fig. 17 gives the curves for three of the specimens. They are typical of about twenty specimens and show how the observed and computed values of the deflection varied with different struts. In Fig. 17 (*a*) and (*b*), the two curves happened to coincide almost perfectly; in other cases, they crossed each other. Great care was exercised to center each specimen in the testing machine so that the end load would act exactly through the geometric axis. Even then the results were quite unsatisfactory.

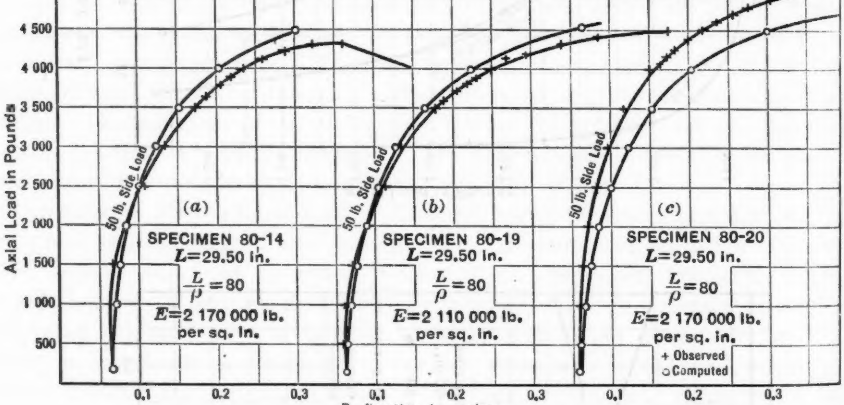

FIG. 17.—VARIATIONS IN OBSERVED AND COMPUTED DEFLECTIONS.

Method Adopted for Eliminating Eccentricities.—About this time the writer's attention was called to the column formula developed by Natalis* and especially to his tests which were made on hollow circular tubes supported between knife-edges and subjected to axial loads. The tube was carefully centered in the testing machine and subjected to an increasing axial load, the deflection of the strut being carefully observed. When a deflection was noted, the load was released; and by means of a pair of sliding plates (the movement of which was controlled by a slow-motion screw), each end of the tube was moved in a direction opposite to that of the deflection. The load was again applied and the effect of deflection corrected. It was found that by moving the ends sufficiently, the axial load could be increased practically to the Euler load without deflecting the strut. Natalis actually exceeded this load considerably before one of his specimens failed, but he accounts for this by the fact that the tube was not uniform and probably had an average cross-section greater than that assumed.

* "Technische Berichte der Flugzeugmeisterei der Flügertruppen," Band III, Heft 60.

This method, when applied to a specimen, gave highly encouraging results. Instead of using plates controlled by slow-motion screws to move the ends, a hammer was tried and found to be very effective. The system finally adopted was to center the specimen carefully in the testing machine and apply an increasing axial load. As soon as a deflection of 0.005 or 0.006 in. was noted, the axial load was reduced to about 50 lb. and the specimen tapped at each end with the hammer to move the ends and introduce corrective moments. The axial load was then increased and this process repeated until a load within 300 or 400 lb. of the Euler value would cause a deflection of only 0.004 or 0.005 in. When this condition was reached the axial load was cut to about 100 lb. and the side load applied. Great care was used, in adding the side load, to avoid jarring the specimen and changing its position in the machine.

This practice of moving the ends of the specimen is a laboratory method for obtaining an "ideal" column, the effect of the motion being to introduce moments by the eccentric application of the end load which will tend to neutralize the "effective eccentricities" in the specimen due to its not being absolutely homogeneous, to its not being of absolutely uniform cross-section, or to its not being perfectly straight.

Table 6 gives the results obtained by using this method on a series of specimens tested without any lateral load. The first five specimens were tested without any effort to adjust the ends, and they show an average difference of 8.8% between the observed and the computed load. The remaining seven specimens of $\frac{L}{\rho} = 80$ were adjusted to eliminate eccentricities, with an average difference of only 2.8% between the observed maximum and the Euler maximum load. Remarkably good agreement was obtained for the specimens having slenderness ratios of 60 and 70, but for the shorter struts, $\frac{L}{\rho} = 50$, the discrepancy was quite large, as would be expected when the observed stress was so close to the actual crushing strength of the material.

The results of these tests on simple columns emphasize the importance of offsetting the eccentricities of the practical column if a satisfactory check of formulas for combined loadings, or for plain axial loads, is to be obtained.

Fig. 18 compares the observed and computed deflections for a specimen tested with combined lateral and axial loads when the method of adjusting the ends was used. This is typical of about ten specimens and indicates that, at least for struts having an $\frac{L}{\rho}$ of 80, the precise formulas are dependable when the actual conditions agree with the assumptions under which the formulas are developed.

Fig. 19 gives curves for three specimens that are representative of the seven tested in the second series. In each case the slenderness ratio is 70. Specimen 70-1 showed the greatest divergence of any in the series, but it

TABLE 6.—COMPARATIVE TESTS WITH NO LATERAL LOAD.

Specimen No.	Slenderness ratio, $\frac{L}{\rho}$	MAXIMUM LOAD, IN POUNDS.		Difference in percentage of computed load.	Crushing strength from test on short specimens, in pounds per square inch.	Maximum stress parallel to the grain, in pounds per square inch.	Ratio of observed stress to crushing strength.	Modulus of elasticity, E.	Dimensions of specimens, in inches.
		Computed.	Observed.						
697-1- 6	80	4 350	4 125	− 5.2	6 100	3 725	0.61	1 715 000	1.28 by 1.28 by 29.50
697-1- 8	80	3 420	3 230	− 5.6	5 330	3 250	0.61	1 348 000	1.28 by 1.28 by 29.50
697-1-10	80	4 290	3 850	−10.3	.....	.....	....	1 694 000	1.28 by 1.28 by 29.50
697-1-13	80	5 170	4 700	− 9.1	6 500	3 970	0.61	2 041 000	1.28 by 1.28 by 29.50
697-1-17	80	4 850	4 175	−13.9	.....	.....	....	1 650 000	1.28 by 1.28 by 29.50
697-1-23	80	3 620	3 375	− 6.8	4 900	2 990	0.61	1 429 000	1.28 by 1.28 by 29.50
697-1-24	80	3 650	3 550	− 2.7	4 900	2 990	0.61	1 440 000	1.28 by 1.28 by 29.50
697-1-25	80	3 850	3 770	− 2.1	4 750	2 900	0.61	1 501 000	1.28 by 1.28 by 29.50
697-1-26	80	3 620	3 400	− 6.1	4 850	2 960	0.61	1 429 000	1.28 by 1.28 by 29.50
697-1-27	80	3 905	3 880	− 0.6	.....	.....	....	1 532 000	1.28 by 1.28 by 29.50
697-1-34	80	1 280	1 290	+ 0.1	4 940	2 260	0.46	1 455 000	0.755 by 0.755 by 17.44
697-1-35	80	1 250	1 300	+ 0.4	4 940	2 280	0.46	1 415 000	0.755 by 0.755 by 17.44
697-2- 8	70	1 630	1 640	+ 0.6	4 770	2 875	0.60	1 421 000	0.755 by 0.755 by 15.26
697-2- 9	70	1 580	1 610	+ 1.9	5 000	2 820	0.56	1 376 000	0.755 by 0.755 by 15.26
697-2-10	70	1 580	1 590	+ 0.6	4 460	2 790	0.62	1 376 000	0.755 by 0.755 by 15.26
697-2-11	70	1 550	1 600	+ 3.2	4 970	2 805	0.56	1 346 000	0.755 by 0.755 by 15.26
697-3- 9	60	4 970	4 700	− 5.4	6 170	3 115	0.50	1 720 000	1.028 by 1.028 by 17.82
697-3-10	60	5 300	5 100	− 3.8	7 350	3 380	0.46	1 830 000	1.028 by 1.028 by 17.82
697-3-12	60	2 110	2 060	− 2.4	4 770	3 610	0.76	1 344 000	0.755 by 0.755 by 13.08
697-3-13	60	2 060	2 100	− 1.9	4 620	3 680	0.80	1 319 000	0.755 by 0.755 by 13.08
697-4- 1	50	4 250	3 150	−22.2	7 400	5 525	0.75	1 885 000	0.755 by 0.755 by 10.90
697-4- 4	50	4 310	3 500	−18.8	7 650	6 140	0.80	1 915 000	0.755 by 0.755 by 10.90
697-4-12*a*	50	2 760	2 450	−11.2	4 320	4 300	0.99	1 225 000	0.755 by 0.755 by 10.90
697-4-12*b*	50	2 760	2 150	−22.1	4 270	3 770	0.88	1 225 000	0.755 by 0.755 by 10.90

will be seen that even that is not great. The scheme of moving the ends to offset eccentricities was used with all struts tested in this group.

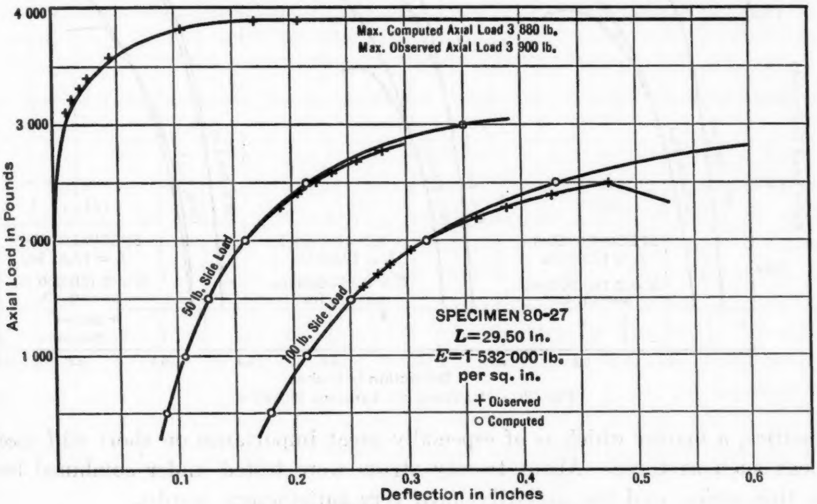

FIG. 18.—LATERALLY LOADED STRUTS.

Fig. 20 shows three sets of curves that are representative of the eight specimens of the third series. The slenderness ratio was 60. The scheme of moving the ends was used on all the specimens with very good results. Specimen 60-8 showed the greatest discrepancy, but even that is satisfactory.

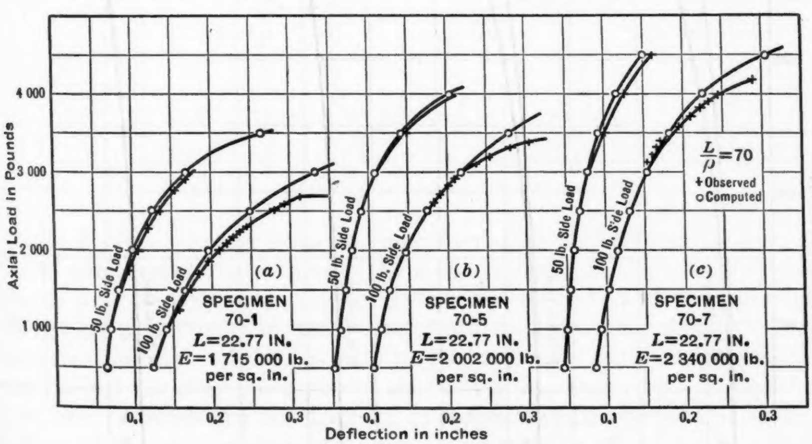

FIG. 19.—LATERALLY LOADED STRUTS.

Fig. 21 gives curves from four struts having an $\frac{L}{\rho}$ of 50. The first two, Struts 50-3 (A and B), do not show a perfect agreement between the curves, but the results from Specimens 50-10 (A and B) are certainly as good as could be desired. All these struts were adjusted to neutralize the eccen-

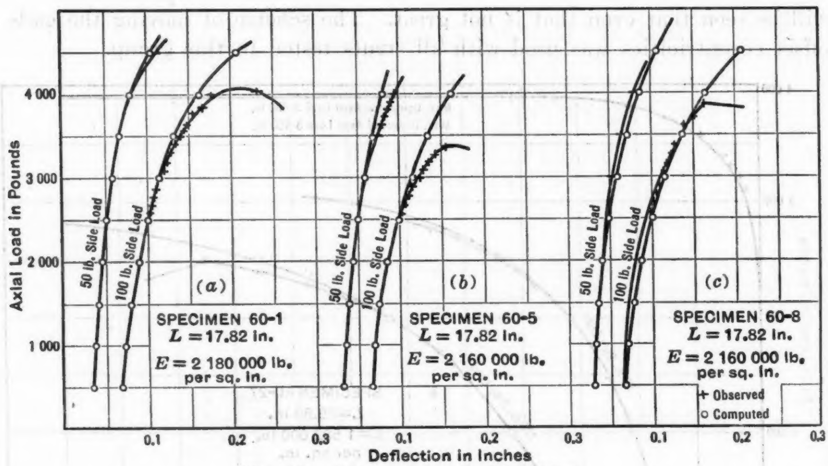

FIG. 20.—LATERALLY LOADED STRUTS.

tricities, a matter which is of especially great importance on short stiff specimens such as these. About twenty struts were tested under combined load in this series, and the majority gave very satisfactory results.

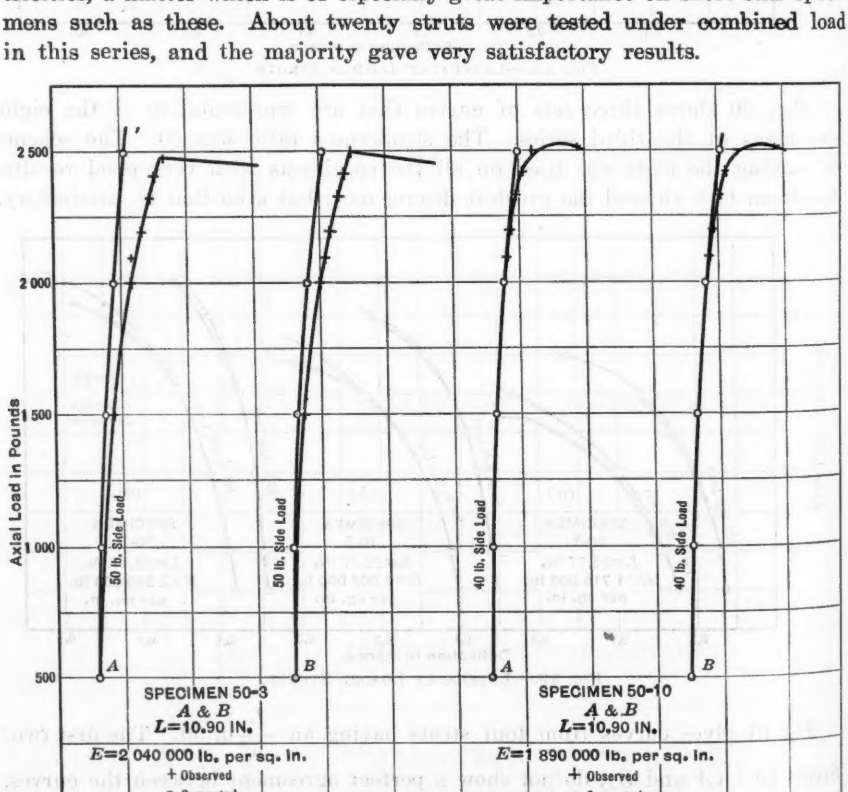

FIG. 21.—STRUTS WITH $\frac{L}{\rho}$ = 50, LOADED LATERALLY.

A study of the foregoing curves leads to the conclusion that the precise formulas are in practically perfect agreement with test results on small specimens as long as the eccentricities of the material are neutralized. At the time these tests were run (1922), the Forest Products Laboratory method for computing the allowable stresses had not been fully developed, therefore, no information could be obtained as to the reliability of the combination of the precise formulas and this method for designing struts subjected to combined loadings.

During 1925 and 1926 tests were made on six spruce beams of a larger size. They were 8 ft. long and were designed to carry an axial compression of 20 000 lb. In addition, there were two transverse loads of 2 000 lb. each applied $22\frac{1}{2}$ in. from the ends of the beam to produce a primary bending moment of 45 000 in-lb. These are the loads carried by the wing spars of an average two-bay observation airplane. The depth of the beams was $6\frac{1}{4}$ in., but their width varied between 2 and 3 in., as shown in Fig. 22. Two solid rectangular sections, two I-sections, and two box sections were designed by the combined use of the precise formulas and the Forest Products Laboratory method for these conditions and tested in the jig shown in Fig. 14. This jig was arranged so that it kept the ratio of side to end load a constant at all times.

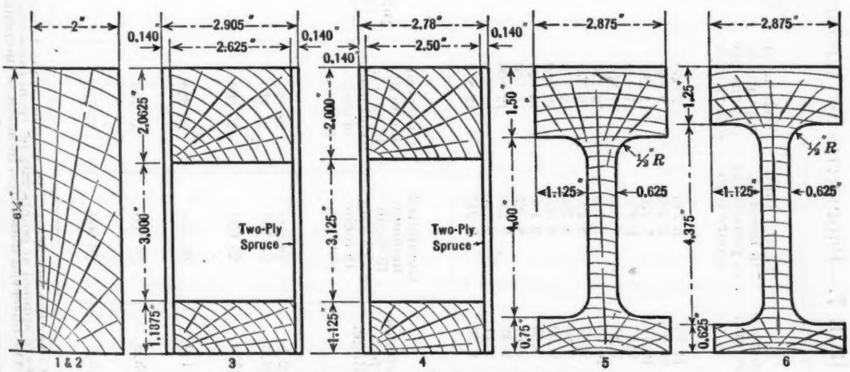

FIG. 22.—SECTIONS OF SPRUCE TEST BEAMS.

Table 7 gives the properties of the material in the different beams, as obtained from tests on minor parts; the stress intensity at the critical section under failing load as computed by the precise method; and the allowable stress intensity indicated by the Forest Products Laboratory method. It is to be noted that four of the six spars tested indicate a variation between the computed and the allowable stress of about 14%, one shows 1%, and another $2\frac{1}{2}$ per cent. Considering the diversity of shapes used, the tests indicate that the form of the cross-section does have an appreciable effect, the agreement between the results with Spars 2, 3, and 5 (one of each shape tested) being approximately equal, that is, 14 per cent.

The agreement between the values of the maximum bending moment given in Table 7 is also worthy of note. The computed moment under an axial load of 10 000 lb. is obtained by the use of Equation (28) of Part I for

TABLE 7.—Properties of Wooden Test Spars.

Spar No.	Specimen from chord.	Specific gravity.	Moisture content.	Elastic limit in bending, in pounds per square inch.	Modulus of rupture, in pounds per square inch.	Compression Parallel to Grain, in Pounds per Square Inch. Elastic limit.	Compression Parallel to Grain, in Pounds per Square Inch. Ultimate.	Modulus of elasticity, (E).	EI_{xx} of spar from test.*
1	Average. ...	0.382	11.95	5 340	7 650	2 950	4 140	1 210 000	49 900 000
2	Average.......	0.385	11.05	6 190	9 840	3 380	5 500	1 480 000	60 221 000
	Compress	0.387	10.18	7 970	11 280	4 710	6 190	1 740 000	
3	Tension........	0.388	9.57	8 150	11 420	3 840	6 060	1 747 000	85 800 000
	Average.......	0.3875	9.85	8 065	11 350	4 275	6 125	1 743 000	
	Compress	0.389	10.05	8 760	12 170	4 730	6 510	1 695 000	
4	Tension........	0.388	9.87	8 420	12 170	4 410	6 610	1 685 000	78 330 000
	Average.......	0.3885	9.96	8 590	12 170	4 570	6 560	1 690 000	
	Compress	0.405	9.99	8 660	12 380	4 710	5 440	1 790 000	
5	Tension........	0.412	9.23	9 190	13 620	4 445	5 810	2 198 000	75 100 000
	Average.......	0.4085	9.61	8 925	13 000	4 575	5 625	1 994 000	
	Compress......	0.413	9.48	9 250	12 800	4 990	6 270	1 663 000	
6	Tension........	0.403	9.11	9 170	12 820	5 000	5 590	1 767 000	66 700 000
	Average.......	0.408	9.30	9 210	12 810	4 995	5 930	1 715 000	

Maximum axial load carried,† in pounds.	Maximum computed stress, $\frac{P}{a} + \frac{My}{I}$, in pounds per square inch.	Maximum allowable‡ stress from Fig. 12, in pounds per square inch.	Difference in percentage.	Computed moment, 10 000 lb. axial load, in inch-pounds.	Observed moment, 10 000 lb. axial load, in inch-pounds.	Difference in percentage.	Maximum observed deflection.	Maximum observed moment, in inch-pounds.§	Maximum computed moment, in pounds.	Difference in percentage.
18 100	6 425	6 580	2.5	28 200	28 130	0.5	1.355	65 200	64 740	1.0
20 625	6 915	7 750	12.0	27 100	27 150	0.0	1.280	73 000	75 100	3.0
21 700	6 577	7 540	15.0	25 650	26 300	2.5	1.050	71 600	66 100	8.0
22 380	7 595	7 660	1.0	26 000	26 320	1.0	1.100	74 900	73 100	2.5
21 320	6 290	7 115	13.0	26 150	27 150	4.0	1.300	75 700	67 800	3.0
18 650	6 565	7 620	16.0	28 900	27 750	3.5	1.200	64 400	63 100	2.0

* The test, EI, was obtained by cross-bending over the full span of 96 in., the loads being applied at the points of application of the transverse loads.
† Two side loads, each 10% of the axial load, were applied at points 22½ in. from the ends. ‡ Values obtained from Fig. 12 are corrected for the strength properties of the beam under investigation. § The observed maximum moment is the primary moment plus the axial load times the observed deflection.

an axial load of 10 000 lb. and side loads of 1 000 lb., or 50% of the design loads for the spars. The observed moment under 10 000 lb. is obtained by taking the primary bending moment of 22 500 in-lb. plus a secondary moment equal to the axial load times the deflection observed during the test. The difference between the moments obtained by these two methods is less than 5% for the worst case which, considering the variations in the type and the stiffness of the sections used, is quite satisfactory. No attempt was made to adjust the ends of these six beams to neutralize eccentricities of material The spars were made under the conditions ordinarily found in airplane factories with, possibly, the little extra care usually given experimental specimens.

The last three columns of Table 7 compare the maximum moments computed by Equation (28) of Part I with the maximum occurring on the beam during the test, the latter being obtained in the same way as in the case of 50% of the design load. It is remarkable how closely the precise formulas agree with the actual conditions at maximum load. It was thought that for stresses beyond the elastic limit these formulas, which assume E to be constant, would give bending moments much below the actual. This is not the case, however, as shown by these six specimens; but it is realized that the data are so meager as to preclude forming any general conclusions. The indications are, though, that the precise formulas give results in sufficiently close accord with actual conditions to be used in the design of members subjected to combined loadings.

The discrepancy between the computed and allowable stresses given in Table 7 is, then, not due to any breaking down of the precise method for stresses beyond the elastic limit, but rather to the under-conservative values given by the Forest Products method. This discrepancy is, however, not so great as it appears at first sight for, with an 8 or 10% increase in both axial and lateral loads, the stresses in the specimen would be increased the 12 or 14% necessary to bring computed actual and computed allowable stresses into perfect agreement. While an error of 8 or 10% in a method of design is rather large, its actual effects are reduced in airplane work by the low properties assumed for standard spruce, because the material that passes Government inspection generally has higher properties than that used in design.

At all events, no method is available with any rational foundation, that is better than the Forest Products Laboratory method for computing allowable stresses under combined loadings, so the use of this method becomes necessary for the rational design of airplane wing-beams.

The range of usefulness of the precise method is being extended at present to the design of metal wing-beams. Such tests as have been made on experimental beams and on full-sized wing panels show very satisfactory results for beams having solid webs; that is, box and plate girder types. Trussed beams, or those having lightening holes in the web-plates, cannot be designed directly by this method due to the increased deflections and secondary moments resulting from the large shear deflections of the web system. By using an $E\,I$-value obtained from a test on a length of actual beam, or by computing the $E\,I$ of a beam that will give the same deflection under lateral load as is ob-

tained by the Williot diagram for a trussed beam, the precise method can be used to give a very good approximation to the actual stresses. It should be borne in mind, however, that the web system accounts for a considerable part of the deflection of a truss and the attempt should never be made to use the moment of inertia of the chord members alone in the precise formulas with the hope of obtaining the correct bending moments under combined loadings.

While the few tests available indicate the probability of a "form-factor" effect on metal beams, no satisfactory numerical values for it have been obtained and it is impossible, therefore, at present to design metal beams as closely as wooden ones. With conventional glued joints, a wooden beam acts as a single unit even if made of many pieces; but with the riveted, pinned, or even welded joints ordinarily used in metal construction, there is some give, deflections and secondary stresses increase, and it becomes extremely difficult to separate the effect of changes in form from that of changes in the method of connection. No data on form factors for metal beams, therefore, can be given. All that can be stated is that the shape of a section does appear to have a material effect on its strength properties so that, for the design of safe but light beams, the allowable stress values, for the present at least, must be determined by test.

It is because of this lack of knowledge as to allowable stress values in metal that, by the use of the methods described in Parts I and II, wooden wing-beams may be analyzed and designed to carry from 10 to 20% more load than metal beams of equal weight. As the strength of duralumin and highly heat-treated alloy steels (when used in the thin gauges necessary for lightness in airplane beams), becomes better understood, this margin in favor of wood will be reduced and probably eliminated; but from the standpoint of direct design and not trial-and-error development, the wooden wing-beam is at present considerably better than the best of the metal ones.

Acknowledgments

Besides the specific acknowledgments made in the body of this paper the author wishes to express his indebtedness to A. S. Niles, Jr., Assoc. M. Am. Soc. C. E., under whose direction as Chief of the Structures Branch of the Engineering Division, U. S. Army Air Corps, the work on the precise method was done. Mr. Niles assisted in the development of several of the formulas. To Lieut. H. A. Sutton, U. S. Army Air Corps, thanks are due for computing the tables showing the movement of points of inflection under various loads. The tests recorded in Part III were run with the co-operation of R. L. Hankinson, Chief of the Wood Branch of the Engineering Division, to whom the writer is greatly indebted for assistance in preparing specimens and testing apparatus, as well as in running the tests.

APPENDIX

TABLES OF FUNCTIONS.

TABLE 8.—FUNCTIONS FOR THREE-MOMENT EQUATIONS, α, β, AND γ FUNCTIONS FOR AXIAL COMPRESSION.*

$\frac{L}{j}$.	α.	$\Delta\alpha$.	β.	$\Delta\beta$.	γ.	$\Delta\gamma$.	$\frac{L}{j}$.
0	1.0000		1.0000		1.0000		0
0.5	1.0300		1.0171		1.0256		0.5
1.00	1.1304		1.0737		1.1118		1.00
		0.0151		0.0085		0.0128	
1.05	1.1455		1.0822		1.1241		1.05
		0.0162		0.0090		0.0138	
1.10	1.1617		1.0912		1.1379		1.10
		0.0175		0.0097		0.0148	
1.15	1.1792		1.1009		1.1527		1.15
		0.0187		0.0105		0.0159	
1.20	1.1979		1.1114		1.1686		1.20
		0.0201		0.0111		0.0170	
1.25	1.2180		1.1225		1.1856		1.25
		0.0216		0.0120		0.0183	
1.30	1.2396		1.1345		1.2039		1.30
		0.0232		0.0128		0.0196	
1.35	1.2628		1.1473		1.2235		1.35
		0.0250		0.0137		0.0210	
1.40	1.2878		1.1610		1.2445		1.40
		0.0268		0.0147		0.0226	
1.45	1.3146		1.1757		1.2671		1.45
		0.0288		0.0158		0.0243	
1.50	1.3434		1.1915		1.2914		1.50
		0.0310		0.0169		0.0260	
1.55	1.3744		1.2084		1.3174		1.55
		0.0334		0.0182		0.0281	
1.60	1.4078		1.2266		1.3455		1.60
		0.0361		0.0196		0.0303	
1.65	1.4439		1.2462		1.3758		1.65
		0.0391		0.0211		0.0327	
1.70	1.4830		1.2673		1.4085		1.70
		0.0422		0.0228		0.0353	
1.75	1.5252		1.2901		1.4438		1.75
		0.0458		0.0246		0.0383	
1.80	1.5710		1.3147		1.4821		1.80
		0.0498		0.0267		0.0416	
1.85	1.6208		1.3414		1.5237		1.85
		0.0542		0.0290		0.0452	
1.90	1.6750		1.3704		1.5689		1.90
		0.0593		0.0316		0.0493	
1.95	1.7343		1.4020		1.6182		1.95
		0.0650		0.0345		0.0540	
2.00	1.7993		1.4365		1.6722		2.00
		0.0137		0.0073		0.0114	
2.01	1.8130		1.4438		1.6836		2.01
		0.0140		0.0074		0.0117	
2.02	1.8270		1.4512		1.6953		2.02
		0.0143		0.0075		0.0118	
2.03	1.8413		1.4587		1.7071		2.03
		0.0146		0.0077		0.0121	
2.04	1.8559		1.4664		1.7192		2.04
		0.0147		0.0089		0.0122	
2.05	1.8706		1.4753		1.7314		2.05
		0.0151		0.0069		0.0126	
2.06	1.8857		1.4822		1.7440		2.06
		0.0155		0.0082		0.0128	
2.07	1.9012		1.4904		1.7568		2.07
		0.0156		0.0083		0.0130	

* From Part II, Appendix to "Airplane Design," by A. S. Niles, Assoc. M. Am. Soc. C. E., and others, U. S. Army Air Corps Publication, McCook Field, Dayton, Ohio. 1926. The plates from which these tables are printed have been furnished the Society through the courtesy of the U. S. Army Air Corps.

TABLE 8.—(*Continued*).

$\frac{L}{j}$.	α.	$\Delta\,\alpha$.	β.	$\Delta\,\beta$.	γ.	$\Delta\,\gamma$.	$\frac{L}{j}$.
2.08	1.9168	0.0161	1.4987	0.0084	1.7698	0.0134	2.08
2.09	1.9329	0.0164	1.5071	0.0087	1.7832	0.0135	2.09
2.10	1.9493	0.0169	1.5158	0.0088	1.7967	0.0139	2.10
2.11	1.9662	0.0169	1.5246	0.0090	1.8106	0.0141	2.11
2.12	1.9831	0.0174	1.5336	0.0091	1.8247	0.0145	2.12
2.13	2.0005	0.0180	1.5427	0.0094	1.8392	0.0147	2.13
2.14	2.0185	0.0181	1.5521	0.0095	1.8539	0.0150	2.14
2.15	2.0366	0.0186	1.5616	0.0097	1.8689	0.0154	2.15
2.16	2.0552	0.0189	1.5713	0.0100	1.8843	0.0157	2.16
2.17	2.0741	0.0194	1.5813	0.0101	1.9000	0.0160	2.17
2.18	2.0935	0.0198	1.5914	0.0104	1.9160	0.0163	2.18
2.19	2.1133	0.0203	1.6018	0.0106	1.9323	0.0168	2.19
2.20	2.1336	0.0207	1.6124	0.0109	1.9491	0.0172	2·20
2.21	2.1543	0.0211	1.6233	0.0110	1.9663	0.0174	2.21
2.22	2.1754	0.0218	1.6343	0.0114	1.9837	0.0179	2.22
2.23	2.1972	0.0222	1.6457	0.0115	2.0016	0.0183	2.23
2.24	2.2194	0.0228	1.6572	0.0118	2.0199	0.0187	2.24
2.25	2.2422	0.0232	1.6690	0.0122	2.0386	0.0192	2.25
2.26	2.2654	0.0237	1.6812	0.0124	2.0578	0.0197	2.26
2.27	2.2891	0.0244	1.6936	0.0126	2.0775	0.0201	2.27
2.28	2.3135	0.0249	1.7062	0.0130	2.0976	0.0205	2.28
2.29	2.3384	0.0256	1.7192	0.0133	2.1181	0.0211	2.29
2.30	2.3640	0.0262	1.7325	0.0136	2.1392	0.0216	2.30
2.31	2.3902	0.0269	1.7461	0.0140	2.1608	0.0222	2.31
2.32	2.4171	0.0277	1.7601	0.0143	2.1830	0.0227	2.32
2.33	2.4448	0.0283	1.7744	0.0147	2.2057	0.0233	2.33
2.34	2.4731	0.0291	1.7891	0.0150	2.2290	0.0239	2.34
2.35	2.5022	0.0298	1.8041	0.0154	2.2529	0.0245	2.35
2.36	2.5320	0.0305	1.8195	0.0159	2.2774	0.0251	2.36
2.37	2.5625	0.0314	1.8354	0.0162	2.3025	0.0259	2.37
2.38	2.5939	0.0323	1.8516	0.0167	2.3284	0.0266	2.38
2.39	2.6262	0.0334	1.8683	0.0171	2.3550	0.0272	2.39
2.40	2.6596	0.0339	1.8854	0.0177	2.3822	0.0281	2.40
2.41	2.6935	0.0352	1.9031	0.0181	2.4103	0.0288	2.41
2.42	2.7287	0.0362	1.9212	0.0186	2.4391	0.0296	2.42
2.43	2.7649	0.0372	1.9398	0.0191	2.4687	0.0306	2.43
2.44	2.8021	0.0382	1.9589	0.0197	2.4993	0.0313	2.44
2.45	2.8403	0.0395	1.9786	0.0203	2.5306	0.0324	2.45

TABLE 8.—(*Continued*).

$\frac{L}{j}$.	a.	Δa.	β.	$\Delta \beta$.	γ.	$\Delta \gamma$.	$\frac{L}{j}$.
2.46	2.8798	0.0406	1.9989	0.0209	2.5630	0.0334	2.46
2.47	2.9204	0.0420	2.0198	0.0215	2.5964	0.0343	2.47
2.48	2.9624	0.0432	2.0413	0.0222	2.6307	0.0355	2.48
2.49	3.0056	0.0446	2.0635	0.0229	2.6662	0.0365	2.49
2.50	3.0502	0.0461	2.0864	0.0236	2.7027	0.0378	2.50
2.51	3.0963	0.0475	2.1100	0.0243	2.7405	0.0389	2.51
2.52	3.1438	0.0493	2.1343	0.0252	2.7794	0.0403	2.52
2.53	3.1931	0.0506	2.1595	0.0260	2.8197	0.0415	2.53
2.54	3.2437	0.0526	2.1855	0.0269	2.8612	0.0431	2.54
2.55	3.2963	0.0545	2.2124	0.0278	2.9043	0.0445	2.55
2.56	3.3508	0.0564	2.2402	0.0288	2.9488	0.0461	2.56
2.57	3.4072	0.0585	2.2690	0.0298	2.9949	0.0478	2.57
2.58	3.4657	0.0605	2.2988	0.0309	3.0427	0.0495	2.58
2.59	3.5262	0.0628	2.3297	0.0321	3.0922	0.0513	2.59
2.60	3.5890	0.0652	2.3618	0.0332	3.1435	0.0533	2.60
2.61	3.6542	0.0678	2.3950	0.0345	3.1968	0.0554	2.61
2.62	3.7220	0.0705	2.4295	0.0359	3.2522	0.0575	2.62
2.63	3.7925	0.0734	2.4654	0.0373	3.3097	0.0599	2.63
2.64	3.8659	0.0762	2.5027	0.0388	3.3696	0.0623	2.64
2.65	3.9421	0.0797	2.5415	0.0404	3.4319	0.0650	2.65
2.66	4.0218	0.0829	2.5819	0.0422	3.4969	0.0677	2.66
2.67	4.1047	0.0867	2.6241	0.0439	3.5646	0.0707	2.67
2.68	4.1914	0.0906	2.6680	0.0460	3.6353	0.0739	2.68
2.69	4.2820	0.0946	2.7140	0.0479	3.7092	0.0771	2.69
2.70	4.3766	0.0991	2.7619	0.0502	3.7863	0.0808	2.70
2.71	4.4757	0.1038	2.8121	0.0527	3.8671	0.0846	2.71
2.72	4.5795	0.1090	2.8648	0.0551	3.9517	0.0888	2.72
2.73	4.6885	0.1144	2.9199	0.0579	4.0405	0.0932	2.73
2.74	4.8029	0.1204	2.9778	0.0608	4.1337	0.0980	2.74
2.75	4.9233	0.1266	3.0386	0.0641	4.2317	0.1032	2.75
2.76	5.0499	0.1336	3.1027	0.0675	4.3349	0.1087	2.76
2.77	5.1835	0.1410	3.1702	0.0712	4.4436	0.1148	2.77
2.78	5.3245	0.1491	3.2414	0.0752	4.5584	0.1213	2.78
2.79	5.4736	0.1579	3.3166	0.0797	4.6797	0.1285	2.79
2.80	5.6315	0.1675	3.3963	0.0844	4.8082	0.1362	2.80
2.81	5.7990	0.1780	3.4807	0.0897	4.9444	0.1448	2.81
2.82	5.9770	0 1894	3.5704	0.0955	5.0892	0.1540	2.82

TABLE 8.—(*Continued*).

$\frac{L}{j}$.	α.	$\Delta\,\alpha$.	β.	$\Delta\,\beta$.	γ.	$\Delta\,\gamma$.	$\frac{L}{j}$.
2.83	6.1664		3.6659		5.2432		2.83
		0.2021		0.1017		0.1643	
2.84	6.3685		3.7676		5.4075		2.84
		0.2160		0.1088		0.1757	
2.85	6.5845		3.8764		5.5832		2.85
		0.2315		0.1164		0.1881	
2.86	6.8160		3.9928		5.7713		2.86
		0.2486		0.1251		0.2020	
2.87	7.0646		4.1179		5.9733		2.87
		0.2676		0.1346		0.2174	
2.88	7.3322		4.2525		6.1907		2.88
		0.2890		0.1452		0.2348	
2.89	7.6212		4.3977		6.4255		2.89
		0.3131		0.1573		0.2543	
2.90	7.9343		4.5550		6.6798		2.90
		0.3402		0.1709		0.2763	
2.91	8.2745		4.7259		6.9561		2.91
		0.3710		0.1862		0.3012	
2.92	8.6455		4.9121		7.2573		2.92
		0.4061		0.2039		0.3298	
2.93	9.0516		5.1160		7.5871		2.93
		0.4466		0.2241		0.3625	
2.94	9.4982		5.3401		7.9496		2.94
		0.4933		0.2474		0.4004	
2.95	9.9915		5.5875		8.3500		2.95
		0.5478		0.2747		0.4446	
2.96	10.5393		5.8622		8.7946		2.96
		0.6117		0.3066		0.4964	
2.97	11.1510		6.1688		9.2910		2.97
		0.6876		0.3446		0.5579	
2.98	11.8386		6.5134		9.8489		2.98
		0.7785		0.3901		0.6315	
2.99	12.6171		6.9035		10.4804		2.99
		0.8886		0.4451		0.7209	
3.00	13.5057		7.3486		11.2013		3.00
		1.0238		0.5127		0.8304	
3.01	14.5295		7.8613		12.0317		3.01
		1.1924		0.5970		0.9671	
3.02	15.7219		8.4583		12.9988		3.02
		1.4063		0.7040		1.1405	
3.03	17.1282		9.1623		14.1393		3.03
		1.6834		0.8426		1.3651	
3.04	18.8116		10.0049		15.5044		3.04
		2.0513		1.0265		1.6633	
3.05	20.8629		11.0314		17.1677		3.05
		2.5547		1.2782		2.0711	
3.06	23.4176		12.3096		19.2388		3.06
		3.2684		1.6350		2.6498	
3.07	26.6860		13.9446		21.8886		3.07
		4.3300		2.1659		3.5103	
3.08	31.0160		16.1105		25.3989		3.08
		6.0084		3.0051		4.8712	
3.09	37.0244		19.1156		30.2701		3.09
		8.8990		4.4503		7.2188	
3.10	45.9234		23.5659		37.4839		3.10
		14.5332		7.2675		11.7808	
3.11	60.4566		30.8334		49.2647		3.11
		27.9956		13.9987		22.6930	
3.12	88.4522		44.8321		71.9577		3.12
		76.2965		38.1491		61.8440	
3.13	164.7487		82.9812		133.8017		3.13
		1 034.4142		517.2088		838.4545	
3.14	1 199.1629		600.1900		972.2562		3.14
		∞		∞		∞	
3.15	−227.1668		−112.9747		−183.8716		3.15
		123.4092		61.7055		100.0325	
3.16	−103.7576		− 51.2692		− 83.8391		3.16
		36.5228		18.2624		29.6049	
3.17	− 67.2348		− 33.0068		− 54.2342		3.17
		17.5035		8.7527		14.1884	
3.18	− 49.7313		− 24.2541		− 40.0458		3.18
		10.2713		5.1865		8.3263	
3.19	− 39.4600		− 19.1176		− 31.7195		3.19
		6.7537		3.3778		5.4750	
3.20	− 32.7063		− 15.7398		− 26.2445		3.20
		4.7787		2.3903		3.8742	

TABLE 8.—(*Continued*).

$\frac{L}{j}$.	a.	$\Delta\, a$.	β.	$\Delta\, \beta$.	γ.	$\Delta\, \gamma$.	$\frac{L}{j}$.
3.21	− 27.9276		− 13.3495		− 22.3703		3.21
		3.5593		1.7807		2.8858	
3.22	− 24.3683		− 11.5688		− 19.4855		3.22
		2.7541		1.3779		2.2330	
3.23	− 21.6142		− 10.1909		− 17.2515		3.23
		2.1940		1.0980		1.7790	
3.24	− 19.4202		− 9.0929		− 15.4725		3.24
		1.7890		0.8954		1.4507	
3.25	− 17.6312		− 8.1975		− 14.0218		3.25
		1.4865		0.7443		1.2057	
3.26	− 16.1447		− 7.4532		− 12.8161		3.26
		1.2548		0.6284		1.0178	
3.27	− 14.8899		− 6.8248		− 11.7983		3.27
		1.0733		0.5376		0.8707	
3.28	− 13.8166		− 6.2872		− 10.9276		3.28
		0.9285		0.4652		0.7533	
3.29	− 12.8881		− 5.8220		− 10.1743		3.29
		0.8111		0.4066		0.6581	
3.30	− 12.0770		− 5.4154		− 9.5162		3.30
		4.6522		2.3367		3.7784	
3.40	− 7.4248		− 3.0787		− 5.7378		3.40
		2.0479		1.0354		1.6681	
3.50	− 5.3769		− 2.0433		− 4.0697		3.50
		1.1477		0.5861		0.9389	
3.60	− 4.2292		− 1.4572		− 3.1308		3.60
		0.7302		0.3785		0.6016	
3.70	− 3.4990		− 1.0787		− 2.5292		3.70
		0.5029		0.2659		0.4179	
3.80	− 2.9961		− 0.8128		− 2.1113		3.80
		0.3647		0.1981		0.3070	
3.90	− 2.6314		− 0.6147		− 1.8043		3.90
		0.2744		0.1544		0.2349	
4.00	− 2.3570		− 0.4603		− 1.5694		4.00
		0.2116		0.1248		0.1854	
4.10	− 2.1454		− 0.3355		− 1.3840		4.10
		0.1662		0.1038		0.1498	
4.20	− 1.9792		− 0.2317		− 1.2342		4.20
		0.1317		0.0887		0.1237	
4.30	− 1.8475		− 0.1430		− 1.1105		4.30
		0.1046		0.0778		0.1036	
4.40	− 1.7429		− 0.0652		− 1.0069		4.40
		0.0826		0.0696		0.0881	
4.50	− 1.6603		0.0044		− 0.9188		4.50
		0.0641		0.0638		0.0757	
4.60	− 1.5962		0.0682		− 0.8431		4.60
		0.0810		0.1169		0.1235	
4.80	− 1.5152		0.1851		− 0.7196		4.80
		0.0238		0.1124		0.0962	
5.00	− 1.4914		0.2975		− 0.6234		5.00
		0.0568		0.1520		0.0938	
5.25	− 1.5482		0.4495		− 0.5296		5.25
		0.1964		0.1975		0.0733	
5.5	− 1.7446		0.6470		− 0.4563		5.5
		0.4898		0.3277		0.0589	
5.75	− 2.2344		0.9747		− 0.3974		5.75
		1.5111		0.8268		0.0482	
6.0	− 3.7455		1.8015		− 0.3492		6.0
		25.3412		12.7331		0.0404	
6.25	− 29.0867		14.5346		− 0.3088		6.25
		∞		∞		0.0048	
2π	±∞		±∞		− 0.3040		2π
		∞		∞		0.0295	
6.5	4.1490		− 2.0242		− 0.2745		6.5

TABLE 9.—FUNCTIONS FOR THREE-MOMENT EQUATIONS, α_h, β_h, AND γ_h FUNCTIONS FOR AXIAL TENSION.

$\frac{L}{j}$.	α_h.	$\Delta\ \alpha_h$.	β_h.	$\Delta\ \beta_h$.	γ_h.	$\Delta\ \gamma_h$.	$\frac{L}{j}$.
0.00	1.0000		1.0000		1.0000		0.00
		0.0284		0.0163		0.0244	
0.50	0.9716		0.9837		0.9756		0.50
		0.0771		0.0446		0.0664	
1.00	0.8945		0.9391		0.9092		1.00
		0.0097		0.0057		0.0083	
1.05	0.8848		0.9334		0.9009		1.05
		0.0100		0.0058		0.0087	
1.10	0.8748		0.9276		0.8922		1.10
		0.0101		0.0060		0.0089	
1.15	0.8647		0.9216		0.8833		1.15
		0.0105		0.0061		0.0090	
1.20	0.8542		0.9155		0.8743		1.20
		0.0106		0.0062		0.0092	
1.25	0.8436		0.9093		0.8651		1.25
		0.0108		0.0065		0.0094	
1.30	0.8328		0.9028		0.8557		1.30
		0.0110		0.0065		0.0096	
1.35	0.8218		0.8963		0.8461		1.35
		0.0111		0.0066		0.0097	
1.40	0.8107		0.8897		0.8364		1.40
		0.0113		0.0067		0.0098	
1.45	0.7994		0.8830		0.8266		1.45
		0.0113		0.0068		0.0099	
1.50	0.7881		0.8762		0.8167		1.50
		0.0114		0.0068		0.0100	
1.55	0.7767		0.8694		0.8067		1.55
		0.0115		0.0069		0.0100	
1.60	0.7652		0.8625		0.7967		1.60
		0.0115		0.0070		0.0100	
1.65	0.7537		0.8555		0.7867		1.65
		0.0116		0.0070		0.0101	
1.70	0.7421		0.7485		0.7766		1.70
		0.0116		0.0070		0.0102	
1.75	0.7305		0.8415		0.7664		1.75
		0.0116		0.0071		0.0104	
1.80	0.7189		0.8344		0.7560		1.80
		0.0116		0.0071		0.0103	
1.85	0.7073		0.8273		0.7457		1.85
		0.0115		0.0071		0.0102	
1.90	0.6958		0.8202		0.7355		1.90
		0.0115		0.0071		0.0102	
1.95	0.6843		0.8131		0.7253		1.95
		0.0115		0.0071		0.0101	
2.00	0.6728		0.8060		0.7152		2.00
		0.0114		0.0071		0.0101	
2.05	0.6614		0.7989		0.7051		2.05
		0.0113		0.0071		0.0101	
2.10	0.6501		0.7918		0.6950		2.10
		0.0112		0.0071		0.0100	
2.15	0.6389		0.7847		0.6850		2.15
		0.0111		0.0070		0.0100	
2.20	0.6278		0.7777		0.6750		2.20
		0.0111		0.0070		0.0098	
2.25	0.6167		0.7707		0.6652		2.25
		0.0109		0.0070		0.0097	
2.30	0.6058		0.7637		0.6555		2.30
		0.0108		0.0069		0.0098	
2.35	0.5950		0.7568		0.6457		2.35
		0.0107		0.0069		0.0097	
2.40	0.5843		0.7499		0.6360		2.40
		0.0106		0.0069		0.0095	
2.45	0.5737		0.7430		0.6265		2.45
		0.0104		0.0068		0.0095	
2.50	0.5633		0.7362		0.6170		2.50
		0.0103		0.0067		0.0093	
2.55	0.5530		0.7295		0.6077		2.55
		0.0101		0.0067		0.0092	
2.60	0.5429		0.7228		0.5985		2.60
		0.0100		0.0066		0.0092	
2.65	0.5329		0.7162		0.5893		2.65
		0.0099		0 0065		0.0090	

TABLE 9.—(*Continued*).

$\frac{L}{j}$.	α_h.	$\Delta\,\alpha_h$.	β_h.	$\Delta\,\beta_h$.	γ_h.	$\Delta\,\gamma_h$.	$\frac{L}{j}$.
2.70	0.5230	0.0097	0.7097	0.0065	0.5803	0.0088	2.70
2.75	0.5133	0.0096	0.7032	0.0065	0.5715	0.0088	2.75
2.80	0.5037	0.0094	0.6967	0.0064	0.5627	0.0085	2.80
2.85	0.4943	0.0092	0.6903	0.0063	0.5542	0.0085	2.85
2.90	0.4851	0.0091	0.6840	0.0062	0.5457	0.0085	2.90
2.95	0.4760	0.0090	0.6778	0.0062	0.5372	0.0084	2.95
3.00	0.4670	0.0087	0.6716	0.0061	0.5288	0.0083	3.00
3.05	0.4583	0.0087	0.6655	0.0060	0.5205	0.0080	3.05
3.10	0.4496	0.0085	0.6595	0.0059	0.5125	0.0080	3.10
3.15	0.4411	0.0083	0.6536	0.0060	0.5045	0.0077	3.15
3.20	0.4328		0.6476		0.4968		3.20

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CHEMICAL-ELECTRIC MEASUREMENT OF WATER*

By A. Barbagelata,† Esq.

Synopsis

One accurate method of measuring flow, called the "salt solution method", depends on the dilution of a salt liquid introduced at a constant rate. This dilution may be measured chemically by titration, but better, perhaps, by its electrical conductivity.

The change in conductivity between the normal and the salt water is found by balancing a Wheatstone bridge circuit, previously calibrated. Difficulties due to air bubbles in the water, and to irregular distribution of the solution, may be overcome.

In its final development this method requires a knowledge only of the total amount and intensity of salt solution introduced, and of the time required for this mixing. Readings of the conductivity are then plotted, the temperature of the water is determined, and the constants for the apparatus are found, from which the flow is readily computed.

The results obtained from this method check closely those found by current meter or bulk measurement. Its simplicity, cheapness, application to varied conditions, and rapidity are points in its favor.

The Chemical (Titration) Method

The chemical or salt titration method for measuring the discharge of water in rivers or canals, has long been known.

If at a given point of a stream, having a discharge of Q liters per sec., a solution containing C_1 grammes of salt (sodium chloride, NaCl) per liter, is introduced at the constant rate of q_c liters per sec.; and if at another given

Note.—Written discussion on this paper will be closed in **August, 1928.**

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point, located down stream, at a sufficient distance for ensuring the thorough mixing of the salt solution with the flowing water, the salt content, C_2 (grammes per liter), is ascertained, the unknown water discharge, Q, can be calculated by the equation,

$$\frac{Q + q_c}{q_c} = \frac{C_1}{C_2}$$

giving,

$$Q = q_c \frac{C_1 - C_2}{C_2}$$

which, for practical use, can be simplified to read,

$$Q = q_c \frac{C_1}{C_2}$$

considering that the salt content, C_2, is very small in comparison with C_1 or, in other words, that the quantity, q_c, of the solution is very small in comparison with the discharge, Q, of the stream.

Discharges as high as 8 or 10 cu. m. per sec. can be measured by introducing 2 or 3 liters per sec. of a 25 to 30% salt solution. As a means of saving time and salt, a few preliminary experiments are advisable to determine the necessary time for the solution to travel from the point of introduction to the sampling station. Some kind of a coloring matter is generally used for this purpose.

After the water samples are taken at the proper time and with due care, the salt content is determined by chemical titration, although other means may be used. In general, the method appears to be rather elaborate and likely to be affected by important sources of uncertainty.

Gauging Salt Content by Electrical Measurements

A considerable improvement, especially in the speed of operation, was found to be possible when the salt content of the water was determined by measuring its electrical conductivity.

It is well known that the electrical conductivity of very diluted salt solutions is proportional, within a wide range, to the salt content. This is shown graphically in Fig. 1. It is possible, therefore, to determine the concentration of a solution by measuring its electrical resistance, provided the temperature is taken into careful consideration, as this factor has an important effect on the conductivity.

Probably this improvement was suggested ten years ago or more. The writer first gathered his information from a paper* by Mr. W. D. Peaslee. The arrangement then suggested (Fig. 2*) was to place the solution in a U-shaped glass tube, *A-B*, and measure the resistance by an ohmmeter, *C*, using direct current from the generator, *D*.

Conductivity by Alternating Current and a Wheatstone Bridge

Having been engaged to test several water turbines with considerable discharges and having decided to try the salt method, the writer at once departed

* *General Electric Review*, February, 1916, p. 132; "The Saline Method of Water Flow Measurement as Used in the Acceptance Test of a Pumping Plant," see, also, discussion by Mr. Peaslee of the paper entitled "Chemi-Hydrometry," by B. F. Groat, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1277.

from current practice by using a Wheatstone bridge and alternating current of industrial frequency, in connection with a set of floating electrodes. This method of measuring the resistance of the water he had already come to appreciate during experiments made for other purposes. By using alternating current of ordinary frequency and a rather low current density at the electrodes, all the difficulties due to polarization are eliminated, together with all the limitations peculiar to the customary methods using telephone currents of small power.

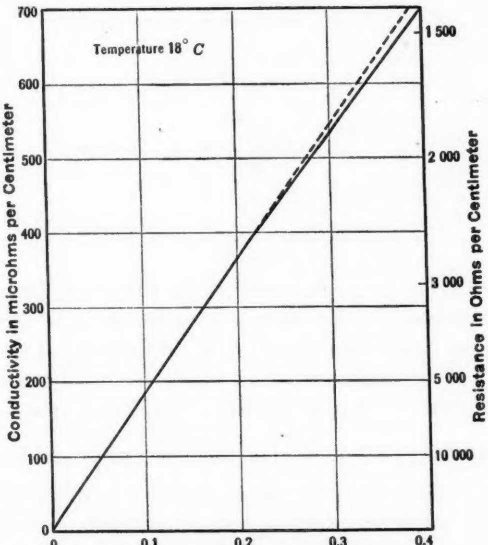

FIG. 1.

A potential of a few dozen volts gives ample sensitiveness, so that this method appears to be unrivaled for measuring any liquid resistance. Of course, the telephone, in connection with a suitable generator of telephone frequency current, is to be adopted in case industrial frequency current is not available.

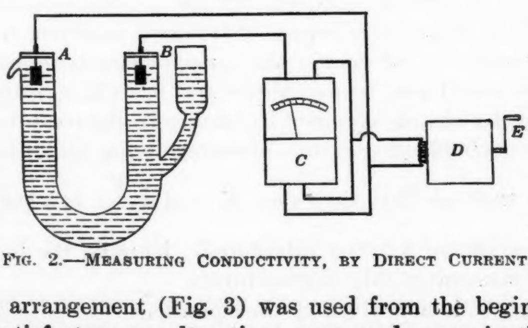

FIG. 2.—MEASURING CONDUCTIVITY, BY DIRECT CURRENT.

The bridge arrangement (Fig. 3) was used from the beginning and as it proved quite satisfactory no alterations were made even in the latest development. The bridge is at first balanced for the natural conductivity of the

water by giving to Branch a a fixed value of 1 000 ohms; to Branch b an arbitrary value, from 1 000 to 5 000 ohms, according to the kind of electrodes and the natural conductivity of the water; and by adjusting Branch c until the electro-dynamometer, e, reads zero. Under these circumstances the electro-dynamometer, having fixed coils excited by the same voltage feeding the bridge, works as a galvanometer.

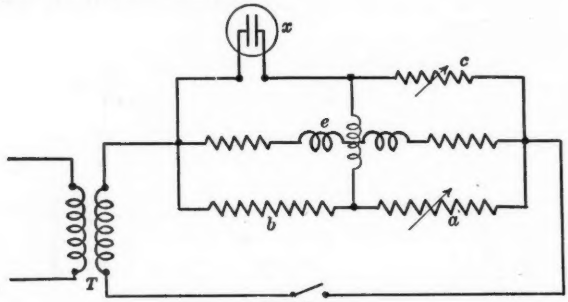

FIG. 3.

When the diluted salt solution arrives at the electrodes and the balance of the bridge is broken, the resistance of Branch a is increased until the electro-dynamometer again reads zero. The difference, $A - 1\,000 = \Delta\, a$, is directly proportional to the increase of conductivity due to the presence of salt, that is, to the salt content in the unit volume of water.

Determining the Constant of the Equipment by Calibration

This procedure gave a reliable equipment for measuring the salt content in the stream. At the same time, another step was taken toward simplicity and accuracy by adopting a new procedure for determining, by a sort of "calibration", a constant of the equipment whereby the discharge, Q, can be readily calculated from the value of $\Delta\, a$.

Having placed the same set of electrodes in a tank containing V liters of natural water, taken from the stream, and having balanced the bridge by assuming that $a = 1\,000$ (obviously, the temperature and the natural conductivity being the same, b and c have the same values as before), small known quantities of the salt solution already used are gradually introduced into the tank and carefully mixed by stirring. After each addition, the value of $\Delta\, a$ corresponding to the zero reading of the dynamometer is recorded.

Of course, it would also be possible to add by trial a volume of solution giving the same $\Delta\, a$ already obtained by testing in the conduit, and thus find the discharge by a simple proportion. However, it was found more convenient to calculate for each addition the value, $K = \Delta\, a \dfrac{V}{v}$, in which, v is, in each case, the total volume of solution introduced. Keeping the temperature constant, K also is constant within narrow limits.

The discharge, Q, is now given by the equation,

$$Q = q_c \frac{K}{\Delta\, a}$$

q_c being the volume of solution introduced in the stream in the unit of time, and Δa, the reading at the Wheatstone bridge from the test in the conduit itself.

By this "calibration"—a sort of "method of equal deviation"—the dosing of the concentrated solution is not necessary. Many factors which otherwise ought to be taken into account, can be neglected. In addition, many sources of uncertainty are eliminated, as, for instance, the possible interference, or reactions, of the salts contained in natural water, with the salt introduced.

Analyzing Sources of Uncertainty—Air Bubbles in the Water

The writer's attempts to measure water by the salt method began in 1919 and was continued at infrequent intervals. Many of these attempts met with considerable difficulty and resulted in failures. Some of the sources of trouble were of local character and need not be discussed. Other sources of uncertainty, connected with details of the method followed in each test, can be summarized as follows:

1.—Presence of air bubbles in the stream;
2.—Irregular distribution of the solution as to the total volume of flow (space);
3.—Irregular distribution of the solution as to time.

Often, in case the sampling station is located in the tail-race of a power plant, troubles are likely to arise because of air being carried by the water in considerable quantity, in the form of bubbles of various sizes. These bubbles, by passing between the electrodes, not only disturb the reading, but are likely to affect the results materially, giving a lower apparent conductivity and, consequently, a discharge greater than the true one.

Many different floating or fixed electrode arrangements have been designed with the idea of freeing the water from air bubbles before it arrives at the electrodes. The final arrangements adopted will be shown subsequently.

Distribution of the Solution in Space

The irregular distribution of the solution in space or, more exactly, in the cross-section of the conduit, is nothing else than the imperfect mixing of the salt solution with the flowing water.

Relying on the information taken from an American source, to the effect that passage through a turbine would be sufficient to secure a perfect mixing, the writer met at first with utter failure. Finally, he was convinced by actual tests that, even after running through a pipe line, 70 m. long, a turbine, and about 10 m. of turbulent tail-race, the solution, introduced in a single jet, had not yet become thoroughly mixed. Differences were found of as much as 10% in concentration at different points of the sampling cross-section.

The best method of overcoming this difficulty is to introduce the solution as uniformly as possible over the whole area, for instance, by using one or more perforated pipes as shown in Fig. 4, and, also, to take samples in many different points of the cross-section, in order to obtain an average of the conductivity over the whole area.

Longitudinal Distribution

To overcome the difficulties due to the irregular distribution in point of time has been a greater problem. As the results obtained are to be considered the most interesting novelty in connection with the writer's work, this point deserves a more detailed discussion.

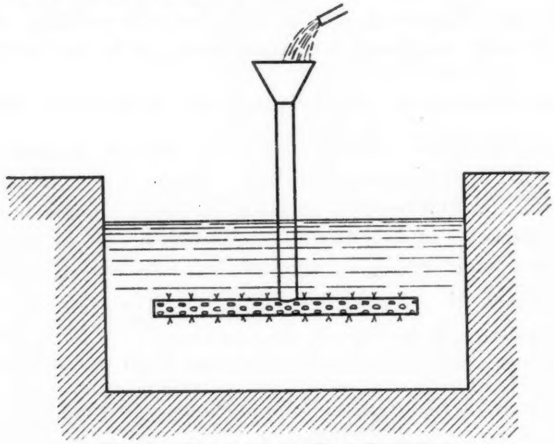

Fig. 4.—Diffusion of Salt Solution.

Consider a canal having a regular section and a constant discharge, Q, and imagine a salt solution to be introduced during t min. at the constant rate, q_c. Assuming that an instantaneous and perfect mixing of the solution in the flowing water would be possible, the salt wave in the stream would have a vertical front, that is, with rectangular longitudinal section, so that, at the dosing station, the conductivity of the water would suddenly increase from the original condition to the highest value corresponding to the presence of salt. It would have this highest value for a time, t, equal to that during which the solution has been introduced and then would abruptly decrease to the natural value. The readings of Δa at the bridge, plotted against time in a diagram, would give a rectangle as the real image of the salt wave.

It is obvious that, owing to the small diffusion velocity of the salt solution and the unequal velocity of the water at different points of the cross-section, even in the ideal case just considered, the wave fronts would not be vertical planes, but, instead, more or less distorted surfaces.

In case, between the point where salt is being introduced and the sampling station, the canal or conduit shows important changes in its cross-section, such as enlargements, basins, etc., the salt wave or its equivalent curve, $\Delta a = f(t)$, will be distorted, owing to the lingering of part of the solution in the enlarged spots.

The phenomenon being similar to that of an electric conductor having capacities in parallel, an exponential profile of the wave front is to be expected (Lines BB in Fig. 5), an assumption fully confirmed by actual tests.

Obviously, the value of Δa is to be taken after the transient conditions have become steady in order not to derive too low a concentration, giving a

greater discharge than the true one. Where the irregularities in the cross-section of the conduit are very important, or when the salt solution has been introduced during too short a time, it may happen that the conductivity at the dosing station begins to fall before it has attained the value correspond-

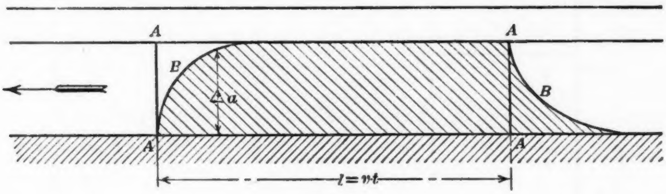

FIG. 5.

ingly to steady conditions, as shown by Fig. 6, so that, by deducting the discharge from the maximum $\Delta\ a$ observed, a considerably greater volume than the true one would be found. The failures of the first attempts probably were due mostly to this fact not having been apparent and not having been considered.

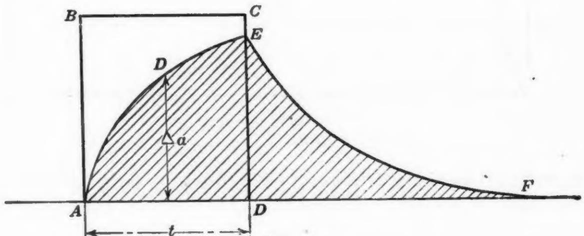

FIG. 6.

As soon as the source of the trouble was detected, the remedy became apparent. It is evident that the area enclosed by the curve, *A D E F*, in Fig. 6, must be equal to the area of the ideal rectangular wave, *A B C D*, having as its abscissa, *t* (the time during which the solution has been introduced), so that the correct value of the concentration to be used in subsequent calculations will be found simply by dividing the area of the curve, *A D E F*, by the time, *t*.

In other words the value, $\Delta'\ a = \frac{1}{t}\ \Delta\ a\ .\ d\,t$, is that which should have been observed at the bridge if the time of dosing, *t*, had been long enough. Fig. 7 shows a curve plotted from an actual test in a conduit in which the influence of the retarding factors was particularly important. This shows how much the maximum ordinate of the plotted wave may differ from that to be taken for an exact calculation of the discharge. Fig. 8 shows, on the contrary, the curve plotted for a conduit in which the influence of the disturbing factors was slight, so that both ordinates coincide.

Final Development of the Method

The main difficulties having thus been overcome, an important principle a recording instrument giving directly the profile of the salt wave. However,

constant rate, but only to know the total volume, Q_s, of solution introduced in any manner whatever into the stream.

This can be demonstrated by considering that the values of $\varDelta a$, being continuously recorded, are proportional to the concentration of the water passing,

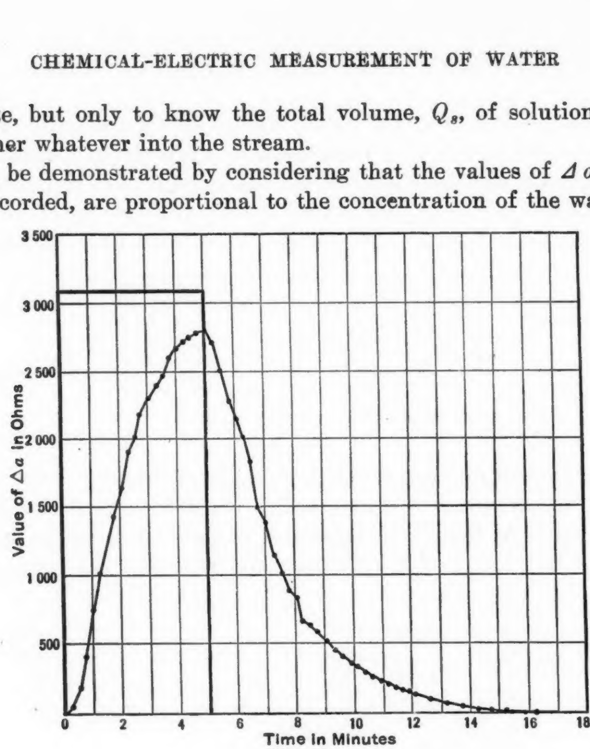

FIG. 7.

at the corresponding instant, through the dosing section according to the equation,

$$\varDelta a = K \frac{q}{Q}$$

q being the quantity of concentrated solution carried along by the constant discharge, Q. The area enclosed by the curve, $\varDelta a = f(t)$, covering the whole time, T, of the wave, can be expressed by,

$$A = \int_0^T \varDelta a \, . \, dt = \frac{K}{Q} \int_0^T q \, dt$$

The integral, $\int_0^T q \, dt$, however, is nothing else than the total quantity of concentrated solution introduced into the stream, so that it follows,

$$A = \frac{K}{Q} Q_s$$

and, consequently,

$$Q = K \frac{Q_s}{A}$$

in which, K is the constant to be determined by the "calibration" already described.

The procedure which now has but little resemblance to the original salt titration method, is very simple and practical in use. For each test a volume, Q_s, of concentrated solution is to be prepared and introduced at the right time

into the stream. Obviously, it is convenient to introduce it uniformly, during a predetermined length of time, in order to obtain the best conditions for plotting the profile of the salt wave, but without any special care for securing a constant rate of introduction. A hand pump may be used for this purpose. However, the injection by two men taking the solution from a tank with buckets and pouring it alternately into the stream is quite satisfactory.

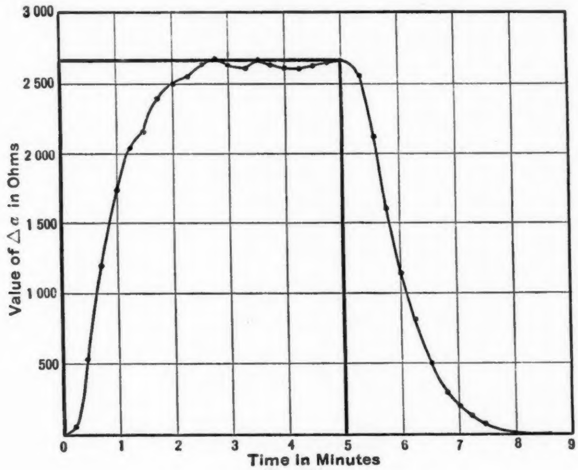

FIG. 8.

At the sampling station the conductivity of the water is continuously checked and the curve, $\Delta a = f(t)$, is plotted. The "calibration" for determining the constant, K, is then made, using a tank of known content, full of natural water and a small sample of the concentrated solution, care being taken to record every change in temperature for the necessary corrections. Having measured the area, A, of the curve, $\Delta a = f(t)$, the discharge of the stream, $Q = K \frac{Q_s}{A}$, is immediately found.

EQUIPMENT FOR INTRODUCING THE SALT SOLUTION

Thus, no special care is required in introducing the salt solution into the stream, with the exception of the arrangements best suited for securing a thorough mixing. However, Fig. 9 shows an equipment giving constant discharge, which was devised for the first tests and which the writer still uses, as it gives a good element of control.

The arrangement involves no new ideas and the diagram is self-explanatory. The salt solution, contained in the upper tank, is delivered to the tank, V, at the bottom of which nozzles of various sizes can be adjusted, R being a cock for stopping the flow. This cock being open, the salt solution is admitted to Tank V in such a quantity as to fill it or to give a slight overflow at the top; the discharge at the nozzle follows under the constant head, H. The overflowing solution is collected by the outside tank, U, and stored in an auxiliary tank. During the short time necessary for attaining conditions of constant

discharge, the pipe, T, is diverted to the auxiliary tank. The quantity, q_c, being the constant discharge of the apparatus (in liters per second) and t, the introduction time (in minutes), the total quantity, Q_s, of solution introduced into the stream will obviously be, $q_s = 60\ q_0\ t$.

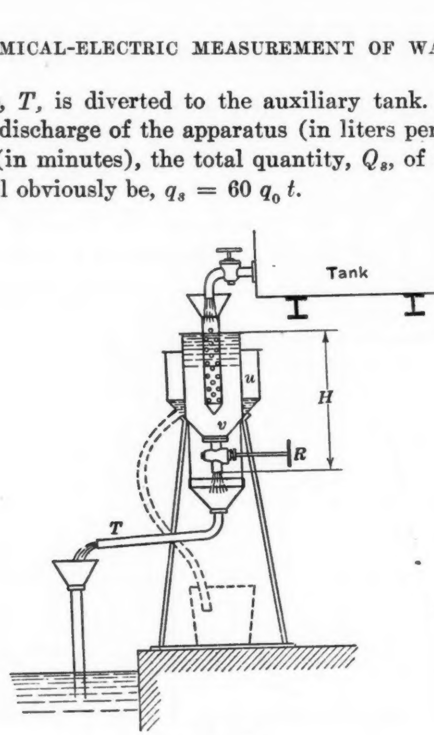

FIG. 9.

EQUIPMENT FOR MEASURING THE WATER RESISTANCE

In Fig. 10 is shown the final arrangement for measuring the conductivity of the water at the sampling station. The various floating electrodes having been discarded, the conductivity was measured in a water circuit taken from the main stream. Water is taken from the conduit by the hand pump, P, and circulated to a pair of fixed electrodes, E. The cock, a, being open and Cock b being closed, the flexible suction pipe, T, is moved continuously, so as to take water from various points over the whole area and secure a fair average of the concentration in case of imperfect mixing.

The water is first delivered to an upper open tank, V, in which air bubbles may easily escape and leaves or other solid matter may be screened by a sieve, g. Thence it flows to the cylindrical vessel containing the electrodes and discharges again into the canal by means of the pipe, t_s. In order to secure a correct profile of the wave, the pump must be operated with sufficient regularity to maintain in Tank V a fairly steady water level.

After plotting the salt wave, the discharge pipe, t_s, is shifted to the tank, S, and the necessary volume of natural water for the "calibration" is collected, due account being taken of the water volume contained in the pump and pipes. After having closed Cock a and opened Cock b, the same hand-pump, P, continuously circulates between the electrodes the water contained in the tank, S, into which, by means of a burette, p, small quantities of the concentrated solution are gradually introduced. All operations are thus conducted very quickly and easily.

By means of a thermometer, t, immersed in the tank, V, the temperature of the water is continuously observed. During the plotting of the salt wave, the water being renewed constantly, the temperature does not change appreciably, but during the calibration, when the same volume of water is circulated from 5 to 6 min., appreciable changes of temperature are likely to occur, especially if there is a difference between the temperature of the water and that of the surrounding air. The writer's assistant, Mr. Bottani, has theoretically analyzed all the particulars of the process, especially the influence of the temperature variations. There is no question as to the great importance of temperature corrections on the accuracy of the results.

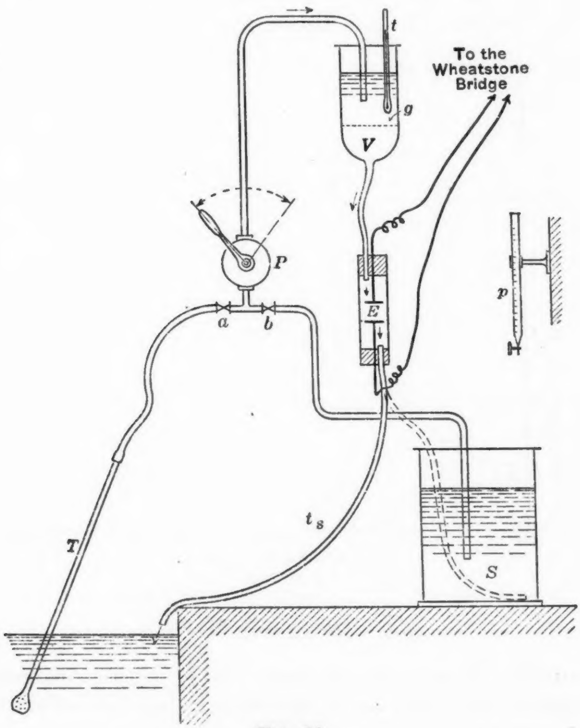

FIG. 10.

The ordinary arrangement of the apparatus for using alternating current has already been shown in Fig. 3, including a small insulating transformer, T. This transformer has the double purpose of preventing any direct or indirect grounding of the line from which the current is taken and also of preventing stray currents, affecting the accuracy of the measurement, from circulating in the instrument. On these principles a complete, portable instrument has been built, having a great sensitiveness and enabling any change in conductivity to be followed easily. It would be quite possible to design an apparatus giving the values of a by direct reading, instead of by reducing it to zero, or even a recording instrument giving directly the profile of the salt wave. However,

the method adopted is quite satisfactory and is unequalled in regard to accuracy.

Experimental Results

In Fig. 11 is shown the discharge curve of a low-head turbine of about 8 cu. m. per sec. The points marked with crosses were obtained from measurements by a calibrated current meter, controlled on the spot by a second apparatus; the points marked with small circles were obtained by the salt method herein described. The agreement is quite satisfactory. Fig. 12 shows the discharge curve of a high-head plant from which measurements have been made by the bulk method, using the forebay as a reservoir. This likewise shows close agreement with the electrical salt method.

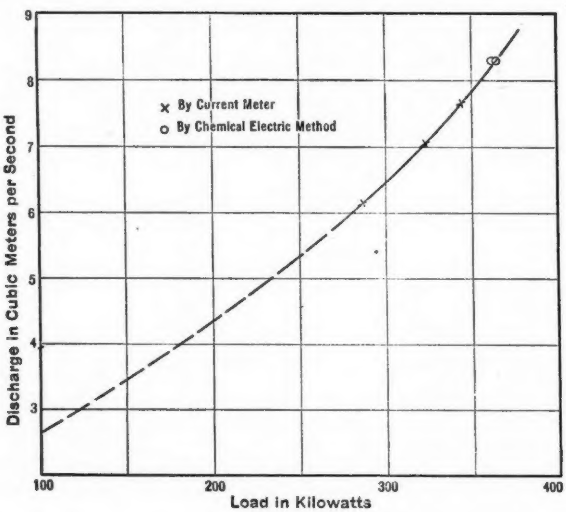

Fig. 11.

Accuracy of the Method

The presumable degree of accuracy can be better estimated from Mr. Bottani's analysis of the sources of uncertainty than from the few tests made. From this analysis an accuracy equal to, or even better than, that of any other method for measuring great discharges may be expected.

Furthermore, this method is not affected by personal factors or coefficients to be predetermined, each test being a self-contained cycle of operations. Thus, systematic errors influencing a whole series of measurements are not to be feared.

Comparative Advantages

Only a few methods are available for measuring large water discharges. In Europe, only the current meter is in practical use. Within recent years two new methods have been developed in America, namely, the Gibson (pressure-time) method and the salt-velocity method. A few simple con-

siderations suffice to demonstrate that the method herein described has many points of advantage in comparison with any one of the others named.

All engineers familiar with current-meter measurements are well aware of the difficulties peculiar to the method. A sufficient length of very regular canal and a constant flow for a comparatively long time are necessary. A bridge across the canal and a very exact sounding of the cross-section are also required. The plotting of the velocity curves is painfully slow, and considerable work is required in subsequent planimetering and computing. The accuracy depends largely on the calibration of the instrument, as well as on many other factors which are not under the operator's control. Under ordinary conditions, two complete current-meter tests in one day may be considered as excellent work.

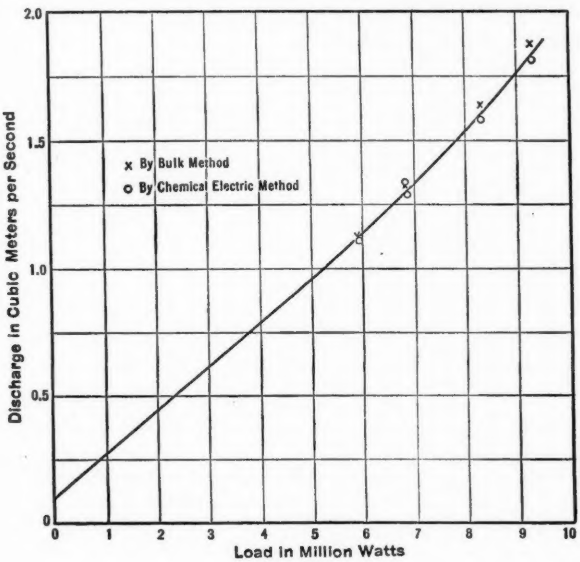

FIG. 12.

The Gibson method is applicable only to plants affording a sufficient length of pipe line or penstock. Special recording instruments are necessary; also considerable care and experience must be exerted in interpreting the pressure-time diagram.

The salt-velocity method, which seems to be now extensively used in America, requires an equipment for injecting the brine under pressure; a special recording instrument, together with a carefully designed set of electrodes; and a careful measurement of the volume of the conduit. This latter is not a simple matter in the case of a conduit with an irregular shape.

The chemical electric method herein described requires only simple equipment, either for introducing the brine or for measuring the conductivity. It is applicable to every water stream, river, canal, or closed conduit. A test can be made in a short time so that it is sufficient if the flow remains constant during only a few minutes. The total time required for a test, including the

so-called calibration, is not more than 30 min. Tests can be made on a running water-power plant without disturbing in any way the regular operation of the plant itself.

Another important advantage is that the method is inexpensive. The quantity of salt required for each test depends largely on the natural conductivity of the water. In the most unfavorable conditions of highest natural conductivity, 400 kg. of salt will be sufficient for measuring discharges to 10 cu. m. per sec. Based on the actual price of salt in Italy (the sale of which is a State monopoly), the expense for each test is less than 100 lire, a mere trifle considering the enormous saving in time and engineering work.

It is hoped that all hydraulic engineers will be helped by this description of water measurement by means of a salt solution. This development seems to be successful in giving a real contribution, however small, to the development of hydraulic engineering practice.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

PAPERS AND DISCUSSIONS

PROBABILITY OF FLOOD FLOWS

Discussion*

By F. G. Switzer, Assoc. M. Am. Soc. C. E.†

F. G. Switzer,‡ Assoc. M. Am. Soc. C. E. (by letter).§—The writer wishes to thank those who have participated in the discussion of his paper, and in particular Mr. Hazen for his interest as evidenced by considerable correspondence.

To answer some of the criticisms of the application of the theory of errors to this problem, the writer asked Professor W. A. Hurwitz, of the Department of Mathematics at Cornell University, to pass his judgment on this phase of the problem. He replies as follows:

"Several of your critics have expressed skepticism about the value of the theory of errors in dealing with your data at all. The objections are of two exactly opposite kinds, and hence rest on very different grounds. We use these methods in the first place because we are ignorant of the underlying causes producing the phenomena we wish to study, and because we hope that the measurements of these causes are distributed in a sufficiently accidental way to justify their being regarded as due to chance. It is therefore correct to say that results based on a twenty-year observation can not with any show of reason be made the basis of prediction for a single specified much shorter period. However, application can be made to average or 'long run' predictions over a new twenty-year period, provided the assumption of distribution of causes by chance is approximately correct. Whether this assumption is correct, nothing but experience can determine. In the enormous majority of problems which do arise in other fields, it is correct. On wholly different grounds rests the statement that twenty-year observations can not be made the basis of a ten thousand-year prediction. This objection has absolutely nothing to do with the application of statistical methods. The objection would have as much or as little bearing if we knew very precisely the laws governing

* Discussion of the paper by F. G. Switzer, Assoc. M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Author's closure.

‡ Cons. Engr., and Prof. of Hydr. Eng., Sibley School of Mech. Eng., Coll. of Eng., Cornell Univ., Ithaca, N. Y.

§ Received by the Secretary, December 20, 1927.

the underlying causes of floods. For the question at issue here is whether these laws themselves undergo change and what kind of change. Of course, you are not trying to assert that your tables and curves will tell an approximately true story for a time when the Coosa River in Alabama is submerged under a glacier. It is much like the fallacy that naming a speed of 60 miles an hour for a train implies that the train must run for an hour."

The writer likes to think of these long periods of time in much the same way that a designer thinks of his so-called factor of safety.

As for the use of what Mr. Creager* calls the "basic stage method", Professor Hurwitz has this to say:

"Let me now come to such practical opinions as I have about the treatment of your data. It seems to me that the lack of knowledge of underlying causes makes it desirable to take as extensive a range of the known factors as possible into consideration. Suppose you have agreed to disregard all observations below a stated measurement. Then a period which happened to contain a number of observations a little below this limit would not contribute at all to your result, while in fact, it might be more effective than a period containing just one observation slightly above the selected limit. That is to say, very tiny changes in the underlying causes might have given several utilized measurements in the one case, and none in the other. My opinion is therefore, that if the labor is not too great, it would be well, at least in some cases to make calculations on the basis of all the observations on hand, large or small. If this method is too onerous or not sufficiently promising, I should think the next best would be to use a 'basic flow' much lower than what you would consider essential to prediction of danger points; the reason for this of course is to include the effect of numbers of near-floods as a basis of prediction of actual floods.

"While I must speak hesitatingly in view of my theoretical rather than practical trend, I feel very dubious about the reliability of the annual peak method. This is somewhat borne out by the results you get from it. In spite of seasonal variations the average result over a sufficiently extended period of time, ought, if the interpretation is a satisfactory one, to depend very little on the choice of the dates for beginning and ending a year. Whatever occurrences are cut off the beginning of one year, ought, in the long run, to be found at the end of another year."

Further work has shown in two cases that the basic-stage method gives results practically identical with those obtained by the use of all available daily discharge records. Also, in these cases, a change in the value of the basic flow has made no substantial change in the final result as long as the number of occurrences to be included is at least one hundred. If a record of at least 100 years was available, the writer would expect the annual peak method to give the same results as the basic stage method. The fact that the basic stage method gives consistent results for a large change in the basic flow considered is evidence that the time element is properly estimated. For the Tallapoosa River a range in basic flow from 5 000 to 20 000 sec-ft. gives a value for what is here called the time period ranging from 0.0124 to 0.27 year. Nevertheless for all cases computed within these limits, the curves shown for that river on Fig. 4† are practically duplicated. There were only 86 storms which exceeded 20 000 sec-ft. When the basic flow was increased

* *Proceedings*, Am. Soc. C. E., August, 1927, Papers and Discussions, p. 1298.

† *Loc. cit.*, April, 1927, Papers and Discussions, p. 568.

to 25 000 sec-ft. only 55 storms could be included, and the resulting curve began to depart from the other curves.

In more recent work on this subject the writer has used a new form of logarithmic probability paper (designed by Mr. Hazen) to excellent advantage. This cross-section paper has values of time scale extended to a probability of 1 in 1 000 000. This is the same limit which appears in Mr. Foster's tables of "Duration Curve Factor".*

In reply to Mr. Jarvis,† it appears that he has fallen into the same error which the writer once made. Plotting the points at the center of their respective sections of the time scale is the correct method. If the curve happens to pass exactly through the highest flow thus plotted, this flow is likely to occur once in a period of time equal to twice the length of record. That any curve happens to pass exactly through any specified point determined by observation is, of course, pure chance. If, for instance, the maximum recorded flood greatly exceeds the next largest flood, the curve could not well be drawn through the point representing that maximum flood, and its frequency would be one in a much longer period of time. If Mr. Jarvis will replot his data it is probable that he will find himself in substantial agreement with the data presented in the paper for the Tennessee River.‡

Through a generous grant from the Heckscher Research Foundation at Cornell University an assistant has been engaged to carry on this work. Interesting results have already been obtained, incomplete at this time (December, 1927). It is hoped to be able to report again on this subject during 1928.

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* *Transactions,* Am. Soc. C. E., Vol. LXXXVII (1924), pp. 196 and 199.

† *Proceedings,* Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2027.

‡ *Loc. cit.,* April, 1927, Papers and Discussions, p. 567.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

TIDES AND THEIR ENGINEERING ASPECTS

Discussion*

By G. T. Rude, M. Am. Soc. C. E.†

G. T. Rude,‡ M. Am. Soc. C. E. (by letter).§—The writer is indebted to the Society for publishing this paper, and to the members of the Society who have contributed useful information and criticism in the discussions. In submitting the paper the writer had in mind a brief, general outline of the fundamentals of the subject of tides and currents for those engineers who have neither the time from their own particular sphere of work, nor the inclination to digest mathematical treatises on the subject.

Mr. Haupt‖ in his discussion refers to the "resacas" of the Bay of Rio de Janeiro and by letter has very kindly furnished the writer a reference to the subject.¶ The "resacas" are not a part of the tidal phenomenon, but are storm waves, due to local winds or to distant storms, which enter the Bay. They do not have a vertical rise and fall of "100 ft." in the meaning of the rise and fall of the tide; but these wind-produced waves, on striking against the vertical protective walls along the cities' fronts, are at times thrown 100 ft. into the air. It is quite probable that their force is increased by the topography of the Bay and perhaps, too, by the fact that the period of oscillation in the Bay, depending upon its dimensions, may coincide very closely with the periods of the waves.

Captain Faris** and Mr. H. De B. Parsons†† have emphasized the importance to the engineer of rational datum planes. The writer is in hearty accord that this matter is one of basic importance in the subject of tides and their

* Discussion on the paper by G. T. Rude, M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Author's closure.

‡ Commander, U. S. Coast and Geodetic Survey, Washington, D. C.

§ Received by the Secretary, December 24, 1927.

‖ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2055.

¶ *The Engineer*, April 11, 1924, pp. 380–388.

** *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2056.

†† *Loc. cit.*, p. 2055.

relation to the engineer. There can be no argument against Mr. Parsons' recommendation in regard to mean sea level as a datum for elevations on topographic maps. This is recognized by the U. S. Coast and Geodetic Survey, as evidenced in the system of levels which that Bureau is spreading over the country for the use of the engineer and for datums for the topographic mapping of the country. For this whole system mean sea level is used as a datum.

The use of high water as the datum for elevations on a nautical chart undoubtedly had its origin in the fact that the high-water line represents the demarcation between land and sea as visible to the mariner at practically all times, being usually marked by débris, or by other objects along the shore, and by discoloration of rocks, etc. As to the land depicted on the chart, the mariner is interested in the outline of the harbor or shore as it appears to the eye, and high water more nearly fills this requirement, for in low, flat country the grass or tree line usually ends very close to the high-water line; in hilly, rolling country, eroded bluff lines coincide very closely with the high-water line for the purposes of the mariner; in rocky country, such as in New England and Alaska, cliff lines are practically the shore line shown on the chart. The mariner, therefore, quite often is able to locate his vessel by bearings on the visible shore line, tangents to well-defined points, islands, etc., plotting the bearings to the charted high-water line.

For these reasons the high-water line was taken as the shore line and the zero contour from which to reckon elevations on the land bordering the sea and harbors. In general, the mariner has little interest in mean sea level, but is more inclined to estimate the elevations of shore objects in their relation to the visible high-water line.

The writer is glad to have Colonel Brown's discussion,* particularly regarding currents in canals. It happens that the straits and canals connecting two differently tided bodies of water, for which the U. S. Coast and Geodetic Survey has had occasion to make predictions, have been of a length and depth in which the hydraulic feature has predominated to the extent that thoroughly satisfactory results have been obtained by considering only the hydraulic gradient, with a constant for the inertia of the mass of water. It is quite probable, as pointed out by Colonel Brown, that, when occasion arises for predicting slacks and velocities for points in a long, shallow canal, this method will not suffice, and an analysis of actual current observations will have to be made.

Colonel Brown had charge of the lowering of the Chesapeake and Delaware Canal to sea level, and any studies he has made of wave propagation in the Canal will be awaited with interest by the profession. Several years ago he furnished the U. S. Coast and Geodetic Survey with records of eighteen months of continuous tide observations at Chesapeake City, Md., the western terminus of the Canal, and at Delaware City, Del., the eastern terminus. Using these data for determining the hydraulic gradient, predictions of the times of slack water were made for Chesapeake City. These times were in very close agreement with the times of observed slack waters obtained over a

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2058.

short period of actual current observations at Chesapeake City during the summer of 1927.

From information recently received from the office of the U. S. Engineers in Wilmington, Del., it appears that the times of slack waters vary considerably in different parts of the Canal, indicating that these predictions are satisfactory only for the western end of the Canal.

Mr. Godfrey's discussion* is, as he states, "a challenge of the universally accepted theory of the tides."

The basic theory of the tides, pertaining to the astronomical forces involved, which has been universally accepted by scientists in the past, has been found to afford a solid foundation for practical work in this subject; and tidal predictions for all parts of the world, made not only by the United States Government, but also by other maritime nations, have agreed remarkably well with actual observations. It is to be understood, of course, that these predictions are based upon both theory and observations.

The manifestations of the astronomical tide-producing forces are complicated by terrestrial conditions which give rise to various secondary theories upon which scientists have not been in entire agreement. The stationary-wave theory of Harris, discussed briefly by the writer,† seems to be in accord with most of the tidal phenomena as observed.

Col. William Barclay Parsons has referred to an interesting and economically important characteristic of the tide in a short strait connecting two independently tided bodies of water, deduced from his studies of the Cape Cod Canal—the curved instead of straight line of low-water elevations through the canal.‡ These deductions for the Cape Cod Canal likewise hold good in the case of the East River, New York, the same characteristic in the low-water plane existing also in that strait.§

Mr. Thomson refers to the probable results which would be brought about in the tidal régime of Long Island Sound by a new East River from Flushing to Jamaica Bay and a new Harlem River from Hell Gate to the Hudson, $2\frac{1}{2}$ miles long instead of $6\frac{3}{4}$ miles, if each of the new rivers were 50 ft. deep and of ample width.‖ Since there is no comparable project available, the matter is debatable, and, too, the results would depend somewhat on the conception of "ample width". Two such rivers, 50 ft. deep and of a width comparable with the present rivers, would probably not reduce the difference in the tide levels appreciably at Governors Island and Hell Gate.

Mr. Thomson also brings up the question whether the Cities of New York, Philadelphia, Pa., and Baltimore, Md., have a comparatively small range of tide due to the fact that the bodies of water on which they are located are "at right angles to the direction of Long Island Sound, Bay of Fundy, and the St. Lawrence, and also whether, if all these rivers, bay, and sounds were

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2341.

† "Tides and Their Engineering Aspects", *Proceedings*, Am. Soc. C. E., August, 1927, pp. 1081-1083.

‡ *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2346.

§ *Special Publication No. 111*, U. S. Coast and Geodetic Survey, "Tides and Currents in New York Harbor", pp. 73–74.

‖ *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2347.

running parallel to the Equator, the tidal ranges would be greatly increased."* It is doubtful whether the azimuth of these bodies of water affects to any extent their ranges of tide.

The tidal graphs† from observations by Mr. Dunham are very interesting, showing oscillations in Green Bay, Wisconsin, with a range as great as a foot at times and with a distinct tidal period. It is generally assumed that the fluctuations in the levels of the Great Lakes are due to wind and barometric pressure, with a periodic tidal oscillation of possibly a few inches at most, largely masked by meteorological conditions.

Mr. Bennett brings out the need for self-recording current meters for observations in waters such as Santa Monica Bay, California.‡ To be fair to instrument designers and manufacturers it should be borne in mind that, while a degree of performance similar to that required has been attained in the modern curve-tracing tide gauges, the work required of the tide gauge and also its installation are much simpler than that required of the current meter. In fact, the difficulties of holding the current meter on station without considerable cost is a much more troublesome matter than the development of the meter itself, particularly in water of considerable depth on an open coast such as the Pacific Coast of the United States.

Self-recording current meters have been developed. One of these, which makes a photographic record of both the velocity and the direction of the current, is described by the writer.§ Another self-recording current meter, which records, electrically, the velocity and the direction of the current,‖ was developed on the *Maud* while that ship was drifting in the Arctic.

The photographic-recording meter (Pettersson's) is already on the market,¶ and it is understood that the electrically-recording meter (Sverdrup) is being further developed.

Mr. Gelineau calls attention** to the requirement of location of the high-water line forming the boundary between State-owned and privately-owned property in the sale or lease of riparian lands flowed by tide water. In this connection the engineer can be of considerable assistance to legislative bodies in the wording of proposed laws covering such cases; that is, to have the law state definitely the high water or other tidal plane in the meaning of the law, so that a proper interpretation can be made, which will be recognized by the Courts in deciding cases of litigation. As an example, the writer recalls a case in which the term, "ordinary high water", was used. That might be taken as meaning "mean high water". This interpretation, if accepted by the Courts, might apply to the Atlantic Coast of the United States, but again that definition would not hold good on the Pacific Coast. Due to the diurnal

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, pp. 2347–2348.

† *Loc. cit.*, pp. 2351–2352.

‡ *Loc. cit.*, p. 2353.

§ *Loc. cit.*, August, 1927, Papers and Discussions, pp. 1126–1129. Also, "A Recording Current Meter for Deep Sea Work," by Dr. Hans Pettersson, *Quarterly Journal*, Royal Meteorological Soc., Vol. XLI, No. 173, January, 1915.

‖ "Two Oceanographic Recorders Designed and Used on the *Maud* Expedition," by H. U. Sverdrup and H. Dahl, *Journal*, Optical Soc. of America, Vol. XII, May, 1926, pp. 537-545.

¶ Those purchased by the U. S. Coast and Geodetic Survey were obtained from Stockholm, Sweden.

** *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2713.

inequality in the tides on that coast the law should be even more specific as to whether "mean high water" or "mean higher high water" is meant, since these are two comparatively widely separated tidal planes. Mean high water is the mean of all the high waters, and mean higher high water, the mean of all higher high waters.

General Black refers to the use of the terms, "progressive" and "stationary", in connection with the classification of tidal waves.* The writer agrees with him that non-uniformity in the nomenclature of a particular subject is likely to be misleading. It was for this reason that the writer was careful to employ the nomenclature used by the majority of tidal students and writers on the subject of tides, both in the United States and abroad.

General Black has made a strong point in favor of a study by engineers as to what changes would ensue in the tidal régime of a body of water from certain engineering projects as, for example, in the deepening and widening of the East River. Such studies, however, would have to be predicated on the nature and extent of the proposed projects. In the case of the East River it is doubtful whether any engineering project within reason, excepting locks, would have any considerable effect on the velocity of the currents. The removal of rocky shoals in Hell Gate and the removal of jutting points in the immediate vicinity would probably have the effect of eliminating, to some extent, the swirls and eddies in the currents which constitute a much greater menace to navigation than an equally high current velocity in a straight channel.

General Black is of the opinion that the characteristics of the tidal movement in the East River, New York, are due to the interference of two tide waves, one entering from New York Bay and the other from Long Island Sound, and he refers to data for the East River in the U. S. Coast and Geodetic Survey Tide and Current Tables as evidence in support of this opinion.† The predictions of slack waters in the East River contained in the Current Tables, however, furnish the strongest evidence that the movement of the water in that strait is without question primarily hydraulic. The predictions, made two years in advance, are based entirely on hydraulic movement, and thoroughly satisfactory results are obtained. The velocities of the current, also predicted in a similar manner, have been compared with observed velocities with like results. Beginning with the Current Tables for the calendar year 1930, the predicted velocities of the currents in the East River, based on hydraulic movement, will be published, in addition to the times of slack water.

In further support of the idea of the combined action of two tidal waves in the East River, General Black makes the statement: "Other evidences of this action could be named", with a footnote reference to *Special Publication No. 111*, U. S. Coast and Geodetic Survey, "Tides and Currents in New York Harbor", by H. A. Marmer, page 77.† The writer is unable to find any evidences in the reference (page 77) of combined action of two tidal waves in

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2715.
† *Loc. cit.*, p. 2722.

the East River. On the other hand, the text (pages 77-78) of the publication to which reference is made and the diagram of slope line both specifically bring out evidences of hydraulic movement, as follows:

"It is obvious that in the East River the tidal movement is conditioned by the fact that it is open to the tides of Upper Bay and also to the tides of Long Island Sound. It has therefore been customary to ascribe the peculiar characteristics of the tidal movement in East River to the interference of two tide waves—the one entering from Upper Bay and the other from Long Island Sound.

"However, the mechanism of the tidal movement in East River can best be understood by regarding the phenomena from the hydraulic point of view; that is, we may regard the changing height of the water in East River as brought about by the fact that part of the time the level of the water in Upper Bay is higher than in Long Island Sound, and part of the time it is lower. In other words, we may regard East River as a channel through which the water flows from the body having temporarily the higher level to the one having the lower level. The height of the water at any point in East River is therefore due to the relative elevations of the water at the two ends.

"If we plot the simultaneous heights of the level of the water at the two ends of East River, the lines joining these simultaneous heights will represent the slope of the water surface in the river and thus the height of the water at those times. These slope lines will then permit the time and range of the tide throughout the river to be determined, since the time and height of the water at any place will be represented by the highst point in the slope lines passing that place, while the time and height of the low water will be indicated by the lowest points in the slope lines."

Then follows a page of comparisons (page 78),* showing that actual observations of the tides in the East River agree very closely with times and heights as read from the slope line diagram, and ending with the following paragraph:

"It is obvious that the simple hydraulic considerations upon which Fig. 25 [*Special Publication No. 111*] is based can give only a first approximation to the tidal conditions existing in the East River. The slope lines are drawn as if the channel from Governors Island to Willets Point ran straight for the stretch of 14 miles with unvarying width and depth. No account has been taken of the varying depths in the waterway, of the differences in width, changes in direction, nor of the effect of the water coming through Harlem River. These factors must obviously bring about modifications; but, notwithstanding this, it is seen that the principal tidal phenomena are easily derived by considering the movement of the water in East River as hydraulic. In the chapter [*Special Publication No. 111*] devoted to the currents in the East River it will be seen that the characteristics of the current give further proof of the correctness of the view that the tidal movement in the East River is primarily hydraulic in character."

In ordinary tidal rivers, such as the Hudson River, the observed current velocities may be corrected to mean values by a factor, which is the ratio of the mean range of tide divided by the range for the period of observations.† In the case of the East River, however, and in similar straits connecting two differently tided bodies of water, such as Seymour Narrows, British Columbia,

* *Special Publication No. 111*, U. S. Coast and Geodetic Survey, "Tides and Currents in New York Harbor," by H. A. Marmer.

† *Loc. cit.*, p. 141.

and Deception Pass, Washington, this reduction factor to mean values is not the ratio of the mean range of tide divided by the range at the time of observations, but the square root of this ratio. This fact furnishes further evidence of primarily hydraulic motion in the East River, since in hydraulic motion the velocity varies as the square root of the head.

General Black refers to *Special Publication No. 111,* U. S. Coast and Geodetic Survey, "Tides and Currents in New York Harbor".* Attention should perhaps be called to the others of a series of such publications, one of which the Coast and Geodetic Survey is issuing each year, covering the tides and currents in the principal waterways of the country.† Besides those already issued, the publication covering Boston Harbor is now in press and the manuscript of a publication covering Portsmouth Harbor is being prepared. Each of these publications is being issued following a comprehensive current and tide survey of the harbor in addition to the observations made in past years. As an example, the publication for New York Harbor discusses the data from 149 tide stations, of which 63 are in the East River; and 322 current stations, of which 93 are in the East River. This publication discusses in considerable detail the tide and current conditions as they exist in that harbor at present. The data can be used for predicting the results on the tidal regime by engineers interested in a specific project of harbor improvement. Any analysis, discussions, or predictions of results on the tides and currents in a harbor based on one or more probable projects of harbor improvement is beyond the purpose and scope of such publications. The data are furnished for tide and current conditions as they are at present; and the field is open to engineers interested in any specific project for further analysis based on that particular project.

Mr. Gaston states:‡

"The conclusion seems to be offered from the text that those monthly variations [in tidal planes], although not always unimportant, cannot be determined on account of being due to differences of atmospheric pressure."

The writer did not intentionally mean to convey the idea that these variations are limited to differences in atmospheric pressures only. They may be due, in addition, to other meteorological conditions, such as fresh-water run-off, prevailing winds, etc. It is possible, too, that variations in atmospheric pressures over wider areas than those shown by local pressure observations may play a part in bringing about these variations in the tidal planes.

The writer is glad to have the references‡ of the mareographical and meteorological observations at the Port of Isabela de Sagua furnished by Mr. Gaston. These data will be extremely valuable in any studies of the variation in mean sea level.

Mr. Scheidenhelm refers to the fact that, during the period covered by the tidal records to which he had access in connection with the Musquash water-

* *Proceedings,* Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2722.

† *Special Publication No. 116,* "Tides and Currents, San Francisco Bay"; *Special Publication No. 123,* "Tides and Currents, Delaware Bay and River"; and *Special Publication No. 127,* "Tides and Currents, Southeastern Alaska".

‡ *Proceedings,* Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2724.

power development, there was no occurrence of an extreme spring tide between 4:00 and 6:00 P. M.* This is due to the fact that in the vicinity of the Musquash River the high-water luni-tidal interval, that is, the time elapsing between the moon's passage across the local meridian and the occurrence of local high water, happens to be about 11 hours. Spring tides occur at the times of new and full moon, at which times the moon crosses the meridian at noon and midnight. Therefore, since the high-water luni-tidal interval for this vicinity is about 11 hours, high water occurs at that place at about 11:00 A. M. and 11:00 P. M., during the time of spring tides. In fact, during this time, low water occurs in the vicinity of Musquash Bay between 4:00 and 6:00 P. M.

Messrs. Lee and Gray have submitted an interesting discussion* on their work in determining the rates of discharge and volumetric renewal of water in the Oakland Estuary, California, in connection with the question of sewage pollution in that tidal canal. The U. S. Coast and Geodetic Survey is now obtaining a series of current observations in this estuary, the data from which will be incorporated in the current tables issued by this Bureau for the use of the mariner.

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* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 211.

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PAPERS AND DISCUSSIONS

THE PLANNING OF THE INDUSTRIAL CITY OF LONGVIEW, WASHINGTON

Discussion*

By Messrs. Brookes Baker and S. Herbert Hare.†

Brookes Baker,‡ Esq. (by letter).§—The plans for Longview, Wash., appear to cover the requirements for a well-balanced community so completely that little remains to be said. As the designer states,|| the planning of an entire new community, with an assured future, occurs so infrequently that there are no precedents by which to judge. Most engineers who have been connected with matters pertaining to land developments have at times staked out new town sites in unsettled territory, this work being generally the mere execution, on the ground, of the promoter's desire to secure the greatest possible number of lots out of a given number of acres, without any regard for the future welfare of the purchasers of the lots. All who have done this will join in congratulating the designers of Longview on the happy solution of their problem.

The placing of the central park one block removed from the business district is a most excellent solution of the need for a park which is of the business district, but not in it. The diagonal streets are indeed convenient; the plan of adjusting the cross-streets to right angular intersections retains the most convenient rectangular spacing on the main streets, where values are greatest, and puts the angular and point lots in the less desirable locations.

The long parkway, occupying the old slough bed, with its many pools set in their banks of green; some shaded by magnificent trees, some spanned by attractive bridges; and all skirted by pleasant drives, affords an excellent

* Discussion on the paper by S. Herbert Hare, Esq., continued from October, 1927, *Proceedings*.

† Author's closure.

‡ Civ. Engr., Fort Worth, Tex.

§ Received by the Secretary, November 14, 1927.

|| *Proceedings*, Am. Soc. C. E., August, 1927, Papers and Discussions, p. 1177.

example of the adaptation of an offensive waste into an object of beauty, which will contribute throughout the years to the pleasure, comfort, and refinement of the growing community.

S. HERBERT HARE,* ESQ. (by letter).†—In the discussion by Mr. Lambuth,‡ the important point of co-operation is again emphasized; not only the co-operation of the engineer, the city planner, the landscape architect, the architect, and others, but that of the realtor, who can render a valuable service in connection with the development of such a plan as was produced for Longview, Wash. The real estate operator who is to have the responsibility for marketing the land, if he has a broad understanding of the problem and of the aims of the city planner, should very properly have a voice in the physical arrangement and development of this land.

Mr. Lambuth has referred to the building of Longview in various nuclei as a "skeletal" plan of development and has pointed out the saving which results from such a program. The success of such a plan, of course, depends somewhat on the prophetic vision of the planner, engineer, and real estate operator in judging the market and rate of growth, and weighing the somewhat greater initial expense involved in such a procedure against the immense losses which result from converting the uses of property early in the life of constructions. The results at Longview have undoubtedly justified the decision.

The provision for suburban garden tracts to which Mr. Lambuth refers was, of course, a part of the carefully considered general plan for the city, wherein provision was made, in its proper place, for every class of property which a city would normally require.

Mr. Baker refers to the Lake Sacajawea development by saying that it "affords an excellent example of the adaptation of an offensive waste into an object of beauty". Almost every city, new or old, has land which is a menace under private ownership but which, with proper development, would be a great asset under public ownership. It has been interesting to note, in the many interviews that have been given at various times by visitors to Longview, that the majority have seemed to appreciate, but accept as a matter of fact, the convenience, order, and systematic arrangement of the city, but few have failed to make special comment regarding the beauty of the city and particularly the park developments. This seems to indicate a growing demand for, and interest in, beauty in civic development.

It is only proper in closing this discussion to pay tribute to Mr. R. A. Long, who had the vision to lead this enterprise and who donated so generously to its success. Without such men, the skill of engineers, architects, and city planners would be wholly ineffective.

* Landscape Archt. and City Planner (Hare & Hare), Kansas City, Mo.

† Received by the Secretary, January 20, 1928.

‡ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2069.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS AND DISCUSSIONS

PRECISE WEIR MEASUREMENTS

Discussion*

By R. L. Parshall, Assoc. M. Am. Soc. C. E.

R. L. Parshall,† Assoc. M. Am. Soc. C. E. (by letter).‡—The measuring weir has been one of the most popular subjects investigated by hydraulic engineers. The authors' valuable paper adds very materially to the vast accumulation of data on this subject. However, much work remains to be done and additions to existing information may be expected in the future.

Flowing water is not always what it appears to be from the standpoint of consistency of motion. Engineers are apt to make certain assumptions as to the line of travel and rate of displacement of the filament in the water mass, that conditions do not fully justify. All parts of a mass of water moving under the influence of gravity do not advance uniformly. For instance, that portion touching the confining surfaces is slowed down, while at some point within the mass the rate of motion may be either accelerated or retarded. This unequal movement disturbs the flow and sets up cross-currents which result in boils or eddies. This disturbed condition of flow becomes less apparent, however, with slower velocities, and at an extremely slow rate of translation the effects of the cross-currents are inappreciable, although they are present. When the stream is led over a weir notch, the velocity of the water is then sufficient to produce unequal variations of movement in the approach section, causing a departure from the ideal. The contracting vein introduces other complications which, in combination, set up a condition far from the ideal conception of stream lines curved gracefully over the crest and reaching through into the nappe of the falling sheet of water.

Flowing water is endowed with still another very troublesome characteristic. The water surface is considered fixed and constant as to elevation or depth, when in reality this is not its condition. An oscillating pulse is set

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* Discussion on the paper by Ernest W. Schoder, M. Am. Soc. C. E., and the late Kenneth B. Turner, Esq., continued from February, 1928, *Proceedings*.

† Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.

‡ Received by the Secretary, December 15, 1927.

up when the mass of water begins to move in an open channel. This changes the depth or pressure and causes an unsteadiness of movement over the crest of a weir. Screens, baffles, barriers, skimmers, and all known mechanical means of damping this effect do not totally eliminate the incessant rise and fall of the surface. Experience seems to indicate that under laboratory conditions and refinement of control, certain combinations of flow conditions very greatly affect the accuracy or dependability of measurement, and for this reason exactness in weir measurement is extremely difficult to attain, if attainable at all.

The authors' experience, as indicated by the various diagrams, is that all observations do not plot along the ideal curve, and the writer's experience with weirs under laboratory conditions shows the same difficulty. It might even be stated that it is doubtful if there is any such thing as precise weir measurements.

The writer is not in full accord with the authors' method of determining the effective head on the weir crest to the limit of accuracy stated. Two methods are described.* In one, a sharp-pointed plumb-bob is attached to a steel tape graduated to hundredths of a foot; in the other a float-gauge is used. The least known graduation marked on either tape or gauge is 0.01; by estimation the nearest 0.001 part is determined, and the nearest 0.0001 part is recorded as a mean. In the original notes (Test 55) the observations of heads are shown where the variation is 0.013 ft. for the plumb-bob determination, while the float-gauge readings varied between limits of 0.023 ft. The attempt to determine the mean head to 0.0001 ft. for this range of fluctuation appears to be going beyond precision, as the word should be understood in such work. However, it is to be noted in this case that the agreement of the means of the two independent measurements is remarkably close. As shown in the original notes (Test 14), both float and plumb-bob gauge readings are recorded to the nearest 0.0005 ft., which calls for the estimation of one-twentieth of the 0.01 division of the scale. This seems to claim an accuracy not to be credited.

Examination of the recorded data shows that the variation or trend of the law of discharge is not perfectly uniform. The variations are not due to inaccurate measurement of the physical constants, but to the inherent characteristics of the flowing water. In Table 42† are given results of discharge over a sharp-crested weir for heads of less than 0.02 ft., and a corresponding discharge, in second-feet, to a limit of 0.00001, with a direct comparison to a computed discharge by means of Francis' formula. There seems to be no logical reason in setting up a comparison of observed and computed free-flow discharge over a sharp-crested weir for a head of only 0.02 ft. The writer's laboratory experience with weirs indicates that about 0.07 ft. for a rising head is required to prevent adherence to the down-stream face of the weir crest. A stream which adheres to the crest certainly can not follow the same law called for by the Francis formula. Perhaps it might

* *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1404.

† *Loc. cit.*, p. 1491.

be said with equal propriety that there is no justification for setting up comparisons of discharges, thus determined, with those computed by the Francis formula, where the heads exceed one-third the crest length.

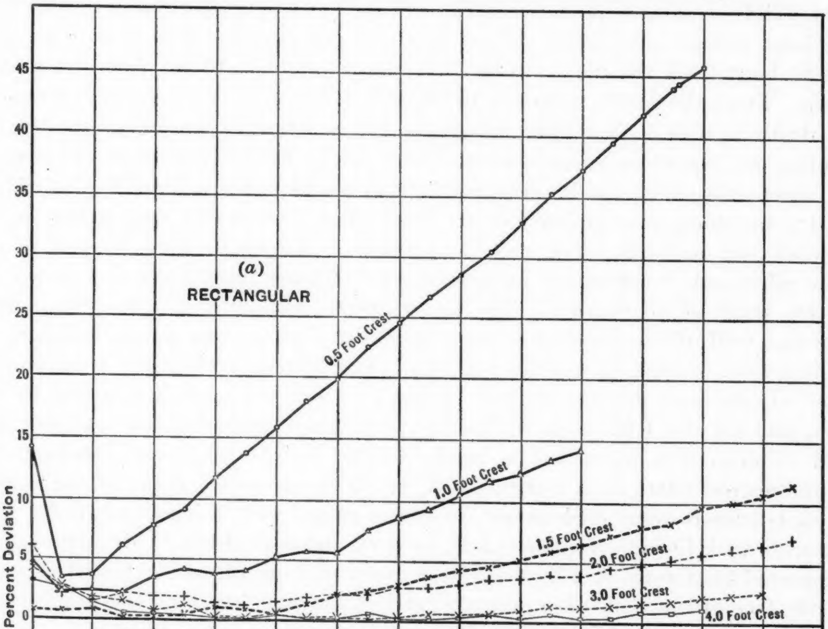

FIG. 55.—PERCENTAGE OF VARIATION OF DISCHARGE, SHARP-CRESTED WEIRS.

An extensive series of tests on various types of weirs was conducted at the Hydraulic Laboratory of the Colorado Agricultural Experiment Station, at Fort Collins, for the purpose of extending the law of flow over weirs

for depths greater than one-third the length of the crest, as recommended by Francis. The general features of this laboratory and method of conducting the tests, together with a complete report of the results, have been described.* The weirs tested were mounted in the end wall of a concrete channel about 80 ft. long which leads to the source of the water supply. For a distance of 20 ft. back from the plane of the weir this channel is 10 ft. wide and 6 ft. deep. From the supply reservoir to the weir box proper the channel increased in depth as well as in width, thus causing a constant expansion of its cross-section as the water approached the weir box. The weir plates were made of sheet steel $\frac{1}{8}$ in. thick. The notch, formed by a brass plate $1\frac{3}{4}$ in. wide and $\frac{1}{4}$ in. thick, was so fitted to the steel sheet that the up-stream face was smooth and uniform. The crest and sides of the notch were dressed to a true adge and the down-stream corner of the brass crest plate was beveled to an angle of 45 degrees. The actual crest was about $\frac{1}{64}$ in. wide. In the end wall of the channel a heavy angle-iron frame was set, to which the weir plates, seated on a rubber gasket, were bolted. When the weir plate was set, the crest was about 42 in. vertically above the level floor of the weir box, and for the 4-ft. weirs the horizontal distance from the side-wall to the end of crest was 36 in. The heads on the weir crests were determined to the nearest 0.001 ft. at a point 10 ft. up stream from the plane of the weir, by a Gurley-Boyden hook-gauge, the hook-gauge well being constructed of concrete with inside dimensions 1 ft. by 2 ft. and 4 ft. deep. This gauge was supported by a vertical 2 by 6-in., well-seasoned stick of timber bolted to the inside face of the well. Through the wall of the weir box 1-in. tubes led into the gauge well at a point below the crest elevation.

The data submitted by the writer are intended to show only the deviation of the Francis and Cippoletti formulas from that determined by carefully observed tests on full contracted weirs with complete aeration of nappe under refined conditions at the Fort Collins Laboratory. In these comparisons the percentage of deviation shows the disparity between the observed and computed discharges where the observed value is taken as the base.

Fig. 55(*a*) shows the deviation in percentage of the observed discharge over rectangular weirs when compared with the discharge computed by the Francis formula, in which,

$$Q = 3.33\ (L - 0.2\ H)\ H^{\frac{3}{2}}$$

Fig. 55(*b*) shows a similar comparison for Cippoletti weirs where the computed discharge is based on the formula,

$$Q = 3.367\ L\ H^{\frac{3}{2}}$$

The actual discharge differs greatly for short-crested weirs of both types from that indicated by the generally accepted formula.

* *Journal of Agricultural Research*, U. S. Dept. of Agriculture, Vol. 5, March, 1916.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

HOUSING AND THE REGIONAL PLAN

Discussion*

By John Ihlder, Esq.†

John Ihlder,‡ Esq. (by letter).§—The writer has been fortunate—or unfortunate—depending upon one's viewpoint, in the discussors of this paper. With a single exception, each party to the discussion sees practically "eye to eye," so they leave nothing further to be said except by way of expressing appreciation for the greater emphasis and clarity they have given to points made in the paper, and for additional data, which the writer has found instructive and helpful.

The writer takes issue with Mr. Rice|| on two points—which really are one. Believing, as he does, that the family is the essential unit of society (the paper recognizes the fact that there are individuals who must be provided for), the writer cannot agree that the requirements of the individual form a proper basis for a housing program. Consequently, also, he cannot agree that "if a reasonable limitation is made as to area per individual, the multiple house will lose the greater part of its objections." To quote from the paper,¶

"The preservation of the family—meaning parents and children—is essential. * * * The one-family house with generous open spaces about it is the best house for the child."

Mr. Rice apparently believes that proper housing is fully provided if the house is sanitary and has adequate light and ventilation. The writer believes that this is not enough, that the predominant type of house must also be a home which makes adequate provision for children. Therefore, he disagrees with the statement of Mr. Rice that "the single-family house may not be the

* Discussion on the paper by John Ihlder, Esq., continued from November, 1927, *Proceedings*.

† Author's closure.

‡ Mgr., Civic Development Dept., Chamber of Commerce of the United States, Washington, D. C.

§ Received by the Secretary, January 19, 1928.

|| *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2398.

¶ *Loc. cit.*, September, 1927, Papers and Discussions, p. 1519.

ideal one for a large industrial population." An industrial population has children.

It may be that under existing conditions in many American cities the wage-earners must live in two or three-room apartments, but that does not make these apartments ideal. Mr. Rice's supporting argument is that the single-family house "is more expensive than the multiple house." In that statement he raises a question which he does not answer, one which cannot be answered briefly because it involves many factors. One can say, however, that before the World War, when housing conditions were more nearly normal than they have been since, the rent of a six-room one-family brick house in Philadelphia was lower than that of a rear five-room unheated, walk-up tenement in New York, or that of an apartment in a wooden "three-decker" in Boston.

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PAPERS AND DISCUSSIONS

CULTURAL OPPORTUNITIES IN REGIONAL PLANNING

Discussion*

By Messrs. John Ihlder, and Joseph Barnett.

John Ihlder,† Esq.—It seems that in the case of boulevard, parkway, etc., the author and the speaker are talking or thinking of two or three different things, because they do not always supply the same definition to a word.

The increase of heavy truck traffic in American cities has become a matter of serious concern. It not only results in breaking up light pavements, but it is injuring neighboring property. In Washington, D. C., the walls of houses are being cracked, and there are also instances of window panes being cracked by the vibrations of heavy trucks. Something has to be done to regulate that kind of traffic.

Heretofore, apparently, city planners have considered it only from the point of view of facilitating traffic movement, in the case of parkways by forbidding trucks. In Washington, trucks are now forbidden on the main arterial roads, such as 16th Street and Massachusetts Avenue, etc., the idea being that they delay the movement of lighter automobiles.

This, however, is only one phase of the problem. It must also be considered from the point of view of the character of neighborhood. The present prohibition in Washington is causing trucks to use the minor residential streets to the great detriment of the neighborhood. Should not city planners begin to consider the creation of a system of heavy traffic streets that will adequately serve trucking needs in the cities, and not simply prohibit and divert it from one street or road to another?

Joseph Barnett,‡ Assoc. M. Am. Soc. C. E. (by letter).§—This is the day of specialists and in the nature of things, specialization will increase as time

* Discussion on the paper by Andrew Wright Crawford, Esq., continued from February, 1928, *Proceedings*.

† Mgr., Civic Development Dept., Chamber of Commerce of the United States, Washington, D. C.

‡ Chf. Draftsman, Westchester County Park Comm., Bronxville, N. Y.

§ Received by the Secretary, January 19, 1928.

goes on. The sum total of human knowledge increases steadily, and it is becoming more and more difficult for the individual, in one life time, to learn, appreciate, and develop the ability to use even a small percentage of the information on any one subject. Therefore, whether the subject for discussion is regional planning as a whole, such as the laying out of a city, or a great park system, or whether consideration is being given to one of the details, say, the adaptability of a certain type of structure, it seems advisable to get different viewpoints on the subject. If each specialist has a proper appreciation and respect for the outlook of the others, pleasing as well as economical results will be obtained. Mr. Crawford uses the word, "designer," as if a dress or piece of furniture is under discussion. In regional planning the functions of design must be divided. The designing engineer, landscape architect, and other specialists all have proper functions to perform.

The Bronx River Parkway, which is a part of the Westchester County, New York, Park System, is an excellent example of the results which can be obtained by co-operation. The bridges are an important feature of this development. To say that they could have been designed by architects alone would be just as untrue as to say that they could have been as successfully designed by the engineer alone. The fundamental structural designs for these bridges were made by the Designing Engineer. Not just the "plumbing" as Mr. Crawford calls it, the computation of stresses and strains, but the conceptions of proportions which made these bridges possible; the development of the solid section rigid frame with variable moment of inertia. Incidentally, on almost every bridge except one, the architect's name appears alone.

To blame all ugly structures of the past on the engineer is a refusal to recognize the fact that such structures were the products of the times. The people more or less controlled the funds for public improvement and any expense over and above the bare requirements formerly met with considerable opposition. Monuments, art museums, and city halls were generally excepted but to place cut stone facing on a highway bridge of twenty years ago would have been considered the height of extravagance. The modern trend, however, is toward a greater appreciation of the esthetic value of those things with which people come in daily contact and this fact makes Mr. Crawford's paper timely indeed, because designers should keep just a little ahead of the tastes of the times. It would be an excellent thing if all civil engineers, especially those engaged in design, would read, carefully, Mr. Crawford's paper; but regional planning would derive just as much benefit from a similar paper illustrating the necessity of architects acquiring an appreciation of things engineering.

Engineers need not fear having architects called in to lend advice on the esthetics of structures on account of the cost because pleasing structures can be built at very little extra expense, provided the architect has a proper appreciation of the engineering aspects of the problem. Likewise, architects should not assume that engineers have no esthetic taste and that they persist in violating esthetic rules when insistence on extreme economy is not pressed

on them. At times, engineers can improve the appearance of a structure very materially. The writer has in mind a case in which outlines for a number of bridges were laid out having intrados curves resembling ellipses. The architects properly recognized that multiple-centered curves would not be smooth, but when the designing engineer suggested curves following fixed mathematical laws instead of the free-hand curves at first laid out, the suggestions were rejected although fundamental clearances and heights were retained. After the first bridges were built they were found to have the appearance of sagging at the crown. On later bridges, mathematical curves were used with perfect effect. One of the structures was a two-span bridge with different span lengths and by applying the same formula to both intrados curves, matched curves resulted. Incidentally, the formula used was,

$$\frac{x^{2.2}}{a^{2.2}} + \frac{y^{2.2}}{b^{2.2}} = 1$$

The slight variation from the true ellipse provided greater clearances at the haunches. This bridge is now finished and the intrados curves are probably the most graceful in this set of bridges.

Therefore, to obtain the best results in regional planning or any allied project, it is not necessary to have the engineer learn architecture, or *vice versa,* but to teach each an appreciation of the other and the public an appreciation of both.

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PAPERS AND DISCUSSIONS

PROBLEMS IN CONCRETE DAM CONSTRUCTION ON THE PACIFIC COAST

Discussion*

By R. A. Hill, M. Am. Soc. C. E.

R. A. Hill,† M. Am. Soc. C. E. (by letter).‡—The author very aptly calls attention to the fact that no two dams are identical, and that they all usually require radical differences in construction methods. He emphasizes justly the necessity of a painstaking, thorough study of the site and all its conditions, and the further need for the contractor to be able to discard all precedent in his visualization of the processes involved from the excavation of the foundation to the final cleaning up of the work.

With rare exceptions, engineers responsible for the design of dams, spend a great deal of time in the compilation of data regarding the conditions which will affect the cost of construction. Generally there is furnished to bidders all available knowledge, such as the frequency and time of occurrence of floods. In some instances, such information is not available, and it is regrettable that in others the engineer does not realize his responsibility and gives extremely limited information to bidders. Yet, how many contractors make a careful study of the information which is furnished them?

Unfortunately, the basis of estimation set forth by the author§ is not followed by a large proportion of contractors. Not long ago bids were asked on one of the most unusual and difficult pieces of dam construction yet to be attempted. It was the boast of one entirely responsible contractor that he arrived at the site of the work at 10:00 A. M., investigated the possible sources of aggregate, and returned to a near-by town in time for lunch. Another contractor bidding on the same work assumed that his plant would cost a fixed sum per cubic yard, and at the time he submitted his bid did not even

* Discussion on the paper by Arthur S. Bent, Affiliate, Am. Soc. C. E., continued from November, 1927, *Proceedings*.

† Cons. Engr. (Quinton, Code & Hill), Los Angeles, Calif.

‡ Received by the Secretary, January 27, 1928.

§ *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1722.

know where he would locate his construction plant in event he was awarded the contract. This contractor did not start to estimate his costs (which ran into millions of dollars) until two weeks before the opening date of the bids.

A short time ago a number of bids ranging from less than \$10 000 000 to more than \$15 000 000, were submitted on a tremendously large dam. It is generally known that a few of the bidders followed the ideal practice, set forth by the author, and spent several months analyzing all the factors affecting the cost of such a tremendous project. On the other hand, there were certain bidders who practically ignored some of the very important considerations. For example, one contractor expressed the opinion that transportation was the only item which required special study, and that the plant and methods of pouring several million cubic yards of concrete were of relatively slight importance. In his opinion, this was purely a manufacturing process and entirely straightforward, although a variation in cost of only 10 cents per cu. yd. would make a difference of \$400 000 in his profits or losses.

These instances, unfortunately, are typical of the procedure followed by many who are deemed responsible contractors. Under press of other work, contractors are prone to postpone the preparation of estimates, so that the limitation of time usually requires that detailed studies be omitted. Yet, it is common to hold engineers' estimates in scorn and to view contractors' bids with reverence. The writer does not wish to imply that he is a proponent of force account work, as it is his opinion, that, in so far as practicable, all construction of this character can be best handled by contract. He does wish to stress, however, the fact that the procedure outlined by the author is an ideal one and is not customarily followed, except by a minority of contractors.

In most cases, however, it would be advisable to segregate the hazardous and uncertain items of construction from those subject to no especial contingencies, rather than include all the work in a lump-sum contract by which the owner must pay for all contingencies which the contractor thinks might arise. This is particularly applicable in the case of foundation excavations within the limits of the stream channel, which, as it involves river control, is one of the major uncertainties. The amount and character of wet excavation are almost indeterminable in advance, in spite of the number of borings which may be made, except in a few cases where bed-rock is exposed for practically the entire width of the river channel. The borings can do little more than determine the depth to rock, and only by actual excavation is it possible to fix on the exact amount of excavation into rock which is desirable.

There is one important step in the preparation of bids, and in the execution of the work, which the author has not stressed. The time available for the construction of a dam is usually fixed by the contract, and, consequently, the entire work must be scheduled so as to be finished within the allotted time. To be sure, extensions are granted, but these are in themselves an expense to the contractor, due to the ever-present item of overhead.

A careful study of the climatic conditions, and the occurrence of floods in the river, immediately brings out the necessity of beginning certain operations at

specific times. In the western part of the United States, it is generally possible to determine with reasonable accuracy the number of days during which work must be stopped due to adverse climatic conditions. It is also possible to determine the time at which floods may be expected and the probable magnitude and duration of such floods. The writer has recommended to contractors of his acquaintance the advisability of preparing a chart covering the entire period of operation, upon which should be shown a typical occurrence of storms and a typical number and grouping of floods in the river. Obviously, the actual conditions will not correspond with this typical weather chart and river hydrograph, but the total number of off days and their general occurrence will correspond quite closely to the actual conditions. With such a chart available, it is then possible to analyze each operation, such as river diversion, excavation in the river channel, cleaning of foundations, pouring of concrete up to river level, re-diverting the river, and closing temporary outlets. The time required for any of these operations is subject to considerable variation. The number of men and amount of equipment necessary to complete each operation in the time available is then a matter of calculation, which, while not absolute, at least materially reduces the element of uncertainty.

The author discusses* the necessity of a superintendent who can foresee such contingencies and who can plan his work a long time in advance. It would seem that this procedure results in shifting the responsibility for the plan of execution from the contractor to his superintendent. A very careful scheduling of operations, from the inception to the completion of the work, might better be prepared by the contractor in conjunction with the superintendent. A schedule so prepared covering the entire job, and prepared as the result of the co-ordination of the best thought of the contractor's organization, certainly is to be preferred to the limited foresight of one man directly engaged on the work.

The advisability of constructing a railroad to the dam site is a matter to be determined in each specific case. Some contractors so much perfer the use of railroads that they are inclined to disregard the possibility of truck transportation. Other contractors, particularly those whose early experience has been on highway work, or on work in the close vicinity of a city, are disposed to overlook the saving which can be effected by railroad operation. The reliability of motor-truck transportation is steadily increasing, as are the number of suitable highways, so that the construction of a railroad is becoming of less importance than heretofore.

The concrete plant should be regarded as a step in the flow of material from the source of supply to the completed work. Usually there are certain conflicting considerations. If a plant is placed so that it is easy of access it usually is difficult to distribute the materials from this plant. On the other hand, if the plant is so located that the delivery of mixed concrete is easiest, it becomes extremely complicated to get material to the plant. On account of the fact that there is a single product sent out from the plant, it is generally simpler to provide this single line of outflow than several lines of approach for materials which go to make up the product.

* *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1725.

The writer agrees that for the construction of a dam in a narrow canyon it is generally advisable to bring in materials to a point above the crest of the dam, and then let them flow downward to the plant and from the mixer downward into the dam. If such a procedure can only be accomplished by hoisting all materials to a point well above the crest of the dam before they enter the plant, the advantage of this location immediately disappears.

The author favors distribution by a trestle for a long low dam, with the plant located at one end. A typical layout of this character was analyzed some time ago, and it was found that if the concrete plant were located at one end there was an average haul of 1 400 ft., with a maximum haul of almost 2 000 ft. By locating the plant on the bank of the river down stream from the dam, and by hoisting directly from the plant to the trestle, the average haul was reduced to less than 500 ft. and the maximum haul was reduced to about 1 200 ft. In either case, the material had to be lifted to the same height.

No hard and fast rule can be followed, and as much consideration must be given to the ease of access to the plant as to that of distribution of the mixed concrete after it leaves the plant. Irrespective of the position in the line of flow of materials, the same tonnage must be handled. In other words, the shortest and easiest route from the source of supply to the finished structure should be followed, and the concrete plant should be placed along the line of flow wherever it will produce the smoothest operation.

A great many dams which have been built, or which are proposed for construction, are located in relatively inaccessible places, so that each job must be a self-contained unit. As the author points out,* the success of the work depends on maintaining a high morale, thereby reducing the labor turnover to a minimum. Not only is turnover expensive in itself, but the desired smoothness of operation can only come from the work of men familiar with their tasks. In order to create and maintain this desirable "elan", the environment in which the men live must be maintained at a high standard. For a great number of years, construction camps were, to say the least, undesirable places. Far-sighted contractors have brought about a change in this condition, and it is safe to say that the future will see even better and more complete facilities provided for the men. It is noteworthy that the camps on force account work are frequently of a lower standard than those provided by contractors.

Summarizing, the writer is of the opinion that far less study is given to the conditions surrounding the construction of a dam than should be given by any one risking his capital or his reputation. This, of course, does not apply to all contractors, but, unfortunately, it does apply to a great number of contractors, and likewise to a great number of engineers responsible for the execution of work of the magnitude of a dam. Most of the uncertainty and contingencies, which contractors provide for by arbitrarily adding 10, 15, or 20% to their cost estimates, could be eliminated by a detailed analysis of all conditions, and by careful scheduling of operations and co-ordination of the different steps involved in the construction of any large concrete dam.

* *Proceedings*, Am. Soc. C. E., September, 1927, Papers and Discussions, p. 1724.

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PAPERS AND DISCUSSIONS

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NEW THEORY FOR THE CENTRIFUGAL PUMP

Discussion[*]

By F. Theodore Mavis, Assoc. M. Am. Soc. C. E.

F. Theodore Mavis,[†] Assoc. M. Am. Soc. C. E. (by letter).[‡]—The author states repeatedly that the "present" theory of the action of centrifugal pumps as presented in standard textbooks is incorrect, but he does not explain his reasons very clearly. He states that Equations (*B*)[§] and (*C*)[§] "form the basis of all present theories", whereas the equations follow as a consequence of the hypothetical properties of an ideal fluid and its orderly behavior as it flows through an imaginary pump. The assumptions and corollaries from which these equations follow, regardless of the mathematical tools which are used in deriving them, may be summarized somewhat as follows:

I.—The Fluid:

(1) Homogeneous.
(2) Incompressible.
(3) Non-viscous.

II.—The Flow:

(1) Vortex free (that is, there is no flow in closed paths within the pump).
(2) No energy losses.
(3) Relative velocity at every point on a circle concentric with the axis of rotation is the same.
(4) Steady; the rate of discharge through the pump does not change from instant to instant.
(5) Continuous; all pump passages are at all times filled with the fluid.

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[*] Discussion of the paper by A. F. Sherzer, Assoc. M. Am. Soc. C. E., continued from February, 1928, *Proceedings*.

[†] Urbana, Ill.

[‡] Received by the Secretary, January 20, 1928.

[§] *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1779.

III.—The Pump:

(1) Frictionless.
(2) The surfaces of all fluid passages coincide with stream lines.
(3) There are an infinite number of impeller blades.

IV.—The External Forces:

(1) Gravitational.
(2) Mechanical.
(3) Non-frictional.

Water is a homogeneous fluid which can be considered incompressible, but its viscosity cannot be neglected. Inseparably linked with the viscosity are resistances opposing every motion of this otherwise ideal fluid and, in the case of higher velocities, turbulence, eddies, and the inevitable consequence, loss of useful energy. The water passages of the actual pump are not perfectly smooth and frictionless and there are not an infinite number of infinitely thin impeller blades, each exerting no pressure on the water in the impeller. Since neither the water nor the actual pump meet the requirements laid down by the elementary theory, it is quite unreasonable to expect agreement between tests of pumps and calculations made according to this theory.

The equations of the elementary theory are very useful, however, in making first approximations. They lead to the familiar straight line characteristic passing through $H = \frac{C^2}{g}$ for $Q = 0$, which may be considered as the first approximation of the actual characteristic curve.

Professor Pfleiderer* shows that a curve, very similar to the experimental characteristic, can be estimated for the purpose of design. Subsequent corrections are necessary to allow for the following factors:

1.—The effect of a finite number of impeller blades in the actual pump.
2.—Shock losses, or losses due to sudden changes in velocity.
3.—Friction losses.
4.—Leakage and recirculation losses.

The first may be made experimentally and the others, which involve losses of useful energy, can be made only by consideration of experimental data. Correction 1 may also be made analytically (assuming an ideal fluid) and as a consequence a newer theory of pumps and turbines has been developed which differs from the elementary theory before mentioned in the following details:

1.—Circulation around each propeller blade.
2.—Between two impeller blades the velocity at no two points on a given circle concentric with the pump axis is the same.
3.—There are a finite number of impeller blades and their position is known.

The mathematical analyses of the flow through pumps according to the newer theory, which considers a finite number of impeller blades, are interesting as problems in applied mathematics, but at present they are so difficult and so restricted in their application to ideal cases that they will not be likely to appeal to the designer. From a purely mathematical standpoint.

* See Pfleiderer, "Die Kreiselpumpen," p. 71 and 123 *ff*, J. Springer, 1924.

Professor Spannhake* studied the flow through a pump with from one to eight radial blades and compared the absolute discharge velocity, C, for the pump having a finite number of blades with the corresponding absolute velocity, C_∞, for the pump with an infinite number of blades as assumed in the elementary theory. He also compared the corresponding moments, M and M_∞,† applied at the shaft of the pump and presented curves showing the ratios, $\frac{C}{C_\infty}$ and $\frac{M}{M_\infty}$, as functions of the number of impeller blades and the ratio of the radii, $\frac{r_1}{r_2}$. Table 1 shows the results of these studies.

TABLE 1.—Tests Showing the Effect of Varying the Number of Impeller Blades.

$\frac{r_1}{r_2}$.*	Two Impeller Blades.		Four Impeller Blades.		Eight Impeller Blades.	
	$\frac{C}{C_\infty}$.†	$\frac{M}{M_\infty}$.‡	$\frac{C}{C_\infty}$.	$\frac{M}{M_\infty}$.	$\frac{C}{C_\infty}$.	$\frac{M}{M_\infty}$.
0.3	0.27	....	0.46	....	0.73	....
0.4	0.21	2.50	0.38	2.10	0.68	1.60
0.5	0.16	2.00	0.30	1.80	0.59	1.45
0.6	0.12	1.65	0.23	1.55	0.50	1.35
0.7	0.08	1.40	0.16	1.35	0.38	1.25

* r_1 = inner impeller radius, and r_2 = outer impeller radius.

† C = absolute velocity of discharge for impeller with finite number of blades; and C_∞ = absolute velocity of discharge according to the elementary theory.

‡ $\frac{M}{M_\infty}$ = ratio of corresponding moments.

Kucharski‡ shows that the relative flow of an ideal fluid in a rotating canal may be resolved into two components: (*a*) A flow through the canal without circulation (hence vortex free and in agreement with the assumptions of the elementary theory); and (*b*) a circulation or rotation within the canal at a constant angular velocity which, relative to the canal, is equal, but opposed in sense, to the absolute angular velocity of the canal about the axis of rotation. The circulation can be determined from measured deformations of an elastic membrane stretched (under constant stress) across a plane section of the canal perpendicular to the axis of rotation and inflated under constant (air) pressure. The deformations should be small so that the stresses in the membrane are not materially changed as a result of the deformation. Contours on the deformed membrane correspond to the stream lines; the inflating pressure corresponds to the angular velocity of the vortex relative to the canal; and the slope of a tangent to the surface of the membrane corresponds to the velocity of flow in a direction at right angles to that tangent.

* Wilhelm Spannhake, "Die Leistungsaufnahme einer parallel-kraenzigen Zentrifugalpumpe mit radialen Schaufeln," Festschrift zur Hundertjahrfeier der Technischen Hochschule, Karlsruhe, 1925.

† $M_\infty = \frac{Q\,w}{g}\,(C_{p2u}\,r_2 - C_{p1u}\,r_1) = \frac{H_\infty}{\omega}$.

‡ Kucharski, "Stroemung in rotierenden Kanälen," *Zeitschrift für das Gesamte Turbinenwesens*, XIV Jahrgang, Hefte 21–22 (1917).

This elastic membrane or soap-film analogy is well known and very helpful in its application to problems of torsion and it is equally applicable to problems of hydrodynamics. This is true from a mathematical standpoint because the differential equation that describes the deformation of the thin elastic membrane is of the same form as the one which describes the vortex motion of an ideal fluid. The differential equation in rectangular co-ordinates, x and y, that expresses the deformation, f, of a thin elastic membrane from a plane position, is:

$$\frac{\delta^2 f}{\delta x^2} + \frac{\delta^2 f}{\delta y^2} = \frac{P}{T} \qquad (16)$$

This is the same as the differential equation for vortex motion in an ideal fluid:

$$2\,\omega = \frac{\delta v}{\delta x} - \frac{\delta u}{\delta y} = -\left(\frac{\delta^2 \psi}{\delta x^2} + \frac{\delta^2 \psi}{\delta y^2}\right) \qquad (17)$$

in which,

ω = the angular velocity about an axis perpendicular to the plane of x-y,

u = the velocity in the direction of x,

v = the velocity in the direction of y,

ψ = the so-called stream function.

If the stream lines are drawn for constant differences in the values of the stream function, ψ (or by analogy equal contour intervals on the inflated membrane), the same quantity of fluid flows between each pair of stream lines. The velocity, which is tangential to the stream lines, is therefore inversely proportional to the distance between these lines.

The flow through the canal, or component, a,* of the relative flow, may be determined in a few ideal cases by the methods of classical hydrodynamics, that is, by conformal representation, using functions of complex variables.† These methods, are certainly to be classed with the most difficult ones in the field of applied mathematics and are quite outside the field of design. However, the determination of the flow of an ideal fluid through any given canal can be made by an experimental process suggested by Hele-Shaw which is more generally usable and vastly simpler than purely mathematical processes. This investigator has shown that, if a fluid dye is introduced through capillary tubes into water flowing between two glass plates placed close together, stream lines are traced by the threads of dyed fluid which, for any boundary conditions, agree with those stream lines calculated by the methods of mathematical hydrodynamics.‡ If a stream-line diagram is available for a given set of boundary conditions, or for a given shape of impeller blade, it can be "twisted" into another stream-line diagram with a different set of boundary conditions, by graphical methods of conformal representation discussed by Prasil.§

* Kucharski, "Stroemung in rotierenden Kanälen," *Zeitschrift für das Gesamte Turbinenwesens*, XIV Jahrgang, Hefte 21–22 (1917).

† Lamb, "Hydrodynamics," or, Lorenz, "Technische Hydromechanik," p. 275 *ff*.

‡ Hele-Shaw, "Investigation of the Nature of Surface Resistance of Water and of Stream Line Motion under Certain Experimental Conditions," *Transactions*, Inst. of Naval Archts., 1898; and "Distribution of Pressure Due to Flow Around Submerged Surfaces," *Transactions*, Inst. of Naval Archts., 1900. Photographs of such stream lines are shown in "Die Wasserbau-laboratorien Europas," p. 240 (V. D. I. Verlag).

§ Fr. Prasil, "Technische Hydrodynamik," First Edition, p. 61 *ff*.

Oertli's studies* conducted in the hydraulic laboratory at the Technische Hochschule, in Zurich, Switzerland, show pictorially how the flow of water through a pump with a finite number of impeller blades differs from the flow of an ideal fluid through a pump with an infinite number of blades, as assumed in the elementary theory. He experimented with a special radial flow pump which had plate-glass windows in the upper impeller disk so that photographs could be made of the flow of the water between the blades. The impeller disks were 400 mm. in diameter and 40 mm. apart. The impeller blades were curved backward on a radius of 196 mm. and were placed so that the forward tangents to the blade and the outer impeller circle intersected at an angle of 30 degrees. A motion picture camera was mounted on the vertical axis of the pump and the relative flow of the water through the pump was recorded by photographs. Aluminum filings served to indicate the movement of the water in some of the tests and a concentrated solution of potassium permanganate introduced into the water through syringe needles indicated the stream lines in others.

In the first series of tests the entrance and discharge circles were closed with plates and one of the cells thus formed between the blades was filled with water while the adjacent cell was filled with oil. The photographs of these fluids, taken as the impeller rotates, show no appreciable eddy in the oil cell, but they do show an eddy in the water with an angular velocity, relative to the rotating disk, nearly equal to the angular velocity of the impeller. If the impeller is rotating clockwise the relative eddy motion is counterclockwise, and conversely. Before the eddy is disturbed by the effect of the viscosity of the water it bears a very close resemblance to the eddy determined by the elastic membrane method.

In the second series of tests, in which the pump was operated against a closed discharge valve, eddies were again formed which behaved much like those mentioned. The viscosity of the water soon disturbed the results as before, and violent eddies, nearly circular in shape, formed between the blades at the inlet and the discharge circles of the impeller blades. Evidently, these eddies transform a large amount of the energy input into heat and Oertli calls attention to the fact that the pump becomes warm when it is operated against a closed discharge valve.

In the third series of tests the pump was operated at three different speeds, but discharged the same quantity of water at each speed: (1) At "normal speed" which was that speed at which the water entered the impeller relatively tangential to the blades at entrance; (2) at one-half the normal speed; and (3) at twice the normal speed. The stream lines were indicated in each test by means of concentrated potassium permanganate solution introduced through syringe needles. Oertli concludes from these tests: (1) That the relative flow through the impeller is not steady, but the pulsations which occur do not seem to have any definite period. They are caused primarily by friction and consequent eddies, etc.; (2) that for two conditions of normal

* H. Oertli, "Undersuchung der Wasserstroemung durch ein rotierendes Zellen-Kreiselrad," Dissertation, Technische Hochschule, Zurich, 1923.

flow, that is, for two equal ratios of $\frac{Q}{n}$, the directions of the middle stream lines are the same. The flow at the discharge end of the impeller is less quiet for a smaller discharge than for a larger discharge since larger eddies are formed at small flows; (3) that the absolute direction of flow at the entrance to the impeller is radial regardless of the value of $\frac{Q}{n}$; and (4) that the relative velocity at the discharge end of the blades is deflected from tangential to the blades (as indicated by the elementary theory) clockwise if the impeller rotates in a counterclockwise direction, and conversely. The angle of deflection increases as $\frac{Q}{n}$ decreases.

The results of Professor Spannhake's work are in agreement with Oertli's Conclusion (4). This is evident if one constructs the conventional discharge velocity diagrams for different quantities of flow through a given pump and reduces the absolute velocity vector in the ratio, $\frac{C}{C_\infty}$ (see Table 1), for the given quantity of water flowing through the pump.

The problems of hydrodynamics are of such a complex nature that even a study of an ideal fluid entails mathematical difficulties which have not been surmounted. The problems are infinitely more complicated if the viscosity of the fluid is taken into consideration in the primary analysis and it is not to be expected that any theoretical equation, similar to the author's Equations (B) and (C), can be developed for the flow of water through a pump. The elementary theory as a first approximation of the solution of a given problem is by far the most logical and satisfactory one that has been proposed. The newer theory, which considers the effect of a finite number of impeller blades, indicates corrections to the velocity diagrams of the elementary theory which bring them more nearly into agreement with those determined from actual performance tests. Subsequent corrections for friction, sudden changes in velocity, and recirculation can be made only on the basis of experimental work. The characteristic curves that follow from the elementary theory and the corrections that are logically necessary if a real fluid is considered, are not unlike those determined from tests of centrifugal pumps.*

* Pfleiderer, "Die Kreiselpumpen," p. 71 and 123 *ff*, J. Springer, 1924.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS AND DISCUSSIONS

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WATER-POWER APPRAISALS

Discussion*

BY LE ROY K. SHERMAN, M. AM. SOC. C. E.

LE ROY K. SHERMAN,† M. AM. SOC. C. E (by letter).‡—This paper is valuable in presenting a subject upon which there exists a diversion of opinion as to procedure and still wider diversion in conclusions even when the same fundamental procedure is accepted. The author criticizes§ the "Steam Substitution Method", the "Real Estate Sales Method", and the basis of relative values presented by the New England Water Works Association.

It is true that, in general, the value of water-power rights are best indicated by the Capitalized Net Earnings Method. The paper conveys the impression, by the example cited, that the Capitalized Net Earnings Method furnishes a simple, inflexible rule whereby the value of potential water-power rights may be precisely measured.

Value, however, cannot be determined by any arithmetical rule, because many elements of present value are dependent on future conditions subject to more or less exact forecast. The rule may be an indication of value but it is not a final and exact measure. For this reason it is desirable, in arriving at the value of a property, to use all the available rules and procedures within their limitations and to secure all the possible indications of value as a guide to final judgment.

The author points out a wide variation in the value of potential water powers indicated by the rule set forth by the New England Water Works Association. This is not a valid criticism. Wide variations in value of potential water powers do exist and they will also be indicated by the Capitalized Net Earnings Method.

It is the purpose in what follows to point out some of the pitfalls, precautions, and need for experienced judgment in applying the Capitalized

* Discussion on the paper by William H. Cushman, M. Am. Soc. C. E., continued from February, 1928, *Proceedings*.

† Pres., Randolph-Perkins Co., Chicago, Ill.

‡ Received by the Secretary, January 11, 1928.

§ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, pp. 1843–1844.

Net Earnings Method to secure a reasonable indication of the fair market value of a water-power right. The writer has in mind especially the valuation of a prospective or undeveloped power right.

At the outset it is well to bear in mind that the Capitalized Net Income Method is based on the simple fact or definition, expressed as an equation:

Net Annual Income = a Percentage of the Investment or Gross Income — Operation = % (Each Item of First Cost).

If all the factors but one are known, that one can be determined whether it be the value of power rights, turbines, percentage on investment, an item of operation, or the value of coal in a steam plant.

Factors Affecting Value by the Capitalized Net Income Method

Capitalized Errors.—Any error in the estimate of annual net earning is increased from eight to thirteen times by the process of capitalization in arriving at the indicated value. In the example given by Mr. Cushman (Table 1 *) an error of 5% in the estimated annual net earning would make an error of 11.43%, or $76 785, in the indicated value of undeveloped water rights.

It is essential, therefore, in applying the Capitalized Net Earnings Method, to prepare all estimates in complete detail with the utmost possible accuracy. Approximations may lead to highly erroneous conclusions. In general, the actual procedure in tabulating the estimate must be analyzed in far more detail than is shown in the example set forth in the paper and should include in the cost of development the cost to make the industry a going concern and the cost of preliminary engineering investigations.

Such cost of speculative investigation is often equivalent to the discovery of a water power theretofore unknown to the riparian owner.

Flowage.—The forecast for dependable flowage through future years materially affects the indicated value. In some cases the available record of stream flow is sufficient for a very precise and reliable forecast. In other cases, these data are meager. In the latter case the power-rights value can be predicated only upon very conservative figures for flow, or else the percentage figures for capitalization must be made high to offset the risk involved to the invested capital.

The author criticizes current practice of averaging monthly flowage in the order of their dryness.† This procedure gives lower figures than the calendar month averages. It also eliminates an error from the apparent available power during the year. For example, a 10-year average of the September flowage might contain nine dryest months and one heavy flood, such as occurred in Illinois in 1926. It may be that the one September flood discharged more water than the capacity of the power plant. If averages were taken on a calendar-month basis the September average would show a higher available rate of flow than really existed.

Load Factor.—The recorded yearly load factor of many power companies ranges from 25 to 90%, with a large number operating at from 40 to 50%

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1847.
† *Loc. cit.*, p. 1846.

load factor. In a plant with no reservoir storage, the annual power delivered is the load-factor percentage of the useful power of the stream to the limit of economical installation. In a plant with reservoir storage capacity sufficient to equalize this natural flow to the demand there is no loss of useful stream power, but the total machinery installation, as before, is in operation only a part of the time. The forecast of what the load factor will be, particularly in the case of undeveloped powers, is important and error in its determination affects the estimated annual net earnings and, consequently, the indicated value of the potential water power.

In the example given by Mr. Cushman, if the load factor turned out to be 65% instead of 75%, the indicated value of the undeveloped water rights would be $397 460, or $274 310 less than the value predicated upon a load factor of 75 per cent.

Market Price for Power.—This is another element of the estimate that requires foresight into the future in case of proposed power developments. Primary power market value can generally be ascertained within 10 per cent. The probable sale of secondary power and its market value is less determinate and is subject to wide variation. The income figure for secondary power, therefore, should be used conservatively.

Of course, the "market rate" should be coincident with the point to which the estimated cost of development is carried, whether this be at the power-house switch-board, at the end of a transmission line, or to the ultimate consumer. The market value changes radically with the point of delivery of power. Sometimes, this coincidence of rate and location has not been rigidly followed, and the indicated value of power rights are consequently erroneous.

Public Ownership.—Potential water power may be owned wholly, or in part, by the State or by the public. This has nothing to do with the value of a particular water right. That value is the same whether there is one owner or two and whether one of these owners is the State or an individual. If there is State ownership, or part State ownership, then only a part of the value of the water power rights belongs to the riparian owner. Is there any public ownership of part of the value of water-power rights as against an individual riparian owner? Unquestionably, certain acts or powers of the State do affect the value of a potential water power. The State may often determine whether the potential power has any value whatsoever.

The following recognized powers of the State affect the value of an individual's riparian water-power holdings:

1.—The right of eminent domain and the various "mill acts."
2.—A permit to erect a dam.
3.—The paramount right of navigation against the development of power.
4.—A franchise to a public utility power company granting it a monopoly of the adjacent market.

The right of eminent domain may create a continuous railroad right of way, the value of which is in excess of that of the adjacent farm lands. The excess value, however, is owned entirely by the railroad company. The public utility franchise may affect the sales value of adjacent potential powers. A permit for a dam at one site decreases or destroys the value of any dam

sites for some distance down stream, even if it does not affect the value of flowage rights.

A public utility franchise is generally accompanied by rate regulation which limits or reduces the possible net earnings and which, in turn, has its effect in reducing the value of water power rights.

In general, it would appear that any indicated value of water power rights computed on the basis of State-regulated rates, belongs wholly to the riparian owner. In this case the State recovers its share of any publicly owned value through the medium of reduction in rates. There is a natural limit to such reduction which the State may effect, regardless of the fact that in some cases high net earnings from a water power may exist; for if the public service rate is unduly low, the water power will be used by the owner for private industrial operations. The Courts and State Commissions have frequently recognized partial public ownership of indicated water power values.

Steam Comparison.—Mr. Cushman has referred to the fact that the Courts have resisted the Steam Substitution Method.* Probably the inherent reason for this is the recognition that the face value of power in a particular locality (and hence the ultimate market value of power) is conditioned upon a fair return on the investment. The existence of local water power will have its effect on the ultimate market value of power. The Steam Comparison Method is an assumption that market rates for power are governed by the cost of steam power production. While this is generally true (due to the demand for power being in excess of the available production of water power), there are many situations where it is not true, and fair rates for water power production will govern the market.

This does not mean that the Steam Comparison Method is without value to the appraiser seeking indications of the value of a water-power right. Take, for example, the case given by the author. A steam plant could be built and operated to turn out the same power, bring in the same gross revenue, but the return on the investment would not be 10%, it would be 8% or $8\frac{1}{2}$ per cent. This steam comparison in itself indicates that the water-power rights in question do have material value. In other cases the steam comparison may show that the local water-power rights have no value.

An indication of no value for power rights, however, does not necessarily mean that the owner of the property could not develop the power and secure a good income on his investment.

Rate of Interest on the Investment.—This factor is perhaps the most important one in the equation for solving the indicated value of undeveloped water-power rights. It is largely a matter of experienced judgment. The indicated value will not be approximately correct unless the assumed rate for capitalization is the true percentage that in the future is actually going to accrue to the operator and investor.

The fair rate of return for money of an investment depends on the risk and work involved. The rates of interest or the market price for capital ranges somewhat as follows:

* *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 1844.

Investment.	Charge.
(*a*) Little or no risk, responsibility, or work, on Government bonds, good first mortgage securities, etc.	4 to 6%
(*b*) Little risk. Maintenance of value dependent on good main agent and work as a proven going public utility	7 to 8%
(*c*) Some risk, without responsibility, as a proposed public utility of a future apparent value equal to Investment (*b*)	8 to 10%
(*d*) Considerable risk, work and responsibility as a proposed competitive industry. Some elements speculative or indeterminate	12 to 20%

What rate now shall be used to determine the indicated value of a potential power? The answer is, the price which an investor, operator, and developer is willing to pay the riparian owner, the "fair, cash, market value" and not the "speculative value."

The riparian owner, or seller, will receive cash. He carries no future responsibility, work, or risk, as does the buyer and operator. The true cash value of the riparian or power rights, therefore, should be on the same basis as the cash market value of each of the other items of cost which are included in the equation,

$$\text{Net Earnings} = \text{Percentage on Investment}$$

Power rights are in the same category with the cash market value of turbines, generators, concrete, or coal, or any other item of the equation. If the equation is applied to a steam plant and solved for the value of coal, the true market value for coal will be found, provided the appraiser assumes correctly the actual true percentage of net earning on the investment. Likewise, in a water power plant, the true cash market value for power rights will be found, provided the appraiser assumes correctly the true percentage of net earning which will actually exist for future time. If this assumption of percentage is not exact, then the indicated value of the power rights, coal, or other value in question, will be in error.

In order to eliminate speculative value from the indicated value of power rights, two procedures are available. One is to take the assumed interest rate sufficiently high so that all the elements of risk in the equation are discounted in the process of capitalization.

The other procedure is to take, consecutively, figures for all the elements in the equation which involve any forecast or element of uncertainty and then use the minimum allowable interest rate, say, 8 per cent. The elements involving uncertainty are: Market value of power, load factor, flowage of the stream, flood risk during construction, etc.

The Allocation of Values of Different Sections of a Water Power Right

Mr. Cushman has considered the appraisal of a water-power right as an entirety. Frequently, it is necessary to segregate this value among several riparian owners. The writer would like to consider this feature as a logical supplement to the paper.

The total value of a water-power right may be divided into the following elements:

1.—Value of flowage and fall.
2.—Special value of a dam site.
3.—Value of land used as a storage reservoir.

Flowage Value.—In the simplest case, that is, a power development on a section of a stream having a uniform rate of fall and without any special economy or advantage at one point over another for the location of the dam; the value of any riparian frontage is in the proportion of the section or distance between the tail-water of the power house and the upper limit of backwater. If the rate of fall varies, or if the flowage is greater at the lower end of the section, the value of any riparian frontage is in proportion to the potential power that the frontage bears to the total potential power in the section under valuation. This valuation of flowage includes not only the water rights, but also a strip of land equal in width to the average width of the overflowed land through the section. Some frontages, particularly those down stream, would have a greater width of overflow and others less width of overflow land. Some adjustment for this inequality should be made without affecting the average value.

Value of Dam Site.—If one riparian frontage, on a section of stream under consideration, has a site which permits the construction of a less expensive dam than elsewhere in the section, this frontage has a relatively greater value than the other frontages.

The value of this frontage due to the dam site, can be measured by the difference, in total water-power values, between the development at this site and a development at the next best adjacent dam site. This difference is not necessarily the same as the difference in cost between the two dams.

The difference between the total value of water rights and the value of the dam site thus ascertained gives the amount of the total flowage value.

Value of Storage Reservoir Acreage.—If the flow of the stream is regulated by a storage reservoir, the amount of power and gross income is increased at additional construction cost, additional cost of operation, and the overflow of additional land. What is the additional overflowed land worth for this water-power purpose? It can be measured by the same capitalized net income procedure, namely, increased gross earnings (due to reservoir) less increased operation cost equals the income on the sum of additional construction cost plus the value of additional land overflowed.

The rate of return on the additional investment for reservoir is not necessarily the same as that on the development for run of the stream. It is generally less; therefore, the percentage for capitalization should be greater.

The value of the riparian acreage subject to storage and intermittent overflow due to the dam furnishing the head for the power plant is not to be confused with any auxiliary reservoir such as that just discussed. The value of the lands overflowed by high water at the power dam is included in the value originally determined by the writer under the heading of "Flowage Value."

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A GRAPHIC METHOD FOR DETERMINING THE STRESSES IN CIRCULAR ARCHES UNDER NORMAL LOADS BY THE CAIN FORMULAS

Discussion*

By Lars R. Jorgensen, M. Am. Soc. C. E.

Lars R. Jorgensen,† M. Am. Soc. C. E. (by letter).‡—Most engineers who have had to make calculations of arch dams, have no doubt been looking for a method to simplify this tedious work. The first that suggests itself is the plotting of curves to avoid as much of the routine calculation as possible. The writer has seen at various times several sets of curves intended for the simplification of arch dam calculation. Such curves have also been used to great advantage for several years in the office of the Constant Angle Arch Dam Company; but until Mr. Fowler's curves appeared the writer had never seen a set where the result sought could be obtained by looking for it in just one place, on just one curve. When the writer saw them for the first time, he wondered where Mr. Fowler had made a mistake, since he had so easily been able to simplify arch dam calculation to the limit.

The author is to be highly commended on his work in plotting these curves because they are of very great help to any one interested in arch dam calculation. In Figs. 10 to 13,§ inclusive, the principal part of the information necessary for the calculation of stresses in any arch dam is available. Mr. Fowler has seized upon the clever idea of using the ratio, $\frac{t}{r}$, for plotting purposes, which is the main reason for the simplicity.

At and toward the very bottom of an arch dam, arch action is very ineffective since other actions (principally punching shear along the contact area) will resist the load with less deformation required and, consequently,

* Discussion of the paper by Frederick Hall Fowler, M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Cons. Engr., Constant Angle Arch Dam Co., San Francisco, Calif.

‡ Received by the Secretary, December 27, 1927.

§ *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, pp. 1908 *et seq.*

will take the greater part of the load. Inasmuch as it is generally difficult to design an arch dam (using the conventional arch formulas) where the arch is short and thick, as is mostly the case at and near the bottom of a dam, the ability of the punching shear developed along the contact area, to hold the dam in place, should be investigated.

Whenever this punching shear stress is less than 65 lb. per sq. in., for instance, on the contact area between the rock and the concrete, the structure should be entirely safe, assuming all load to be resisted by shear.

If the bottom is calculated as an arch, carrying the full water load and using conventional formulas, such as those of Professor Cain, stresses of 800 lb. compression and as much as 200 lb. per sq. in. tension should be allowable. Such calculated stresses may generally be more conservative in this part of the dam than one-half these values would be at a point higher up where the arch has to take the load acting as an arch. The load, at the bottom especially, divides between punching shear, arch action, cantilever action, etc., with punching shear generally taking the largest share. In calculating the structure as an arch close to the bottom, it is safe to use larger unit stresses, both in compression and in tension, as it is known that the actual stresses will be considerably lower than the calculated stresses.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

SURVEYING AND MAPPING IN THE UNITED STATES

Discussion*

By John C. H. Lee, Assoc. M. Am. Soc. C. E.

John C. H. Lee,† Assoc. M. Am. Soc. C. E. (by letter).‡—This interesting and instructive paper accurately describes the harmonious relation and efficient co-operation which have been developed between the U. S. Geological Survey and what Army engineers like to consider its old parent organization, the Corps of Engineers. The same general specifications and standard requirements of accuracy are recognized, so that the economically important topographical maps of the Geological Survey are becoming, in all essential respects, satisfactory tactical maps for the military forces. Also, the accurate military surveys are being accepted as standard topographic sheets.

The author gives an interesting résumé of the widely needed project for completing the general topographic map of the United States. How important the completion of this survey project is to National defense, can perhaps be appreciated with the realization that among the 60% of uncompleted sheets, there are many vitally important strategic areas. Modern military doctrine recognizes the importance and almost the necessity of correct topographical maps as military plans for large troop concentrations and for successful field operations. Divisions and larger units cannot be rapidly and reliably moved or supplied without good maps. American military forces are all trained under this principle. The use of maps becomes second nature to them. Without maps their operations would be greatly hindered and their success jeopardized. Nevertheless, the fact remains that along wide stretches of this country, including areas of vital strategic value, there are still no maps worthy of the name. From a military standpoint, therefore, the program authorized under the Temple Act should be carried to completion, especially along the borders of the United States.

* Discussion on the paper by C. H. Birdseye, M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Maj., Corps of Engrs., U. S. A.; Dist. Engr., Vicksburg, Miss.

‡ Received by the Secretary, January 16, 1928.

From the Army engineer's standpoint, moreover, the project should be completed within the borders as well. Studies of flood control and waterway improvement not only can be materially facilitated through good topographic maps, but, as was the case recently in the Mississippi Valley, can be seriously hampered without them. This need was brought into bold relief during the high-water fight of 1927. There were no reliable maps available for much of the back-water country over which extremely important relief operations were being conducted.

The Army engineer recognizes also the value and importance of good topographic maps in planning railway and highway communications with which he is charged in time of war within the period of operations. Whereas no one now sees the eventuality which would bring warfare within the borders of this country, it may be recalled that five or ten years prior to the World War, practically no one ever thought that American troops would fight in Europe.

Since this problem confronts the Army—the problem of no maps for important strategic areas—the task of making emergency maps during actual operations must be faced. The solution of this difficulty is being sought through experiments in aerial photographic surveying and map reproduction in the field. The problem of rapid field triangulation that can be carried on without molestation of the enemy; the rapid and reliable means of obtaining the necessary over-lapping photography; and, finally, a swift and certain method of reproducing the completed map in quantity under field conditions, becomes a task worthy of high engineering talent. This work is all under the direction of the Chief of Engineers. It is of the highest importance in a military sense and not without its peace-time economic value.

As stated* by Colonel Birdseye, aerial photographic surveying has become an important adjunct to the general project, the total cost of which would seem to justify the expenditure of sufficient funds to develop the new art in a thoroughly practical sense. This development is being carried on by the War Department, which has, fortunately, a combination of skilled fliers and competent map-makers working together in harmonious co-operation.

* *Proceedings*, Am. Soc. C. E., October, 1917, Papers and Discussions, p. 1926.

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PAPERS AND DISCUSSIONS

AERIAL TRAMWAYS

Discussion*

By Messrs. George Paaswell and Casper D. Meals.

George Paaswell,† M. Am. Soc. C. E. (by letter).‡—While the entire paper is so interesting and unusual, both in its topic and in its lucid manner of presentation that the writer would like to discuss every feature of it, he is confining these notes to a treatment of two minor points, the hyperbolic functions and the cubic equation. The former may be computed to any degree of exactness from the trigonometric functions and tables and the latter may be solved directly. Few engineers seem to be aware of this latter fact.

The hyperbolic functions bear a direct relation to the circular function of a characteristic angle the geometric significance of which need not be discussed herein. The relations may be given as follows: $\sinh u = \tan v$; $\cosh u = \sec v$; $\tanh u = \sin v$; $\coth u = \csc v$; $\operatorname{sech} u = \cos v$; and $\operatorname{csch} u = \cot v$. The relation between u and v is given by the formula,

$$u = 2.3025851 \log \tan \left(45^\circ + \frac{v}{2}\right)$$

The method of computation will be seen in the solution of the cubic equation. Given a cubic equation in its most general form,

$$x^3 + a\,x^2 + b\,x + c = 0$$

it is reduced to a form,

$$y^3 - 3\,p\,y - 2\,q = 0$$

by substituting $x = y - \dfrac{a}{3}$.

The roots of the cubic equation in y are then found from the following, depending upon the signs of p and q:

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* Discussion on the paper by F. C. Carstarphen, M. Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Engr., Corson Constr. Corporation, Brooklyn, N. Y.

‡ Received by the Secretary, November 15, 1927.

(1) If both p and q are plus and q^2 is greater than p^3, use the substitution,

$$\cosh u = \frac{q}{\sqrt{p^3}}$$

The three roots are then,

$$y_1 = 2\sqrt{p}\cosh\frac{u}{3}$$

$$y_2 = -\frac{y_1}{2} + i\sqrt{3\,p}\sinh\frac{u}{3}$$

and,

$$y_3 = -\frac{y_1}{2} - i\sqrt{3\,p}\sinh\frac{u}{3}$$

(2) If p is positive, q, negative, and q^2 is greater than p^3, use the substitution,

$$\cosh u = \frac{-q}{\sqrt{p^3}}$$

and the three roots are,

$$y_1 = -2\sqrt{p}\cosh\frac{u}{3}$$

$$y_2 \text{ and } y_3 = -\frac{y_1}{2} \pm i\sqrt{3\,p}\sinh\frac{u}{3}$$

(3) If p is negative,

$$\sinh u = \frac{q}{\sqrt{-p^3}}$$

and the three roots are,

$$y_1 = 2\sqrt{-p}\sinh\frac{u}{3}$$

$$y_2 \text{ and } y_3 = -\frac{y_1}{2} \pm i\sqrt{-3\,p}\cosh\frac{u}{3}.$$

(4) If p is positive and q^2 is less than p^3, substitute,

$$\cos u = \frac{q}{\sqrt{p^3}}$$

and the three roots are,

$$y_1 = 2\sqrt{p}\cos\frac{u}{3}$$

$$y_2 \text{ and } y_3 = -\frac{y_1}{2} \pm \sqrt{3\,p}\sin\frac{u}{3}$$

Refer to the cubic equation,

$$0.0000457\,t - 0.474 = \frac{666\,666\,667^*}{t^2}$$

and, to reduce the size of the coefficients, take the unit load as 100 000 lb. The equation then reads,

$$4.57\,t^3 - 0.474\,t^2 - \frac{1}{15} = 0$$

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2153.

or,

$$t^3 - 0.1037\,t^2 - 0.01459 = 0$$

reducing to the standard form by placing

$$t = x + \frac{0.1037}{3}$$

$$x^3 - 0.003585\,x - 0.0145 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (115)$$

In Equation (115) $p = 0.001195$ and $q = 0.00725$. This form of the cubic equation comes under Case (1),

$$\cosh u = 175.6$$

Then, from the circular relations already given,

$$\log \cosh u = \log \sec v = 2.2445245 \ldots . \quad v = 89^\circ\ 40'\ 24''$$

From the relation between u and v,

$$u = 2.302585 \log \tan 89^\circ\ 50'\ 12'' \ldots . = 5.8602$$

$$\frac{u}{3} = 1.9534.$$

To obtain v,

$$1.9534 = 2.3026 \log \tan \left(45^\circ + \frac{v}{2}\right) \log \tan \left(45^\circ + \frac{v}{2}\right) = 0.84835$$

and,

$$v = 73^\circ\ 51'\ 40''$$

$$\cosh \frac{u}{3} = \sec 73^\circ\ 51'\ 40'' = 3.5975.$$

The real root of Equation (115) is, therefore,

$$x = 3.5975 \sqrt{(0.001195)} = 0.24873$$

$$t = x + \frac{0.1037}{3} = 0.28363$$

or, reducing to pounds, the real root is 28 363 lb.

These processes illustrate the comparative simplicity of both the computation of hyperbolic functions from trigonometric tables and the direct solution of the cubic equation.

Casper D. Meals,* Assoc. M. Am. Soc. C. E. (by letter).†—This comprehensive paper fills a big gap; the wealth of practical, usable data therein bespeaks the concluding thoughts, namely, that aerial tramways are best designed by those who are fitted by experience and training to do so.

Dumping Cradleways, or Contractors' Suspension Bridges.—When calculating the size of the cables to be used, the live load moving on to the platform is to be taken as the empty car weights only, the material being dumped off the cars as they pass on to the platform. For a 9-car trip, 8 empty cars and

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* Chf. Engr., American Cable Co., Inc., New York, N. Y.

† Received by the Secretary, December 29, 1927.

1 loaded car will be on the platform, but part of the reaction of the latter is carried on the bank end, so that to consider the moving load as 9 empty cars will be sufficiently accurate.

The platforms are usually 14 ft. wide, 75 to 130 ft. long, and weigh from 170 to 250 lb. per lin. ft. and the suspender rope spacings are from 10 to 18 ft. on centers, the average being 12 ft. The empty cars weigh from 5 000 to 9 000 lb., the average being 6 000 lb. The suspenders are generally made up of ½ or ⅝-in., 6 by 19, plow steel rope, and two double sheave tackle blocks; 10-in. blocks for the ½-in. rope, and 14-in. blocks for the ⅝-in. rope.

The hold-down tackle at the end of the platform is usually of ¾-in., 6 by 19, plow steel rope, reeved on 16-in. double sheave blocks. The deflection of the main cables is usually taken at 6 to 7% of the length of the span and the size of the cables may be based on a factor of safety of three as a minimum on the actual breaking strengths of the cables. For the one installation of a dumping cradleway noted by Mr. Carstarphen,* the main cables are given as 2½-in., 6 by 37, plow steel rope. A 2¼-in., 7 by 19, galvanized plow steel, bridge cable of approximately the same breaking strength would have been more economical.

The actual breaking strengths of galvanized plow steel bridge cables are in excess of the manufacturers' standard lists, and data pertaining to suitable size of bridge cables for these dumping cradleways are given in Table 42.

TABLE 42.—Cable for Dumping Cradleways.

Diameter of rope, in inches.	Construction.	Pounds per foot.	Metallic area, in square inches.	Approximate Breaking Strengths, in Tons.	
				Catalog.	Actual.
1½	7 by 7	3.70	1.057	90	98
1⅝	7 by 19	4.34	1.257	106	115
1¾	7 by 19	5.10	1.460	124	140
1⅞	7 by 19	5.90	1.630	144	158
2	7 by 19	6.73	1.816	164	175
2⅛	7 by 19	7.60	2.230	185	195
2¼	7 by 19	8.52	2.436	208	220
2⅜	7 by 19	9.5	2.722	232	250
2½	7 by 19	10.5	3.106	256	275
2⅝	7 by 19	11.6	3.522	283	300
2¾	7 by 19	12.7	3.774	310	330
3	7 by 37	15.5	4.243	360	385

The modulus of elasticity of these cables may be taken at 17 000 000 lb. per sq. in. for new cable, and 20 000 000 lb. per sq. in, for cable that has been repeatedly stressed.

The maximum stress in the main cable span may be determined from:

$$T_{\text{max.}} = \left\{ \frac{w\,S^2 + 4\,\rho\,k\,(S - k)}{8\,d} \right\} \sec\beta \ldots\ldots\ldots\ldots \quad (122)$$

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2108.

The unloaded cable deflection (see Fig. 24) at which it must be erected before any dead or live loads are applied, may be determined from,

$$d_1 = d - \frac{15 \Delta L}{16 (5 b - 24 b^3)} \qquad (123)$$

in which,

$$\Delta L = \frac{H_1 S}{A E} \left(1 + \frac{16 b^3}{3}\right)$$

$$H_1 = \frac{4 p k (S - k)}{8 d}$$

$$b = \frac{d}{S}$$

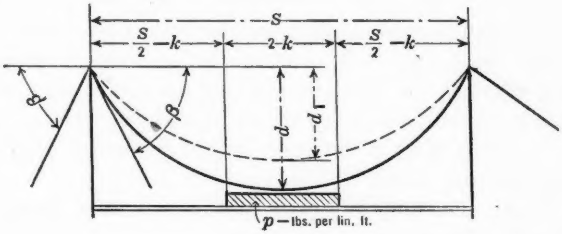

FIG. 24.

Track Cables.—The breaking of a wire in a smooth-coil track strand does give trouble by uncoiling to the nearest tower support or coupling. However there has been developed for this service a pre-formed, smooth-coil, track strand in which this objection has been eliminated, the wires being pre-shaped to their position in the cable and, therefore, being unrestrained in their place in the strand. The outer wires will stay in place and also will wear to a much thinner section before breaking, thus prolonging the life of the strand.

The physical properties of this type of strand (known as the Tru-Lay Brand), correspond to values given in Table 6.* The E-values may be taken at 20 000 000 lb. per sq. in. for a 19-wire strand, and 18 000 000 lb. per sq. in. for the 37-wire strand. For a greater number of wires it will be even less, as there is relatively greater looseness in the strand construction of a 61-wire strand than in that of a 19-wire strand and, consequently, the E-value of the former must be lower.

The deflections of the main cables are usually limited from 2 to 2½% of the span length, s, so as to eliminate steep grades at the towers which would require a larger traction rope and greater engine duty.

The frictional hysteresis loops noted by Mr. Carstarphen* for progressive and retrogressive loadings of the strands have been noted also by the writer in tests made of both strands and wire ropes.

The "rocking saddle", shown in Fig. 5,† should have the merit of reducing the wear and abuse on the track strand or cable at the saddle by pivoting,

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2119.
† *Loc. cit.*, p. 2121.

and thereby reducing the angle that the cable will make when the carriage is approaching or going off the saddle. If this point is attained, then the rocking saddle would eliminate the use of protection hoods over the track cables at the saddles.

Couplings for Joining Together Sections of the Track Cables.—The writer was not aware that couplings to smooth-coil track strands were ordinarily attached in the same manner as the locked-coil and wire strands by means of wedges. Certainly, a zinc socket attachment should be more effective, considerably cheaper, and should eliminate the possibility of loose wires in the strand, because, unless extreme care is exercised in driving the wedges in that particular type of connection, some wires will be disturbed and loosened in the strand.

Traction Ropes.—It has been the writer's experience that plow steel ropes will give as economical service as cast steel or mild plow steel; and for severe service, the plow steel rope will prove more economical because, being of a higher grade of steel, it will better withstand wear and offer as great a fatigue resistance operating over the sheave equipment. Occasionally, 6 by 19 Seale construction and 6 by 8 flattened strand ropes are used, although the 6 by 7 is more generally used.

Lang Lay ropes offer the following advantages over Regular Lay ropes of the same length of rope lay:

> (*a*) Greater wearing surface, each outer wire having approximately three times the length exposed for wear, leading to lower bearing values.
> (*b*) Greater flexibility, because the outer wires of the strand are parallel to the axis of the rope against the manila core and set up less frictional resistance in bending.
> (*c*) Lower bending stresses, the bending stress in a Lang Lay rope being approximately 80% of that in a Regular Lay rope of the same construction.

The bending stress in a wire is greater the farther away it is from the axis of the rope and the nearer it runs parallel to the axis. In a Regular Lay rope, this combination of unfavorable conditions is found. In a Lang Lay rope it is not found, because where the wires do lie parallel to the axis of the rope, they are nearest to the axis and are embedded in the manila core of the rope.

A Lang Lay rope is inherently "cranky," due to the fact that the wires in the strands, and the strands in the rope, are laid up in the same direction. A Lang Lay rope of the Tru-Lay type, fabricated with pre-formed wires and strands, will eliminate entirely the "crankiness" so typical with Lang Lay ropes and will also wear longer and withstand bending better because the individual wires and strands of the rope are in an unrestrained position.

Carriers.—Carriages are usually made up with two sheaves of 10 to 14 in. treads in a pivoting frame attached to the bucket bale as described by Mr. Carstarphen.* The writer would like to know whether the weights given in Table 8† include the weight of the carriage with sheave and the grip? The

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2123.
† *Loc. cit.*, p. 2124.

carriage, with sheaves, weighs approximately 100 to 150 lb., and the grips from 35 to 75 lb., depending on the type and the loads.

Data pertaining to minimum steel tower weights for 8-ft. gauge cables, for towers, 65 to 100 ft. high, are as follows:

Height, in feet.	Weight, in pounds.
65	10 000
70	11 000
80	13 000
85	14 500
100	17 000

Bolts and washers for timber towers of the closed type, including the anchor-bolts and washers, will be approximately as follows:

Height, in feet.	Weight, in pounds.
10	400
20	425
30	500
40	650
50	800
60	1 100
70	1 300
80	1 800

Tension in Traction Ropes.—The coefficient values as given for K include that of a greasy rope on a rubber-lined and leather-lined sheave. Is it implied from this that such driving sheaves are used? Unless the unit radial bearing pressure of the rope on the rubber and leather lining is kept within a value of 65 to 75 lb., the lining will soon be cut out.

The radial pressure of a rope in a sheave groove is a function of the rope tension and the sheave diameter. It should not be confused with the resultant of the tensions in the rope with regard to the angle the ropes make with each other. The unit radial pressure is determined from:

$$u = \frac{P}{r\,d} \quad \text{(124)}$$

d being the rope diameter; r, the radius of the tread of the sheave, in inches; and P, the load, in pounds.

Size of Track Cables.—If the choice is guided by experience, it will pertain to the construction rather than the size; for, after all, the stress requirements, which are calculable, are the governing consideration in the size and quality. The bending stresses need not be considered because the deflections are relatively so shallow and the traveling loads so light, that the bending of the track cables by the carriage wheels or at the tower saddles is not of prime importance. Bending stresses and the reduction or loss in strength of a wire rope bent around a sheave are not analogous.

Bending stresses are applicable only to moving ropes operating over sheaves and have to do with their "efficiency" in service and not to their

reduction in breaking strength. There must be a limit to the smallness of the sheaves for operating ropes in relation to the rope diameter of various constructions. By this is meant that there is a "critical" sheave diameter, smaller than which the strands of the rope cannot slide past each other to adjust themselves to the bend and, consequently, constructional restrictions are set up within the rope. This condition sets up excessive internal stresses absolutely beyond the scope of mathematical treatment, although the writer is convinced that bending stresses in wire ropes are not susceptible to precise mathematical investigation as there are too many variables involved, such as, relation of wire sizes, core condition and size, strand and rope lays, strand constructions, lubrication, etc.

The critical tread diameters of sheaves for various constructions of ropes are given in Table 43.

TABLE 43.—Critical Sheave Diameter Ratios for Operating Ropes.

Construction	6 by 7.	6 by 19 Seale.	6 by 19 Warrington.	6 by 25 Spacer Seale.	6 by 37.	8 by 19 Warrington.	8 by 19 Seale.
Critical ratio, $\frac{D}{d}$	28	20	16	16	14	14	16

For sheave sizes less than those given in Table 43, bending stresses are of little value in any calculations.

The reduction or loss in strength of ropes bent around sheaves (this applies only to non-operating ropes) has been reported* quite fully. It is shown that for 6 by 19 ropes the efficiencies are as given in Table 44, in which D is the tread diameter of the sheave, and d the rope diameter. From these values it will be seen that the loss in strength of a $\frac{3}{4}$-in., 6 by 19, plow steel rope bent around a 6-in. sheave is 24% instead of 8% as noted by Mr. Carstarphen.†

TABLE 44.—Efficiencies of 6 by 19 Ropes Bent Around Sheaves Under Static Load.

Ratio, $\frac{D}{d}$	8	10	12	14	16	18	20	22	24	30
Efficiency, percentage	76	79	81	86	88	92	93	95	95	95

The moduli of elasticity of various ropes, given by Mr. Carstarphen in Table 26‡ are at a variance with values secured by the writer in numerous tests. However, some of this may have been due to a difference in the strand

* "Some Tests of Steel Wire Rope on Sheaves," *Paper No. 229*, U. S. Bureau of Standards.

† *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2162.

‡ *Loc. cit.*, p. 2163.

and rope lays and also in the construction of the strand. Tests made by the writer gave the following values of E, in pounds per square inch:

19 wire strand.......	20 000 000
37 wire strand.......	18 000 000
6 by 7 rope..........	14 000 000
6 by 19 rope.........	13 000 000
6 by 37 rope.........	12 000 000
8 by 19 rope.........	10 000 000
6 by 19 rope with independent wire rope center	16 000 000
7 by 7 and 7 by 19 bridge cables......	17 000 000

These values are for new steel ropes, ½ in. in diameter, and larger. For old ropes that have been in service, the E-values will be approximately 20% in excess of the values given in the tabulation.

To indicate the strand construction variable in the E-values for wire rope: The 2¼-in., 6 by 37, suspender ropes on the Delaware River Bridge between Philadelphia, Pa., and Camden, N. J., were made up of a three operation strand, all wires being laid in the same direction, and the rope had a 7 by 7, Independent wire rope center of Lang Lay. The E-value was only approximately 12 000 000 lb. per sq. in., and for the first run or initial loading it was only approximately 8 000 000 lb.* Incidentally, the monograph from which these values were taken, verifies Mr. Carstarphen's data that the E-values increase with an increment in the load on the rope.

* These values are given in the recent monograph published as a Final Report of the Board of Engineers to the Delaware River Bridge Joint Commission of Pennsylvania and New Jersey, p. 120.

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PAPERS AND DISCUSSIONS

BAFFLE-PIER EXPERIMENTS ON MODELS OF PIT RIVER DAMS

Discussion*

BY MESSRS. B. E. TORPEN, OREN REED, AND HAROLD K. FOX.

B. E. TORPEN,† M. AM. SOC. C. E. (by letter).‡—The authors are to be congratulated for the thorough manner in which they have conducted these baffle-pier tests. The subject of scour below the overflow dams or spillways is becoming more important as more and higher dams are built, and is of especial interest to hydraulic and designing engineers. The scour below the Wilson Dam,§ and many other similar experiences with service dams, show the necessity of better design of spillway aprons. The type of spillway, and the treatment of the apron best suited to any special site are dependent on many factors, such as (1) safety, the factor of prime importance in all structures; (2) economy, a factor second only to safety in many cases; (3) type of dam, whether gravity, arch, earth, etc.; (4) architectural treatment; (5) hydraulic gradient of the stream; (6) geology; (7) topography; (8) climate; (9) proximity of power house or other structures; (10) percentage of time curve for certain flood flows; (11) depth of over-pour; and (12) depth to bed-rock for dam foundation. It cannot be expected, therefore, that a single series of tests on a special phase of this subject can establish the type of treatment best suited to all sites. It is hoped, however, that observed discharges during the next few years will demonstrate the similarity of actual conditions to the results of the tests, and prove beyond doubt the utility of models in illustrating new features for design.

In the Pit River tests the feature of causing the falling sheet of water to strike the mid-height of the splitter piers and in a downward direction, seems especially effective. This was secured by placing a 15-ft. jump-off at the bucket of the dam.

The Pit dams are arched in plan, but of such long radius that no stability can be depended upon from the arch, the dam being of the full gravity section. The bottom of the jump-off, therefore, must be outside the full gravity

* Discussion on the paper by I. C. Steele and R. A. Monroe, Members, Am. Soc. C. E., continued from January, 1928, *Proceedings*.

† Supt., Design and Constr., Bull Run Dam, Water Bureau, Portland, Ore.

‡ Received by the Secretary, December 12, 1927.

§ *Engineering News-Record*, February 3, 1927.

section and all concrete in the bucket and jump-off is chargeable to the spillway, in addition to the massive piers. The total of this concrete is no small item and can not be overlooked when comparing types from the standpoint of economy.

In selecting the type of spillway for the Pit dams the authors have stated their reasons clearly. Their tests indicated the type best suited to meet those conditions, and now actual overflow will be the final test. It is believed that other unpublished data on tests of spillway models may add to the value of these discussions and with this in mind the writer presents, as briefly as possible, the results of a series of such tests for the Bull Run Dam now under construction for the water supply of Portland, Ore.

The dam is 200 ft. high, of the concrete gravity type, and curved in plan as is common practice at sites slightly too wide for pure arches. The maximum flood may be 21 000 sec-ft., the free over-fall is 176 ft. and the minimum flow of the stream is about 160 sec-ft. The water-shed is only 75 sq. miles, but due to winter rains from 500 to 5 000 sec-ft. will spill over the dam half the time.

The topography is favorable for a spillway around the left end of the dam with a concrete apron to the edge of the bluff, 150 ft. above the river. The country rock is basaltic, in horizontal layers varying from 5 to 50 ft., and some flows are considerably altered. Some fear was entertained that the constant over-pour would erode this basalt and eventually require the spillway channel to be concreted to the water's edge, which made this location of the spillway rather costly. The alternate plan chosen was to carry the spillway over the central part of the dam to an apron, which extends down stream 150 ft. to protect the toe of the dam.

Toe Protection.—The question of further protecting the toe was next considered. Special treatment must be given the down-stream end of the apron also unless the velocity is reduced at this point. This has recently been illustrated at the Wilson Dam. To build heavy baffle-piers at the toe of the dam and concentrate the destruction of power up to 350 000 h. p. directly over the toe (the most dangerous part of the dam foundation) seemed undesirable. This type of treatment also requires a "bucket" 15 ft. deep, and much extra concrete is necessary.

Baffle-piers at the bucket cause considerable spray at the toe of the dam and this was objectionable because the outlet-valve house is located immediately adjoining the spillway on one side and a future power-house setting is on the other. The spill water may at times carry large logs (in spite of boom protection) with enormous striking effect when hitting the nose of a baffle-pier. The conditions for destroying energy at the toe of the dam by a standing wave were unfavorable since the river gradient is so steep that water at all stages flows at a depth below the critical. A subsidiary dam below the main dam would have been required to create a deep pool. This, in turn, would have had to be protected from down-stream erosion.

The best plan seemed to be to carry the spillway sheet at a high velocity across the apron to a point 150 ft. down stream from the dam and there deflect the sheet upward so as to secure the maximum distance and spread of

the striking area of the water mass and the river bed-rock. This plan was experimentally tested.

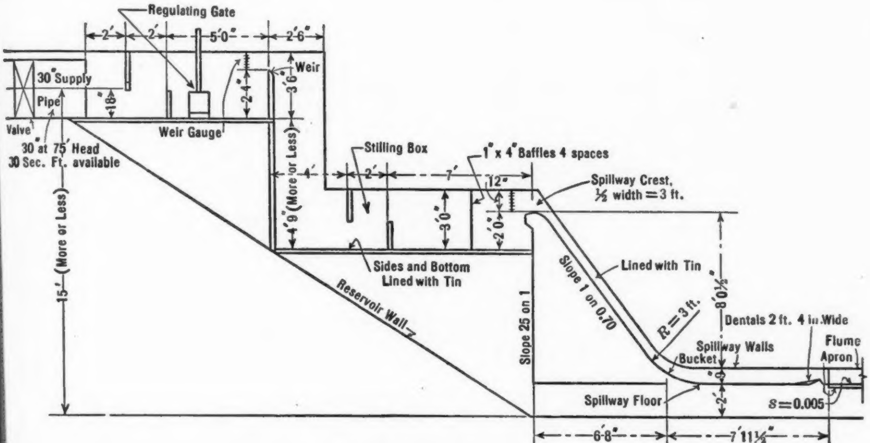

FIG. 33.—ONE-TWENTIETH SIZE MODEL OF SPILLWAY SECTION, BULL RUN DAM.

Model.—A 1 to 20 scale model of the spillway sections of the dam was constructed in the empty Reservoir No. 2 in the City of Portland, where adequate facilities for flow and control of water, head, etc., were possible. (See Fig. 33.) Flow over the model spillway was regulated by valves and gates and was accurately read on gauges to simulate certain discharges over the dam. Tests of various types and combinations of upward deflectors (see Fig. 34), were then conducted for flows representing approximately 1 000, 3 500, 5 000, 10 000, 15 000, and 21 000 sec-ft.; measurements were recorded and characteristics of flow noted. (See Table 10.) An illustration of typical cases is shown in Fig. 35.

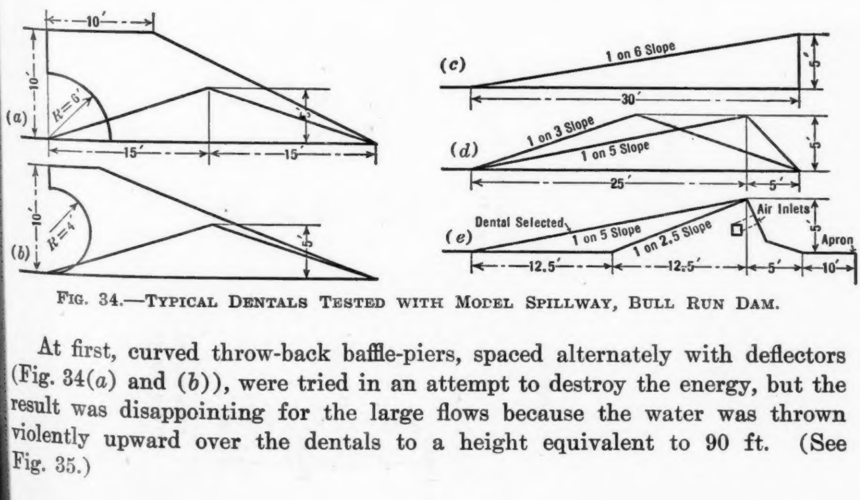

FIG. 34.—TYPICAL DENTALS TESTED WITH MODEL SPILLWAY, BULL RUN DAM.

At first, curved throw-back baffle-piers, spaced alternately with deflectors (Fig. 34(*a*) and (*b*)), were tried in an attempt to destroy the energy, but the result was disappointing for the large flows because the water was thrown violently upward over the dentals to a height equivalent to 90 ft. (See Fig. 35.)

TABLE 10.—WATER DEPTHS OBSERVED WITH USE OF BAFFLE-PIERS.

Station.*	Depth of Water, in Feet.									
	Type (*a*) Dental.†						Type (*b*) Dental.†			
	Q = 1 000 sec-ft.	Q = 3 500 sec-ft.	Q = 5 000 sec-ft.	Q = 10 000 sec-ft.	Q = 15 000 sec-ft.	Q = 21 000 sec-ft.	Q = 1 000 sec-ft.	Q = 3 500 sec-ft.	Q = 5 000 sec-ft.	Q = 10 000 sec-ft.
Left:										
1 + 60	0.3	0.8	1.0	1.6	....	3.8	0.3	0.6	1.0	1.6
1 + 50	....	....	....	....	3.0	....	....	0.6	....	....
1 + 10	8.0	....	....	....	2.8	....	8.0	....	....	....
1 + 0	....	9.2	....	1.4	....	3.8	7.8	8.0	1.0	1.4
0 + 50	....	....	1.0	1.4	....	4.0	7.6	9.2	0.8	1.2
0 + 30	8.0	10.4	....	....	....	....	....	....	....	....
0 + 0	6.0	....	....	....	....	....	....	....	....	....
Right:										
0 + 50	6.0	7.6	....	....	....	....	5.8	7.8	7.2	....
1 + 0	....	....	....	....	....	....	....	....	....	4.0
1 + 50	....	....	....	....	....	....	....	....	....	....
2 + 0	5.8	7.4	8.6	10.0	9.0	11.0	6.2	8.2	8.6	....
2 + 25	5:2	6.2	8.0	9.4	10.0	11.0	7.8	7.0	7.8	9.2

* See Fig. 35.

† See Fig. 34.

TABLE 10.—(*Continued.*)

Station.*	DEPTH OF WATER, IN FEET.									
	Type (*b*) Dental.†		Type (*c*) Dental.†						Type (*d*) Dental.†	
	$Q = 15\,000$ sec-ft.	$Q = 21\,000$ sec-ft.	$Q = 1\,000$ sec-ft.	$Q = 3\,500$ sec-ft.	$Q = 5\,000$ sec-ft.	$Q = 10\,000$ sec-ft.	$Q = 15\,000$ sec-ft.	$Q = 21\,000$ sec-ft.	$Q = 1\,000$ sec-ft.	$Q = 3\,500$ sec-ft.
Left:										
1 + 60	....	3.6	0.3	0.8	....	....	....	3.6	0.3	0.8
1 + 50	....	....	0.3	0.6	1.2	....	....	....	0.25	0.7
1 + 10	....	....	....	....	....	....	....	....	....	....
1 + 0	2.0	3.0	5.8	0.6	1.0	2.0	2.6	3.6	4.8	0.7
0 + 50	....	....	....	....	....	2.0	....	....	....	0.7
0 + 30	....	....	6.4	0.8	....	....	....	3.6	5.6	....
0 + 0	....	....	....	....	....	....	....	....	....	....
Right:										
0 + 50	....	...	6.4	6.4	....	....	....	....	6.0	5.8
1 + 0	....	....	....	....	....	....	....	....	....	....
1 + 50	....	....	....	....	....	....	....	....	....	....
2 + 0	5.8	9.0	5.5	7.2	8.4	6.0	8.0	6.0	6.0	7.4
2 + 25	10.2	11.4	5.0	6.6	8.0	9.0	9.0	10.0	5.6	7.0

* See Fig. 35.

† See Fig. 34.

TABLE 10.—(*Continued*)

Station.*	Depth of Water, in Feet.									
	Type (*d*) Dental.†				Type (*e*) Dental.†					
	Q = 5 000 sec-ft.	Q = 10 000 sec-ft.	Q = 15 000 sec-ft.	Q = 21 000 sec-ft.	Q = 1 000 sec-ft.	Q = 3 500 sec-ft.	Q = 5 000 sec-ft.	Q = 10 000 sec-ft.	Q = 15 000 sec-ft.	Q = 21 000 sec-ft.
Left:										
1 + 60	....	....	2.6	3.2	....	0.6	....	....	....	3.0
1 + 50	1.2	....	....	....	0.8	0.6	....	....	....	....
1 + 10	....	....	....	....	....	....	....	....	....	....
1 + 0	1.0	1.6	2.4	3.2	5.8	0.6	1.0	....	2.0	3.0
0 + 50	1.2	1.4	2.4	....	....	....	....	1.2	2.2	....
0 + 30	....	....	....	3.2	6.6	1.0	....	....	....	2.8
0 + 0	....	....	....	....	....	....	....	....	....	....
Right:										
0 + 50	....	....	....	....	5.8	5.0	....	....	....	....
1 + 0	....	....	....	....	....	....	....	....	....	....
1 + 50	....	....	....	....	....	....	....	2.0	....	....
2 + 0	8.8	4.4	4.0	5.0	6.0	7.2	1.8	....	3.2	5.0
2 + 25	7.8	9.6	8.8	9.0	5.8	7.2	8.6	7.6	5.0	....

* See Fig. 35.
† See Fig. 34.

Next a simple uptilt in the apron was tried. The end of the apron was raised 5 ft. in 30 ft. (Fig. 34 (*c*)), in the direction of flow. This served the purpose of lifting the fast traveling sheet into the air and preventing the rock

FIG. 35.—TESTS OF UPWARD DEFLECTORS.

from scouring close to the end of the apron. The disadvantage was that the attack of the falling sheet was concentrated and was still powerful although at a safe distance from the dam. To split this attack the apron was divided into

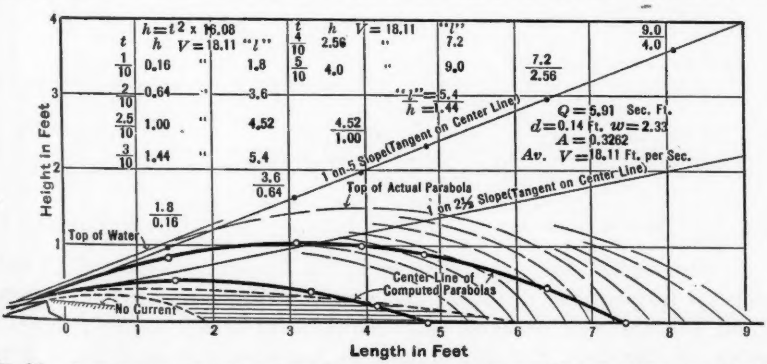

FIG. 36.—COMPARISON OF COMPUTED PARABOLAS AND FIELD MEASUREMENT OF ACTUAL PARABOLAS, SPILLWAY DENTAL TESTS FOR BULL RUN DAM.

dentals of different slopes (Fig. 34(*d*) and (*e*)), which spread the attack over a wide range by creating two separate parabolas. (See Fig. 36.) By varying these slopes the point of a falling jet may be regulated at will, within the limiting velocities of the overfalling sheet, which passes over the spillway floor and over the dentals without any appreciable loss of energy, and dissipates

itself in a well-scattered attack on the river channel far beyond the apron. Types (*d*) and (*e*) (Fig. 34), were both considered good. Type (*e*) was selected because of the ease of providing air inlets, although the necessity of these inlets was not apparent from the experiments.

From the "Percentage of Time Curve" (Fig. 37), it is evident that for 98% of the time the discharge over the spillway will be less than 3 500 sec-ft. which is required to sweep the apron at high velocity. During this long period the discharge down the ogee slope will enter the pool on the spillway floor and form a small standing wave there. Over the dentals the over-pour will be steady and a 10-ft. apron is provided in Type (*e*) (Fig. 34) to prevent erosion of the rock at this point. Fig. 38 shows the dentals (Fig. 34(*d*)) in place and Fig. 39 is a general view of the model, showing the resultant parabola described by the waters under a flow of 21 000 cu. ft. per sec.

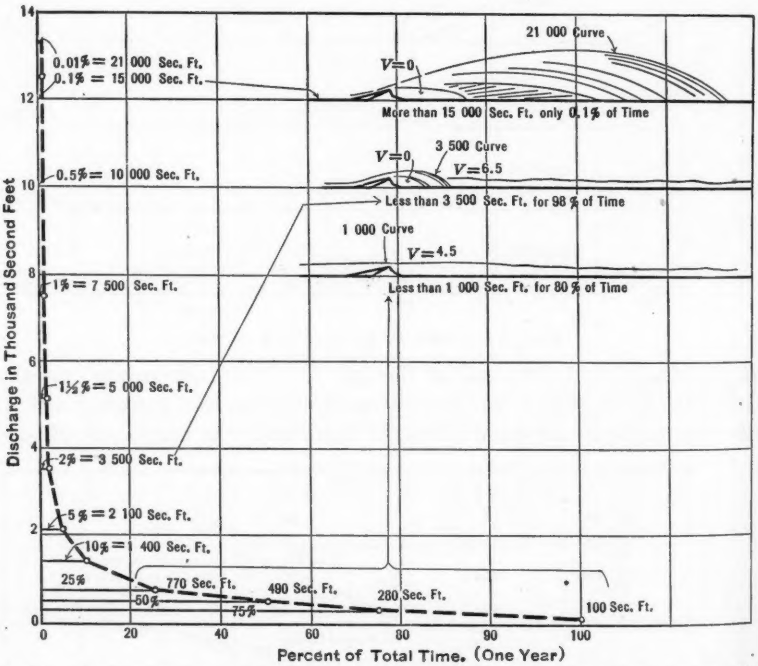

FIG. 37.—FLOW OF BULL RUN RIVER FOR A TYPICAL YEAR AT BULL RUN STORAGE DAM.

At the higher flows the shallow apron bucket is swept clear and the parabolas form at the dentals. The duration of these floods is so short, the falling water so well scattered, and the point of attack so far removed from the toe of the dam, that no danger to the structure need be anticipated. From the tests no appreciable current is expected to exist for about 40 ft. downstream from the dentals under full flood flows.

Whether the concrete apron will stand up under the fast moving sheet of water can be answered after actual overflow, but the time of this action will be so short that little scour should take place as the water is free from silt. The construction of a back-water pond of considerable depth for a stilling-pool

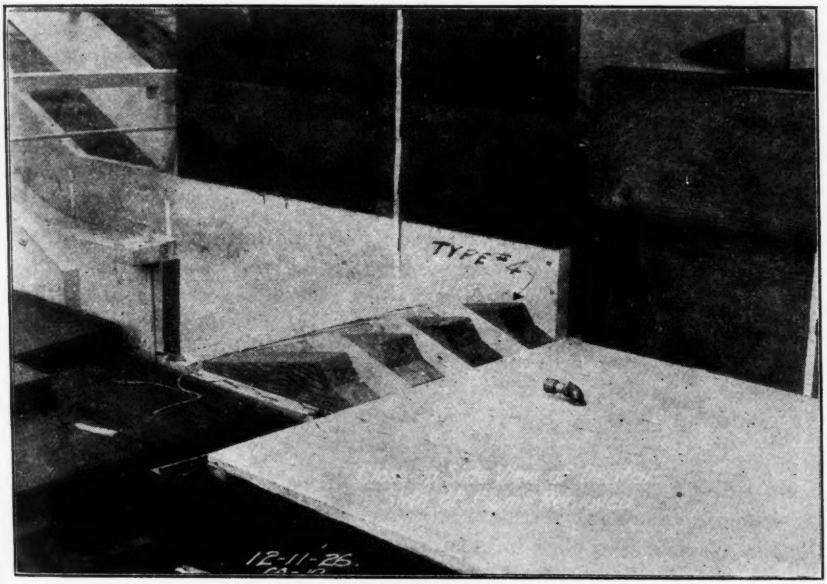

FIG. 38.—BAFFLES, TYPE (*d*), IN PLACE.

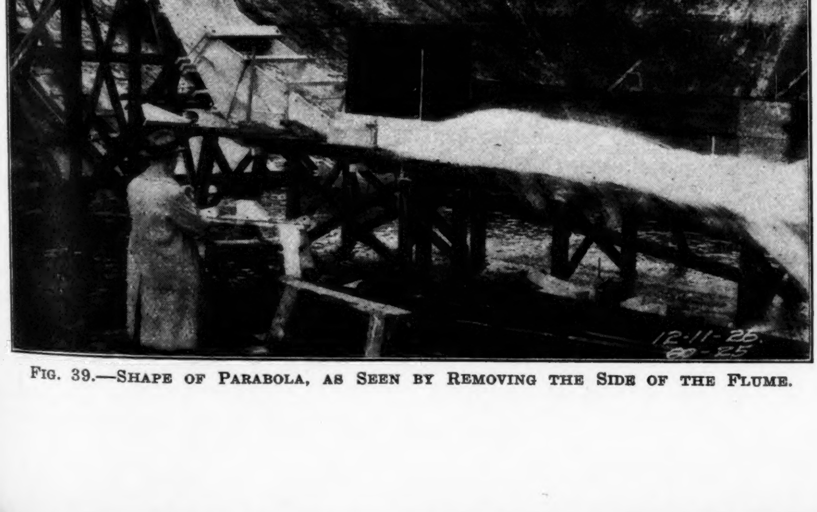

FIG. 39.—SHAPE OF PARABOLA, AS SEEN BY REMOVING THE SIDE OF THE FLUME.

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would, of course, be an additional safety measure, but is not considered necessary. The falling parabolas may excavate a pool, in time, which will serve the same purpose. The dental piers, as constructed, are practically identical with Model (*e*) (Fig. 34), except that the down-stream portion is excavated 10 ft. into the solid rock and poured as one piece with the piers and the whole anchored into the rock with steel rods. The advantages of this type of spillway are simplicity, economy, and safety. For greater economy the fore-apron could be shortened considerably. The cost of the dentals is very low as compared to deep buckets, jump-offs, depressed aprons, or stilling-pools.

The curve at the bucket was first tested for a short radius of 10 ft., but the rebound of the overfalling sheet was very pronounced at the larger flows. A radius of 100 ft. was next tried and gave very smooth delivery, but at quite an expense in extra concrete over a short radius. A radius of 50 ft. was next tried and this gave fairly good results. For additional safety a radius of 60 ft. was chosen.

These tests were carried out under the personal direction of D. C. Henny, M. Am. Soc. C. E., Consulting Engineer, and Ben S. Morrow, Assoc. M. Am. Soc. C. E., Chief Engineer, of the Bull Run Dam.

OREN REED,* ASSOC. M. AM. SOC. C. E. (by letter).†—The authors are to be commended for their presentation of the experimental work on models for the Pit River dams. Although a great number of such problems are difficult, or impossible, to solve by mathematics, opportunity is seldom given to attempt their solution by experimental observation. In Europe, on the other hand, extensive laboratory and field experiments are run on models of many important projects, before the final designs are made.‡ This is particularly true of hydraulic problems.

Difficult foundation conditions introduce a special problem which, unless properly taken care of, forms a great hazard for a dam. The experiments described by the authors have made it possible to insure that the over-pour will leave the concrete apron at a low velocity under the most unfavorable conditions.

The design adopted is very similar to the energy-controlling device at the Gatun Dam. However, the baffle-piers at the Gatun Dam are probably secondary in importance for destroying the energy of the spill. The spillway is in the form of a circular arc, 808 ft. in length along the crest. The discharge converges to a concrete channel, 285 ft. wide. The energy of the converging stream will partly neutralize itself, while the baffle-piers will aid materially. The Gatun Spillway has proved its effectiveness to pass high flood flows.

Before the flood control dams for the Miami Conservancy District were built in the Miami Valley, Ohio, many experiments were made to determine the best form for several features, including the outlet and spillway structures.§ The outflow from certain of these dams may become large and

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* Asst. Designing Engr., San Joaquin Light & Power Corporation, Fresno, Calif.

† Received by the Secretary, January 9, 1928.

‡ "European River-and-Harbor Laboratories Revisited," by John R. Freeman, Past-President, Am. Soc. C. E., *Engineering News-Record*, Vol. 99, December 1, 1927.

§ Technical Report, Pt. III, Miami Conservancy District.

the banks of the channels below the outlets are of soil, which is easily eroded. After thorough trial the use of stilling-pools was adopted as the most practicable method of controlling the energy of the outflow. The pool was constructed so that a hydraulic jump occurred through a wide range of discharge and tail-water conditions. The position of the jump was controlled within narrow limits by sloping the floor downward from the outlets. For the conditions below the Miami Valley dams, the use of baffle-piers was found to give poor results. The flow on the models was not stable under moderate changes of discharge and tail-water level, or in changes of the form of the structures. It was considered that the analogy between the models and the full-sized structures was inconclusive.

Model experiments may often be carried out on complete plants. This was done in 1919 on a 1 to 25 scale model of the Solbergfos Power Project in Norway. At Solbergfos the Glommen River is controlled by a gravity dam with three roller-gates for flood regulation. The power house is parallel to the stream, and the up-stream end forms part of the dam. The reaction units, thirteen in number, take water from an open forebay and discharge into the river just below the dam. By these experiments an arrangement was determined which gave the least interference of the spillway over-flow with the discharge from the power house.

The conclusions from model experiments should be mainly qualitative because, even under the most favorable conditions, the water is in a very turbulent state, and any quantitative measurements will be inconclusive. Caution must be observed because slight peculiarities in the model might correspond to great irregularities in the complete structure. The observation of tendencies is, however, of great importance. More problems should be attacked by this method, and it is hoped that the near future will see a marked change in attitude toward such research.

HAROLD K. FOX,* M. AM. SOC. C. E. (by letter).†—A real experiment such as the authors have conducted and have so ably presented, always adds something worth while to engineering knowledge. The writer visited the project during construction and, therefore, can fully realize the problems encountered.

A description of an emergency job which, at the same time was an experiment, is prompted by this paper. The problem is not in connection with a dam, but is one of the same nature, namely, destroying the energy of water to prevent erosion.

A concrete lined ditch, 3½ miles long, with a capacity of 100 sec-ft., was constructed to divert water into a reservoir. The discharge end terminated above the lake level and about ½ mile from the shore. Water was discharged into a ravine leading to the lake, and the problem was to prevent undercutting at the end of the ditch, as the material was a deep deposit of red clay and decomposed granite which eroded very easily. After some study, it was finally decided to connect the end of the ditch to a steel pipe in order that the water might be conveyed down the steepest part of the ravine and then be made to

* Constr. Engr., San Joaquin Light & Power Corporation, Fresno, Calif.

† Received by the Secretary, January 12, 1928.

ow in the channel. At the end of the pipe where the velocity was 35 ft. per ec., a very crude wooden structure was built for the purpose of destroying nergy and transferring the water to the ground surface at a low velocity.

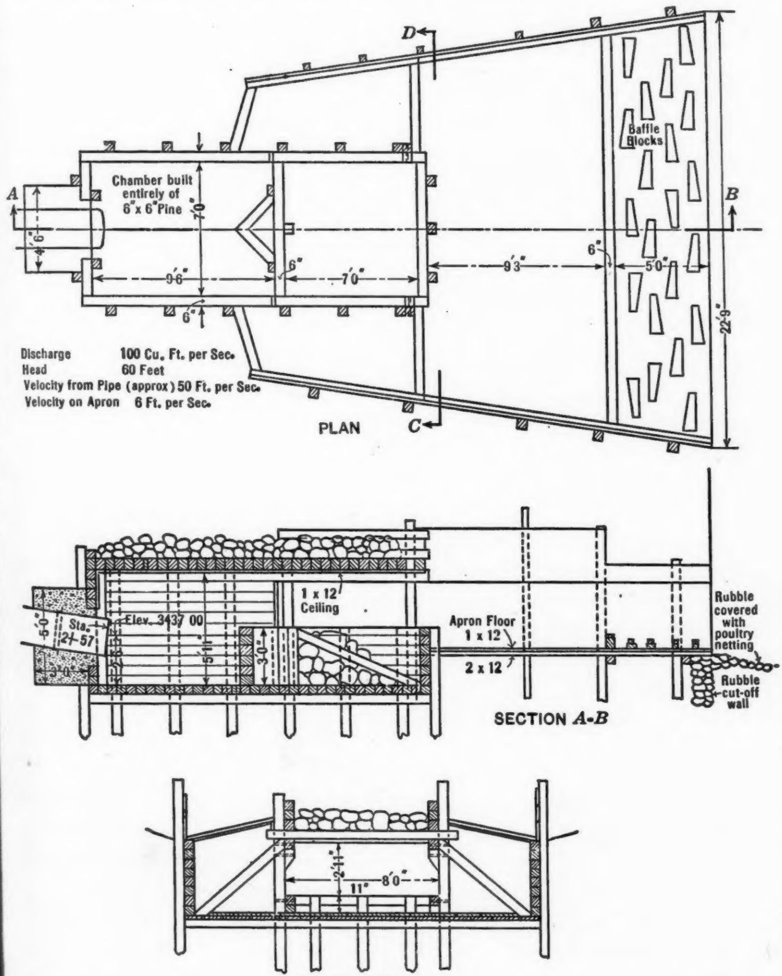

FIG. 40.—PLAN AND SECTIONS, ENERGY DISSIPATING CHAMBER, BROWN'S CREEK DITCH.

ig. 40 shows the method of splitting the water as it left the pipe and creating "boil" within a chamber. It was found necessary to build a roof over this hamber and weight it down with rocks as the spray got behind the retaining alls on the sides and started to cut. From this chamber, the water flowed ver a pile of rock which further reduced the velocity and then, finally, over series of baffles and left the platform with a velocity of about 6 ft. per sec. he structure has stood for five years without repairs and apparently will give ervice for several years in the future.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

THE SCIENCE OF FOUNDATIONS—ITS PRESENT AND FUTURE

Discussion*

By Messrs. J. Albert Holmes and Jacob Feld

J. Albert Holmes,† M. Am. Soc. C. E.—The speaker is greatly interested in the work Professor Terzaghi has been doing for a number of years in the investigation of soil mechanics. Although this paper relates particularly to foundations, it has, together with the author's previous investigations, a bearing on another engineering activity, the investigation, design, and construction of earth dams.

While earth dams of moderate height, built 2 000 years ago and more, are still impounding water and have continued to do so for the greater part of their existence, it is only within the memory of most of the older engineers and within the lifetime of many of the younger ones, that modern engineering thought has been directed toward the problems involved. One of these problems, and the principal one, is the selection and proper placing of materials in a structure. Theory and experience have demonstrated that pressures, tending to disruption, exist in the materials being placed in an earthen dam. To control this tendency a thorough knowledge of the medium used is as essential as in other engineering works. In investigating and studying the character of materials, permeability or ability to resist the flow of water through them, must be determined.

Several factors enter into the phenomena of flow through soils, as actual size of grain, relative sizes of grain, compactness, and colloidal content. A measure of relative size is indicated by the uniformity coefficient, which means a range of sizes such as will produce a dense mass resulting in small passageways and low percentage of voids. In their natural condition (and it is in this condition that use is made of them), soils have a tremendous range in the size of particles. Writers on the subject of colloids have maintained until quite recently that soils contain not more than 2% of material in a colloidal state. To account for the capacity of soils to absorb dyes and gases, it was assumed that they contained absorptive minerals known as zeolites. Upon investigation no zeolite minerals could be identified in soils. Instead

* Discussion on the paper by Charles Terzaghi, M. Am. Soc. C. E., continued from February, 1928, *Proceedings*.

† Hydr. Engr., Pearse, Greeley & Hansen, Chicago, Ill.

it was found that soils contain from 6 to 70% of colloidal material, in which lies their entire absorptive ability. The inorganic colloidal material in soils is chiefly made up of the products of chemical weathering and decomposition of soil-forming minerals, together with organic matter and perhaps some soil minerals of colloidal size.

An average diameter of 1 micron is arbitrarily fixed by the U. S. Department of Agriculture as the upper limit of colloidal size. At and below this dimension the particles cease to be affected by gravity and are subject to Brownian movement when in suspension in a liquid. Also, at the dividing line of 1 micron between colloidal and non-colloidal particles, microscopical control is good, while with less diameter, points of emanating light might be mistaken for the particle itself. In solution, those particles that diffuse slowly, and after being deposited are evaporated and separate into a shapeless jelly, are of one class or group. Those that diffuse rapidly, and under the same process crystallize, are of a second group. Because of their resemblance to glue the substances of the first group are called colloids, from the Greek word, "Kolla", and for the same reason the second group is called crystalloid. Colloidal material occurs in soils in the gel condition.

Professor W. D. Bancroft, of Cornell University, states that colloidal chemistry is the chemistry of everyday life. It certainly touches engineering frequently and at many points: Pavements; water supply; sewage disposal; foundations; concrete and water-proofing of concrete; earth dams; and agriculture.

The U. S. Department of Agriculture, in its Bureau of Soils* and Bureau of Roads, at Washington, has been investigating the subject of colloids for some time as has the Society's Special Committee on the Bearing Value of Soils for Foundations, etc. The Department of Agriculture is interested in the subject because it bears on the fertility of soils. The retention of moisture and the chemical action of fertilizers are dependent on the quantity and character of colloidal matter present in soils. The colloidal condition of soils enters very materially into the problem of dam construction.

One of the recent accomplishments of the U. S. Department of Agriculture in its Bureau of Soils was to determine the chemical composition of forty-five samples of soils and subsoils, collected from widely distributed points in the United States and representing many types. Peats and mucks were not included. The colloidal material was extracted from these samples and the chemical composition of both soils and colloids determined. It was found that soils and extracted colloids alike contained the following simple chemical compounds and elements, but in varying quantities:

Silica	$Si\,O_2$	Soda	Na_2O
Titanium	$Ti\,O_2$	Phosphoric acid	P_2O_5
Alumina	Al_2O_3	Sulfur	$S\,O_3$
Ferric oxide	Fe_2O_3	Chlorine	Cl
Manganese	$Mn\,O$	Water	H_2O
Lime	$Ca\,O$	Nitrogen	N
Magnesia	$Mg\,O$	Organic matter	....
Potash	$K_2\,O$		

* *Bulletins 1122, 1193, 1311, 1452,* U. S. Dept. of Agriculture.

Carbon dioxide, CO_2, was obtained from three soil samples and one sample of colloid.

Silica is the principal constituent of soils, and it was found that the colloidal matter is composed mainly of silica, alumina, iron oxide, and combined water, the percentage varying in the samples as follows:

	In Soils.	In Extracted Colloids.
Silica	51.32 to 93.66	31.84 to 55.44
Alumina	2.57 to 22.92	16.42 to 38.28
Iron oxide	0.93 to 13.82	4.66 to 16.67
Combined water	....	3.33 to 16.56

The colloids in the soils ranged from 6.2 to 57.8 per cent.

The colloidal material differs chemically from the larger soil particles, but separate and distinct compounds were not identified. The colloidal mixtures are very intimate and are without tendency to break up into known compounds. Table 4 illustrates the differences in composition of colloidal matter and whole soil and the coarser particles of soil.

TABLE 4.—Average Compositions of Soils, Colloids, and Coarser Minerals Compared as Percentages.

	SiO_2.	Al_2O_3.	Fe_2O_3.	CaO.	MgO.	K_2O.	Na_2O.	Combined H_2O.
Colloidal matter from 45 samples	43.34	26.84	10.41	1.04	1.72	1.43	0.38	9.93
Coarser mineral particles from 35 of the 45 samples	87.2	6.0	1.9	0.5	0.5	1.5	0.9	0.6
Mineral particles in fine sands and silt of a different series of 26 soils	83.3	7.3	1.5	2.0	1.7	2.9	0.9	0.4

The nature of soil colloids is destroyed by heat. Two samples, one of colloidal or ultra-clay, the other of clay soil, made into a number of test pellets, were subjected to carefully controlled heat. At desired temperatures a portion was removed and the temperature was stepped up on the remainder. As the pellets were removed from the furnace, ammonia absorption determinations were made, with the following results:

Colloid Sample:

Temperature, in degrees centigrade	110	265	374	559	754	1130
Cubic centimeter of NH_3 absorbed per cubic centimeter of colloid	110.3	100.8	80.0	74.1	57.5	2.2

Soil Sample:

Temperature, in degrees centigrade	110	190	265	374	522	673	844	1130
Cubic centimeter of NH_3 absorbed per cubic centimeter of soil	27.7	25.3	24.8	19.7	14.9	13.6	7.4	1.4

The colloids were progressively destroyed, and under the high temperatures quite completely destroyed.

Extracting the colloids from a soil to determine the amount of colloidal material is a long and laborious process. A more rapid method, checked by the extraction process, is the use of dyes, gases, and water vapor.

It was found that practically all the colloids in a soil are associated with the finer particles, and very few or none with the coarser parts. Certain ratios were found between component parts of the colloids which gave a correlation. This correlation, associated with rainfall, temperature, and color, has been presented as an index of the soils for agricultural purposes.

The speaker studied the data presented by the experiments with the object of finding a relation that might be used in the selection of soils for dam building. The most satisfactory is that existing between the colloidal content and loss by ignition. The process of incinerating samples of soil to determine their "organic" content is not a new idea, but a knowledge of the colloidal content, carefully determined, of a series of soil samples that have been incinerated, is new.

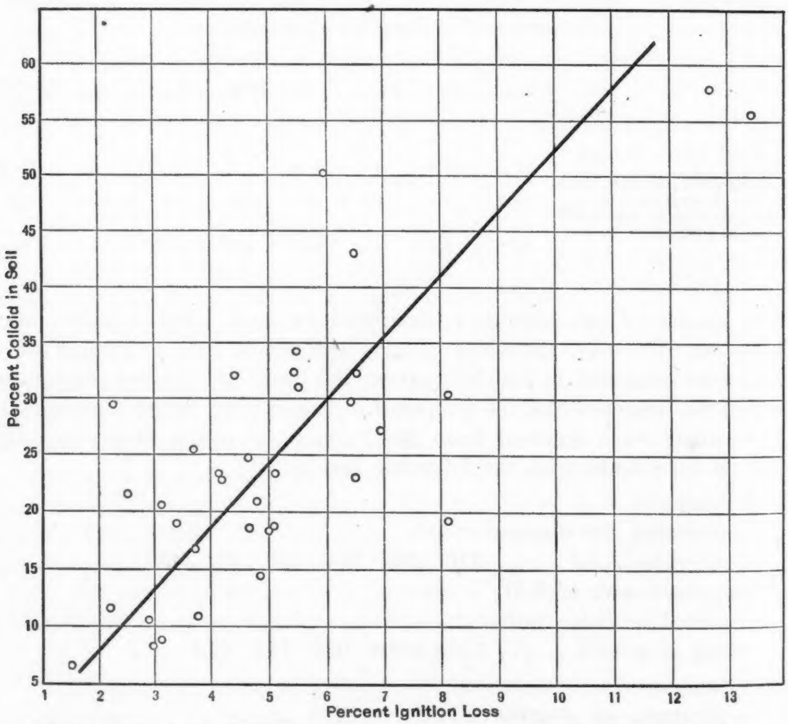

FIG. 20.—RELATION BETWEEN IGNITION LOSS AND COLLOIDAL CONTENT OF SOILS.

The relation between colloidal content and ignition loss is shown in Fig. 20, which indicates the determinations from thirty-five samples. The diagram is offered as a means of determining approximately the amount of colloidal

material in soils proposed to be used in dam construction. The incinerating of a soil can be accurately and quickly done. The extraction of colloidal material from the thirty-five soil samples was almost complete. It may be seen by the diagram that practically all the samples range between 2 to 8% ignition loss and 5 to 35% colloidal content.

The process of sluicing earth by the semi-hydraulic or the hydraulic method, would probably separate less than one-half the amount of colloid that is removed from a soil by laboratory methods; that is, an ignition loss of 5% would not mean that 25% of the colloids could be separated by sluicing, although approximately that amount is in the soil. All of it would be sluiced, with the finer particles, into the core where, by its gel-like condition, and by swelling, it would fill the voids between the particles. It is as undesirable to have too little colloidal material in a dam as too much. Both these extremes should be avoided by the selection of a material containing the right amount of colloids. The colloidal content of core material being placed in a dam may be ascertained by incineration with the assurance that the result is reasonably correct.

TABLE 5.—DATA SHOWING THE RELATION BETWEEN THE COLLOIDAL CONTENT (IGNITION LOSS), AND MAXIMUM RATE OF SEEPAGE THROUGH SOILS.

No. (1)	Material. (2)	Effective size,* in millimeters. (3)	Uniformity coefficient.† (4)	Ignition loss, percentage. (5)	Maximum rate.‡ (6)
1a	Very fine sand	......	......	0.00	514 000
1b	Very fine sand	......	......	0.00	387 000
2	Very fine sand	0.023	2.4	1.36	375 000
3	Very fine sand	0.02	2.0	1.54	256 100
4	Upland soil	0.054	5.1	4.37	26 840
5	Subsoil	......	......	3.32	21 350
6	Fine soil, sandy, roots	0.026	9.3	4.52	21 200
7a	Top-soil and subsoil	......	......	3.67	15 300
8	Sandy soil	0.091	2.9	3.97	15 000
9	Fine sand	......	......	4.12	11 700
10	Top-soil, roots in	......	......	5.74	10 800
11	Top-soil, roots out	......	......	4.80	10 000
7b	Top-soil and subsoil	......	......	3.67	8 600
12	Gravelly subsoil	......	......	3.01	6 300
13	Fine sandy soil	0.04	5.9	6.79	3 550
14	Subsoil and sand	......	......	3.46	3 200
15	Top-soil and subsoil	......	......	5.21	2 200
16	Top-soil and subsoil	......	......	7.00	1 500

* "Effective size" is the size of grain in materials than which one-tenth of the sample is finer and nine-tenths coarser. The finer one-tenth controls the seepage rate.

† "Uniformity coefficient" is the relation, expressed as a ratio, between the grain size which has 60% of the sample finer than itself and that which has 10% finer than itself.

‡ "Maximum rate" is the quantity of water, in gallons per acre per day, that will seep through a material, with the loss of head equal to the depth of the material.

No permeability tests were made by the U. S. Department of Agriculture on the soils investigated for colloidal content and chemical composition. The speaker has compiled data, from various sources, on the relation between ignition loss and permeability. These data are given in Table 5, in Columns (5) and (6) of which it may be seen that the percentage of ignition loss increases in reasonable regular order as the permeability rate decreases.

Having this knowledge and having determined the colloidal content of a soil as indicated by Fig. 20, or by a more precise method, a solution of the problem of selecting a proper material for an earth dam is made more possible. Effective size and the uniformity coefficient of a fine-grained soil have no relation to its permeability rate, although a soil having a high uniformity coefficient insures small void spaces and low rates of flow.

A gramme of colloid, containing particles having an average diameter of 91 millimicrons, has a surface area of particles amounting to 24.2 sq. m. Because of this enormous area the surface forces developed are sufficient to make the colloid a stronger binding agent than Portland cement. This is true only when the material is dry. The experiment cited was not carried beyond a 10% content of the binding agent, although it was sufficient to show that colloidal material is the principal binding agent of soils, giving them cohesiveness according to the moisture content.

In a homogeneous structure, either compacted in layers or consolidated by irrigation, the function of water is to soften the colloids in the dry, hard soil, breaking it up and permitting its particles to re-arrange themselves in a more compact form and by the continued presence of water, in order to keep the colloidal material expanded in the voids of the soil. Colloids have capacities for swelling of from 40 to 150% and a water-holding capacity of 50 to 140 per cent.

Investigation of the permeability rate, combined with a determination of colloidal content and ignition loss, is very much needed. In making a mechanical analysis to determine the uniformity coefficient the "clay" fraction, diameter 0.005 to 0 mm., roughly checks the colloidal content.

The speaker offers the foregoing for consideration and suggests a line of research in soils, physical and chemical as well as mechanical, that will supplement the data given.

JACOB FELD,* ASSOC. M. AM. SOC. C. E. (by letter).†—In 1727, Couplet published his monumental work on soil pressures, giving a "complete theoretical analysis of soil physics," and, now, 200 years later, Professor Terzaghi writes that the science of foundations is of the present and future. From this it must be assumed, that the science of foundations has no past, and there is no possibility of criticizing that statement if the rigid definition of a science is insisted upon. Still, it is hard to slight such announcements as appear in the *Annales* of the French Academy of Sciences (January 22, 1783) to the effect that the problem of determining the lateral earth pressure was susceptible of a rigorous solution for any special case by the application of the theory outlined on that date by Chauvelot.

The design of foundations has never been based on a true science of foundations, and, to disagree with the author, it is not even now (in 1928) based on any scientific principles. There are, in common use, certain methods on which various types of foundation design are based. Such methods are adaptations of principles of mechanics, taken either from statics or pneumatics, with or without correction factors.

* Cons. Engr., New York, N. Y.

† Received by the Secretary, January 11, 1928.

It cannot be expected that the old, although not accurate, methods of design should be discarded in favor of principles from a "science of foundations" when such science does not exist in the common knowledge of the day. As the author states, there is insufficient data available from which even empirical rules may be deduced, and so little is known of the materials which are encountered (soils and rocks) that no skeleton or shelving of general data and hypothesis can be formed as a basis for the accumulation of missing information. Until such a framework can be formed, no one can announce even the birth of a science of foundations.

The present methods for the design of foundations are open to many more criticisms than are enumerated by the author. However, these methods have been used for the preparation of plans for many foundations which, for the large percentage of cases, have been successful. Note that this statement uses the expression "preparation of plans" and not "design". The success of a plan is measured by the result obtained in supporting a superstructure without apparent or measurable danger. The economy of a foundation plan is quite another matter.

It is quite difficult to clear away all this accumulation of precedent and "experience" and suddenly substitute a new "science of foundations." Certainly, it is not a one-man job and no one-man's work can be expected to supply all the necessary frame upon which such a science can be built. The profession should be very thankful to the author for his numerous reports of the past few years, all pointing to the desired goal. It is high time that the Society has produced some constructive results in providing a basis for the development of this much needed science.

The greater part of the work in the subject of soils during the last three centuries has been in the development of the art of foundation construction rather than in the preparation of data for the scientific development of foundation design. The method of shoring or underpinning a structure is far more advanced than the design of the foundation which may be placed under such structure. The chief drawback to the lack of scientific development has been the absence of any authoritative nomenclature or definite standardization of terms. The tendency has been to copy expressions from the mechanics of solids, although it has been many years since soil mechanics was separated from hydrodynamics and from the statics of solids. Some writers have gone far in the opposite direction, coining new expressions which, often after translation, are crude and misinterpreted or entirely incomprehensible to the reader.

The relation between the size and carrying capacity of a loaded area can never be rigidly fixed by any usable expression. The author deals only with the individual case, corresponding to the case of a single pile, and has omitted entirely the discussion of the effect of loaded areas in close proximity on each other. Even in the individual case, the author has omitted a very important factor, namely, the shape of the loaded area. In the bearing of a solid against a solid, within the elastic limits, the usual assumption that the shape of the loaded area has no effect, probably brings in no appreciable

error. The study of loaded plates of various shapes leads one to doubt the accuracy of the assumption. In a perfectly elastic material, with no cohesion and no surface tension, the shape of a loaded area can have no effect. However, in a substance like a clay, there is a resistance to settlement under loads (bearing power, so-called), which is a function of the area and also a function of the perimeter. The latter term is now being disregarded. Some of the author's conclusions* should be modified to suit this perimetral resistance. Two circular areas of diameters in the ratio 1:2, loaded equally per unit area, have equal resultant resistances as a function of the area, but the smaller has twice the length of edge per unit area. The smaller area will settle the least.

An equally important problem is the determination of the effect of one loaded area upon adjacent areas. The complexity of such a usual problem as the design of spread footings for the columns of a frame building with its interior and exterior footings as well as square and non-square loaded areas, is appalling when one thinks of the speed with which plans for such footings are being daily prepared. Many instances are known where unequal settlement is very evident. Unfortunately, few such cases are on record.

Still another disregarded factor is the amount and nature of cover (backfill) above the bearing area of a footing. In a few isolated ideal cases, theoretical methods seem to point to the true values of the side frictional support of piers or caissons, based on the formulas of Coulomb and others, centuries old.†

Edge resistance and side-frictional resistance tend to equalize the distribution of resulting carrying capacity per unit base area. The true state of affairs is much closer to the uniform than to the parabolic distribution of the soil reaction. The diagrams in Fig. 3‡ are an indirect proof of the last statement; for, if the parabolic distribution results in bending moments in continuous footings of more than twice the calculated moment (based on a uniform distribution), it is inconceivable why at least one-half such footings built have not failed. The "factor of safety" of a structure, based on good concrete and a reduced load-summation design and built with concrete "not quite as good as specified", is certainly in the neighborhood of two.

The writer would like to bring up a problem which has interested him in connection with a recent arbitration in which he was involved. If one of two exactly similar six-story buildings is removed, and replaced by a taller building with footings placed at exactly the same depth as had previously existed, but with greater load intensity, should there be expected a settlement of the adjacent building, which has not been disturbed? Is such a design proper, or should some means be devised to prevent the old building from settling, even if the new footings are not built to any level lower than the sub-grade of the adjoining building? The problem has also some interesting legal aspects. The writer might mention his argument that even if the new

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2268.

† *Engineering News-Record*, Vol. 88, 1922, p. 1052.

‡ *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2270.

footings are built no lower than the sub-grade of the old ones, their necessary settlement on receiving load will cause them to take a position sufficiently lower to cause a "sympathetic" settlement of the adjoining footings.

The subject of piles for foundation purposes is very ably treated by the author.* The objection to the use of the recognized pile-driving formula is a special case of the general objection made by the Belgian scientist, Boussinesq, that all soil pressure designs are fundamentally problems in statics and the experimental work on which such designs are based, is dynamic in method.

As to the basis for a soil classification, the writer† is still of the opinion that:

"* * * the classification of soils and other granular materials should be along the lines of strain characteristics rather than those of stress, namely, elastic strain, plasticity, and fluidity, for these phenomena are evident and measurable, and embrace within them the results of all the stresses which may act. Generally speaking, the elasticity factor will take care of the solid, the fluidity of the fluid, and the plasticity of the colloid properties of the material being classified."

The author is to be commended upon this latest contribution to the subject of soil mechanics and it is hoped that the Special Committee on Soils of the Society will extend the ideas included therein and soon report a framework for the development of a "science of foundations", so that an accumulation of data can be begun.

* *Proceedings*, Am. Soc. C. E., November, 1927, Papers and Discussions, p. 2273, *et seq.*
† *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 1567.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

FLOOD CONTROL WITH SPECIAL REFERENCE TO THE MISSISSIPPI RIVER

A SYMPOSIUM

Discussion*

By Messrs. C. S. Jarvis, Edward H. Schulz, Q. C. Ayres, H. S. Gladfelter, and Byron E. White

C. S. Jarvis,† M. Am. Soc. C. E.—It has been the speaker's privilege to glean in broad fields, long since harvested, and to discover values among material habitually overlooked, neglected, or discarded. This opportunity resulted from a co-operative arrangement between the Secretary of War and the Secretary of Agriculture, fostered by the Corps of Engineers, U. S. Army, and the Bureau of Public Roads. In the course of this work it has been demonstrated that the earliest and rarest hydrographic records, far too meager at best, are largely lying dormant as an undigested mass, or as detached and unrelated fragments, although they are capable of being assembled and utilized to great advantage. American engineers have been accustomed to think that the longest periods of meteorological and hydrographic record were for about 60 years, when as a matter of fact they exceed a century.

Fig. 39 depicts the relation of run-off to precipitation on two of the watersheds supplying New York City. The habits of the Esopus area are portrayed from 1906 to 1926, inclusive. The monthly precipitation, which is platted above the axial line, occasionally produces run-off at nearly the same rate; but it lags behind the stormy period as required for percolation, thawing, or concentration. The average monthly precipitation ranges from 3.5 to 4.5 in., and the depth of run-off to be expected during March or April is fully 6 in. Even though the rainfall is greatest during the summer months, the yield

* Discussion on the Symposium on Flood Control with Special Reference to the Mississippi River, continued from January, 1928, *Proceedings*.

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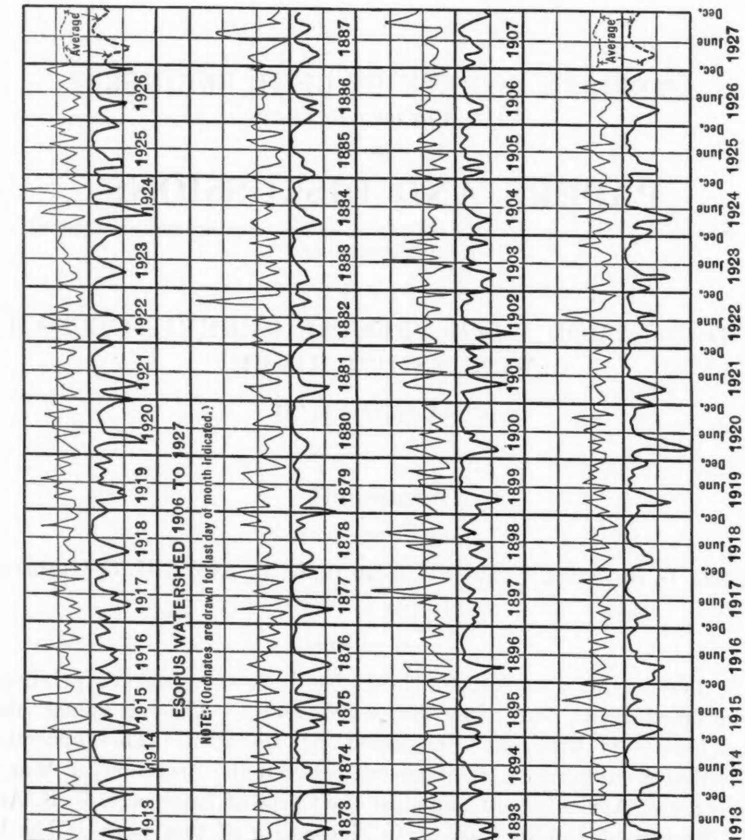

FIG. 39.—MONTHLY PRECIPITATION AND RUN-OFF ON WATER-SHED SUPPLYING NEW YORK, N. Y.

is the least, due to the higher temperature, rates of evaporation, absorption by the soil, interception by foliage, and the demands of vegetation during the season of growth. The run-off from the 257 sq. miles of drainage area during the period of record was 31.2 in. in depth yearly (if considered evenly distributed over the water-shed), or 65.4% of the precipitation.

The hydrographs for the Croton drainage area extend from 1868 to 1926. The orderly behavior of the run-off since 1875 would be an excellent argument for reducing or controlling floods by forestation and wise supervision of the water-shed, were it not for such years as 1902 and 1920. The monthly run-off during the spring thaws in each of these years was double or treble the average monthly precipitation, and this could not fail to produce violent floods above the reservoir. The average annual run-off during the 59-year period of record was equivalent to a depth of 22.6 in. over the 375 sq. miles of water-shed, or 47.2% of the average annual precipitation. Only two meteorological stations were maintained for the first few years on this area, but the number was periodically increased, and for the past 20 years nine stations have given daily reports. The maximum annual rainfall recorded for this entire area was 63.76 in. in 1901; the minimum was 37.62 in. in 1880. As might be expected, those years marked the extremes of annual run-off, 34.4 and 11.8 in., respectively. The observed monthly precipitation frequently reaches double the average and rarely if ever attains three times the average for a given month.

Application of Basic Data to Other Drainage Areas.—The application to the head-waters of the Mississippi or any other river is obvious, if the climatic and soil conditions are comparable in a known degree. Less extensive records of the Drainage Division in the U. S. Bureau of Public Roads, the Miami Conservancy District, and other authorized agencies confirm this opinion. Notably, the Chief of Engineers, U. S. Army, stated in his Annual Report for 1894 (page 1709) that the 562 sq. miles of drainage area above Pine River Dam in Minnesota, with an average annual rainfall of 23 in., yielded 18.4% to the river; while the 3 265 sq. miles of the Pokegama watershed, with 24.7 in. of annual precipitation, yields 16.8 per cent. These results accord very closely with what should be expected when the glacial soil, the relatively flat topography, the extent and shape of each drainage area, the low rainfall, the wind movements and, consequently, high evaporation rates are considered. As early as 1850, or thereabouts, it was definitely known that the usual yield from the Mississippi Drainage Basin is nearly one-fourth of the precipitation each year, and subsequent records, including those reported by Dr. H. C. Frankenfield,* have closely confirmed the findings of Humphreys and Abbot in this regard.

Hydographic Records Over Long Periods.—It is gratifying to find among public documents the early records of daily gauge heights, even though diagrams or manuscripts supply much of this information. Those already located for the Ohio River at Wheeling, W. Va., extend back to 1838; and those of the Mississippi at Natchez, Miss., include the year 1817. The break in the Wheeling record between 1850 and 1860, and in the Natchez record

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2470, *et seq.*

between 1847 and 1861 may yet be closed; but of this there is no certainty at present.

Fig. 40 is an example of a long-period hydrograph, based on daily records of gauge heights at Cincinnati, Ohio, dating from 1858 and extending continuously into 1927. Sheets such as this must be regarded as merely index sheets, with the high and low gauge readings for each month as the principal aim. The minor oscillations occurring from day to day could not be shown accurately on such a small scale. The resulting curves are useful mainly in assembling the prominent data and locating the periods requiring further examination.

Fig. 41(*a*) deals with notable flood years at Cincinnati, St. Louis, Mo., and Natchez. Fig. 41(*b*) portrays similar data for a number of important tributaries of the Ohio and Mississipi Rivers, and incidentally shows the inception of the 1927 flood. The observer at Beardstown, Ill., had little to report during the latter months of previous years; but in September and October, 1926, the Illinois River broke all previous records for gauge heights, although probably not for actual discharge along the valley. Infringements on the lateral storage basins and natural flood-plains, such as have taken place on the lower course of the Illinois River, must inevitably elevate the water surface within the restricted channel, and intensify the effects of record-breaking storms such as occurred in September, 1926.

The depression of the Beardstown hydrograph during the early winter does not mean that the danger had passed, but that the flow had been checked, largely by freezing in the channel, in the soil, and in valley storage.

The Wabash, Tennessee, Cumberland, White, and Yazoo Basins all contributed unusual amounts during the latter part of 1926, setting the stage for the tragedy that was to follow. The same is true regarding the Ohio, the Missouri, and the Arkansas Basins. It is significant that the Ohio River receded to a moderate stage early in 1927; otherwise, additional chapters would have appeared in this year's history of disaster.

Fig. 41 helps to explain the forecast of the U. S. Weather Bureau some months in advance of the happening.

United States Weather Bureau Forecast of Flood Stages.—Early in January, 1927, Dr. H. C. Frankenfield, of the U. S. Weather Bureau, remarked, in the speaker's hearing, that conditions were shaping themselves for a Mississippi River flood that would far surpass all previous records. During succeeding weeks, as he anxiously noted the development of initial stages, he lamented the lack of publicity accorded the warnings of impending danger. When at length the catastrophe was recognized and accorded front-page prominence, the forecast of flood stages proved to be of incalculable benefit and unfailing reliability for the guidance of patrols and rescue organizations. One aggravating factor was apparently beyond the range of accurate prediction, and that was the intense torrential rainfall in the Lower Mississippi Valley. This occurred at the worst conceivable time for depleting the lateral storage and soil moisture capacity, augmenting the flood wave, increasing

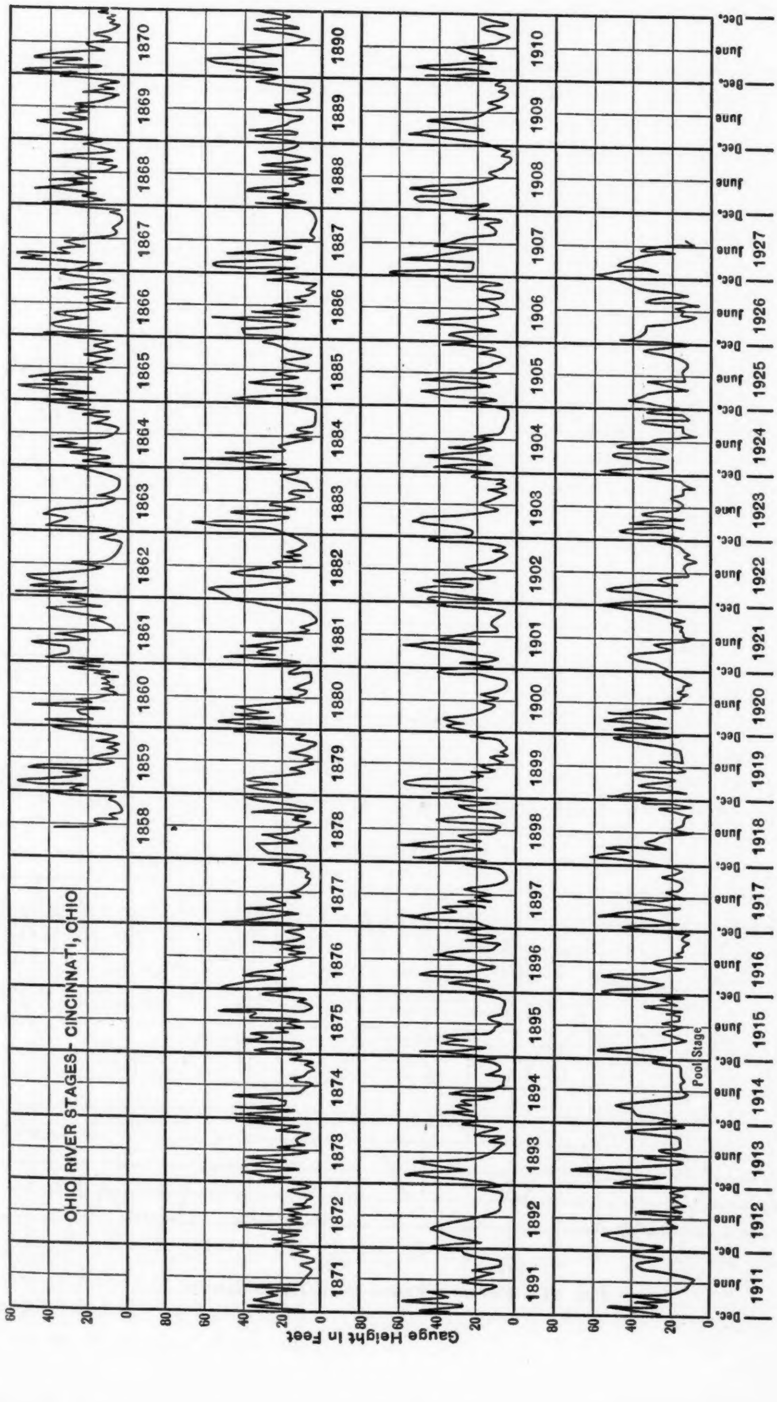
OHIO RIVER STAGES - CINCINNATI, OHIO
Gauge Height in Feet
Pool Stage

FIG. 40.

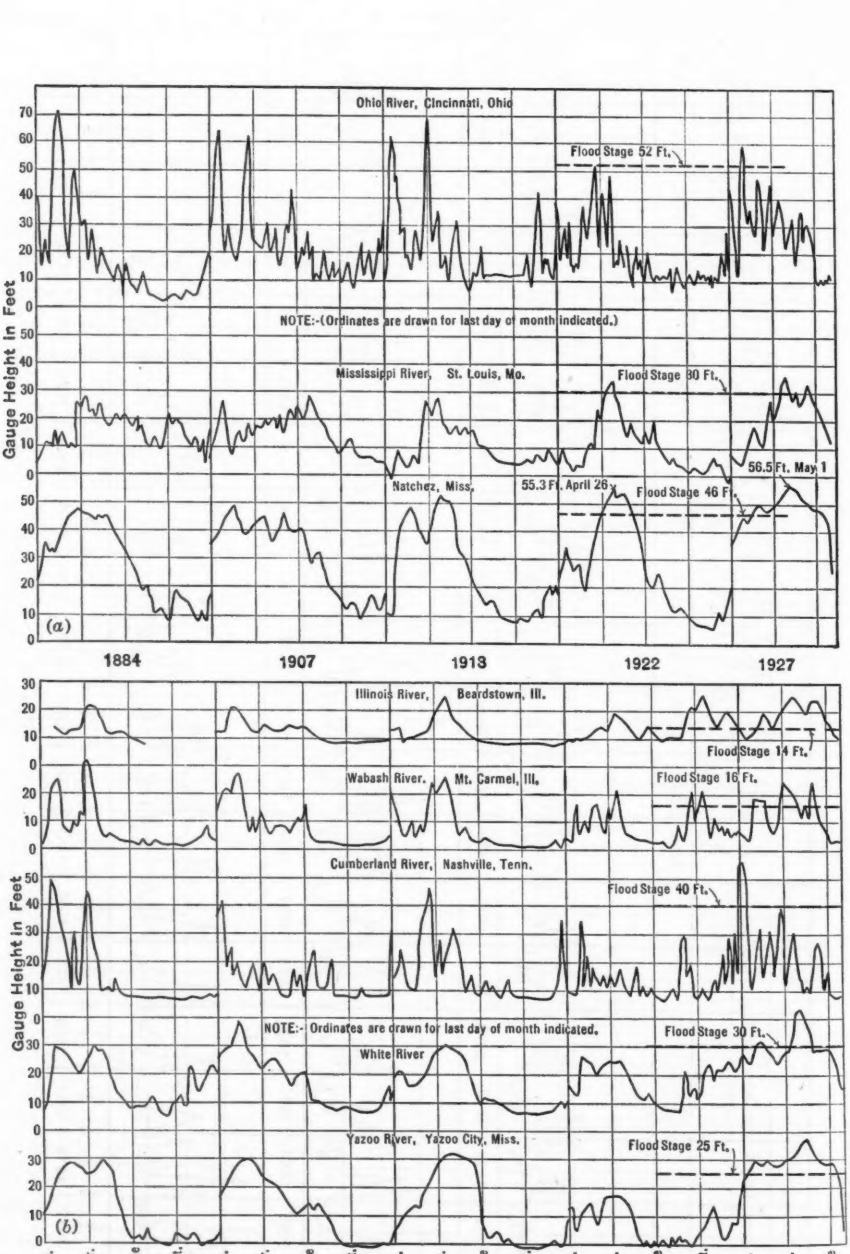

FIG. 41.—NOTABLE FLOOD-YEAR HYDROGRAPHS.

its velocity, and impeding the efforts aimed toward human safety, rescue, and relief.

Relationship Shown by Hydrographs.—Fig. 42 portrays the hydrographs of a river system. The Ouachita at Camden, Ark., has the sharp variations so commonly observed on head-waters, or in semi-arid country, while at Monroe, La., representative of the lower valley, the curve is fairly smooth. The records of the Red River, at Shreveport, La., and at Alexandria, La., show a closer relationship. Those for the Mississippi, at Red River Landing, La., and for the Atchafalaya, at Melville, La., nearly 40 miles distant, as well as for the intervening station at Barbre Landing, seem to be nearly identical in form.

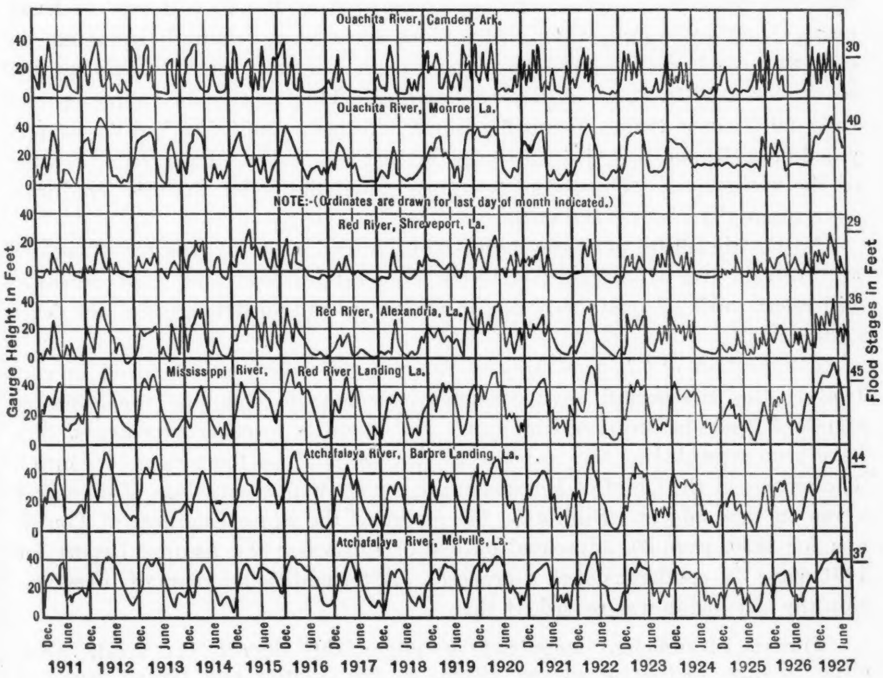

FIG. 42.—HYDROGRAPHS OF A RIVER SYSTEM.

The explanation for such a coincidence is to be found in the Old River channel which conveys water from the Red to the Mississippi, or reverses the flow, according to their relative stages. When the 1927 flood was at its highest, the relief afforded through this one outlet ranged between 100 000 and 200 000 cu. ft. per sec., reducing the flood height at New Orleans, La., from 1.7 to 3.0 ft., an effect nearly equal to that of the artificial crevasse below the city. Recalling the $1 313 000 000 estimated cost of reservoirs capable of lowering the flood crest 3.36 ft. at Red River Landing, the effective relief afforded by the Atchafalaya outlet is worth more than has been expended to date on Mississippi flood control.

According to official current meter measurements,* the discharge of the Mississippi River through or adjacent to Old River channel has exceeded 250 000 sec-ft. at some time during each of the 1912, 1913, 1916, 1917, 1920, and 1922 flood seasons. The same authority indicates that other notable measurements were as follows: 154 000 sec-ft. in February, 1907; 347 000 in April, 1912; 317 000 in April, 1920; 447 000 in April, 1922; and 206 000 in April, 1923.

Examples of Relief Outlets and Automatic Spillways.—In spite of the sporadic notions promulgated during the past eighty years for divorcing the Red and the Atchafalaya from the Mississippi drainage, or for closing the Atchafalaya at Barbre Landing against the Red River, this great relief outlet has been consistently maintained and improved by the U. S. Corps of Engineers. Log rafts and other obstructions have been removed, inducing the current to enlarge its own channel until the capacity now exceeds 400 000 sec-ft. for a 50-ft. stage at Barbre Landing. This represents an increase of fully 50% since 1882. Meantime, some revetment and sills had to be provided near Simmesport, La., to insure against the possible depletion or capture of the main river. Thus, doubtless, originated the belief that the engineers were gradually closing the outlet.

Automatic spillways of the most expensive and wasteful type have operated at intervals ever since the first levees were established; and these crevasses will recur until adequate spillway facilities are provided to protect the levee system.

The Fiction of "Levees Only".—Because the engineers employed at controlling the Mississippi have consistently sought to complete the levees to standard grade before diverting funds and energies to other expedients, they have been regarded as the sponsors of "levees only" as the means of control. The reports of the Chief of Engineers are replete with discussions of divergent views on this subject. Similarly, the report of the State Engineer of Louisiana for 1850, page 39, advocated outlets or spillways; the Annual Report for 1861, page 38, condemns such a proposal. After additional years of investigation the conclusion was recorded† that:

"Outlets of sufficient magnitude to appreciably depress the floods, must either deluge vast areas of our State, as the Ashton and Grand levee or Morganzia, or be most expensive and injurious, as would be the Bonnet Carre and Lake Borgne, and therefore I oppose them, Lafourche and all, and turn for safety to our present levee system."

Despite such sentiments among local residents and officials, it is significant that the main reliance continued, not as "levees only", but levees aided by a great relief outlet and overflow sections or the inevitable crevasse within the uncompleted structure. Furthermore, extensive storage was proposed, accomplished, and operated on some of the head-waters as an aid to flood control and its natural corollary, namely, increased flow during the low-water season.

* "Results of Discharge Observations, 1838–1922," Pamphlet compiled and issued by the Mississippi River Commission, 1925.

† Annual Rept., State Engr. of Louisiana, 1871, p. 20.

The "levees only" policy is fiction at best; for actually other means have passed the experimental stage. A relief channel operated throughout American history has convinced most of the former opponents of spillways.

Regarding Physical Limitations.—Try to imagine the 1927 flood confined between levees, 10 or 15 ft. above the completed embankments of recent years, resisting a current velocity of 12 ft. per sec., or 8 miles per hour, double or treble the safe velocity along unrevetted banks of silt; and then return to reasonable measures.

The proposed location of the levees a few miles back from their present location would encounter the same difficulties because of the lateral slope of nearly 7 ft. per mile away from the main channel. Considerations of foundation pressures, structural security, and the safeguarding of communities, all seem to forbid a great increase of levee heights. Unless adequate detention or spillway facilities are provided, the river assumes the mastery and man becomes the fugitive.

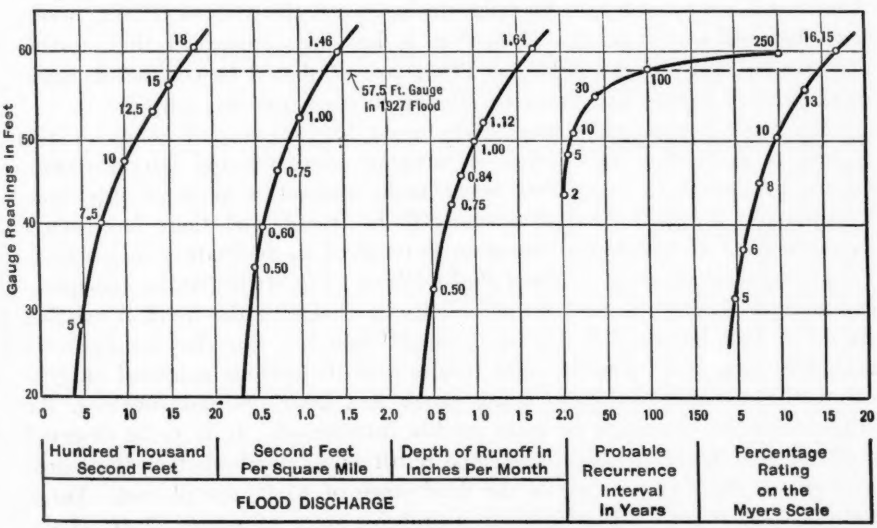

FIG. 43.—ANNUAL FLOOD RATINGS OF THE MISSISSIPPI RIVER AT RED RIVER LANDING, LA.

A Résumé of River Habits.—Fig. 43 illustrates a device originated by the speaker for condensing hydrographic data and transforming it into convenient units. Such a system of parallel scales would present on a single page the most essential elements of voluminous records and extensive research, forecasts as to probable future recurrence, and a convenient means of comparison. Thus, the 1927 flood at Red River Landing with a gauge height of 57.5 ft., discharged nearly 1 600 000 sec-ft., or 1.3 sec-ft. per sq. mile of drainage area; or at the rate of 1.5 in. depth per month over the entire water-shed. The probable frequency of recurrence, judged by the whole unweighted record, is once per century; or in the light of the river's aggravated habits during recent years and their prospects for continuance, it may better be classed as

a 25-year flood. The rating on the Myers scale is 14.5 per cent. Fig. 43 is based on 57 years of continuous record to 1927.

Using the same general method of analysis in 1923 and 1925, the speaker reached the conclusion* that the rating at Helena, Ark., for rare floods should be 25%, corresponding to a discharge of 2 500 000 sec-ft. He also urged the adoption of spillways to afford relief, and suggested a general re-appraisal of the flood situation to meet the changed conditions wrought by Man. Information disclosed after months of research seems to confirm both the estimate and the recommendations. The main significance of the 25% rating lies in the probability of discharges exceeding 2 500 000 sec-ft. at Red River Landing if the levees hold. The net accretion at this station from below Helena, Ark., should be measured approximately by the maximum concurrent contributions from the White, the Arkansas, the Yazoo, and Red Rivers, minus the outflow through the Atchafalaya. As this depends on the stage of the Red River it seems advisable to investigate the feasibility of diversions above Alexandria, into such neighboring drainage as the Sabine River, where local benefits would accrue. Likewise, it has been suggested that, during emergencies, part of the Arkansas River run-off should be conducted southward through bayous and never be allowed to reach the Mississippi.

Need for Topographic Maps.—The need for topographic maps of this region, to facilitate the location of auxiliary channels and lateral storage basins, or detention areas, was never more imperative than at this time. Fortunately, the 1927 flood-plane can still be traced, and there has been a demonstration of how much spreading is required to dissipate a major flood.

A Graphical Solution of Flood Probabilities.—Fig. 44 illustrates the speaker's method of determining flood periodicity by analyzing the trend of existing records. This has been described in detail† and has thus far satisfactorily withstood tests and comparisons of results with the various approved analytical methods. In addition to being much less laborious and involved, the graphic solution seems to be more readily interpreted. It is to be observed that, for Red River Landing, the same result is obtained whether the gauge readings or the distances above the flood stage of 45 ft. are plotted. For a 400-year period both curves indicate a probable stage of nearly 61 ft., if the entire unweighted record is considered.

Maximum Annual Flood Heights.—Fig. 45(*a*) represents the maximum annual stages attained on important tributaries of the Mississippi. The unmistakable upward trend of the Illinois River, at Beardstown, has been observed with alarm for several years; this is also true of the Yazoo and other rivers under similar treatment.

Fig. 45(*b*) indicates that the Ohio River and its principal tributaries are gradually yielding to some form of restraint and regulation. Among these influences, not yet evaluated, but generally under-estimated, are channel

* "Flood Flow Characteristics," by C. S. Jarvis, M. Am. Soc. C. E., *Transactions*, Vol. 89 (1926), p. 1023, Item No. 947.

† Discussion by C. S. Jarvis, M. Am. Soc. C. E., of paper entitled "Probability of Flood Flows," by F. G. Switzer, Assoc. M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., October, 1927, Papers and Discussions, p. 2027.

improvements, detention basins, and numerous reservoirs operated solely for local protection or enterprise.

Fig. 45(*c*) displays the definite upward trend so characteristic of all Mississippi River stations between Cairo, Ill., and the Gulf. It also shows the neutral tendency at St. Louis, and the evident downward trend at St. Paul, Minn. There has been no progressive diminution of precipitation in this area throughout the period of record; the brief cycles of either sub-normal or excessive rainfall are readily traceable (Fig. 45(*c*)). The record of gaugings by current meter between 1866 and 1917 indicates that the cross-section has remained fairly constant, so that the diminution of flood crests at St. Paul means a decreased rate of discharge, not only apparent, but real. The effective lowering amounts to several feet during the past 50 years. Translated into discharge, this reduction is about 40 000 or 50 000 sec-ft., which, at Cairo, would represent nearly 1 ft. on the gauge.

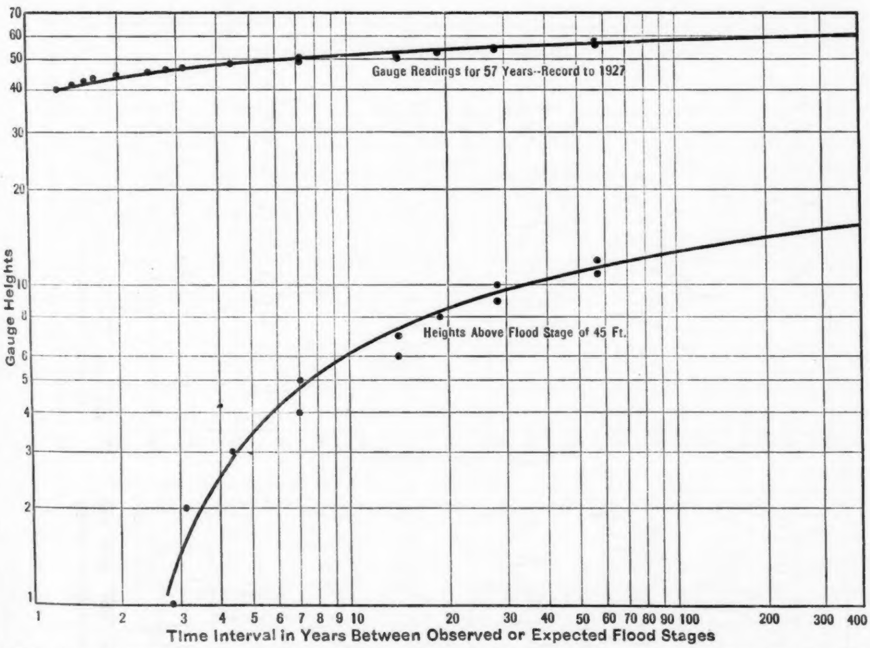

FIG. 44.—TIME INTERVAL, IN YEARS, BETWEEN OBSERVED OR EXPECTED FLOOD STAGES, ON THE MISSISSIPPI RIVER, AT RED RIVER LANDING, LA.

Regulation by Head-Waters Reservoirs.—It is difficult to ascribe the reduction of flood heights at St. Paul to any other influence than the head-waters reservoir system established by the War Department. So well have these reservoirs functioned that observers doubt whether they have yet withstood a severe test. A similar impression has been entertained regarding the structures of the Miami Conservancy District; but examination of their records will prove that they have been called into action more than twenty

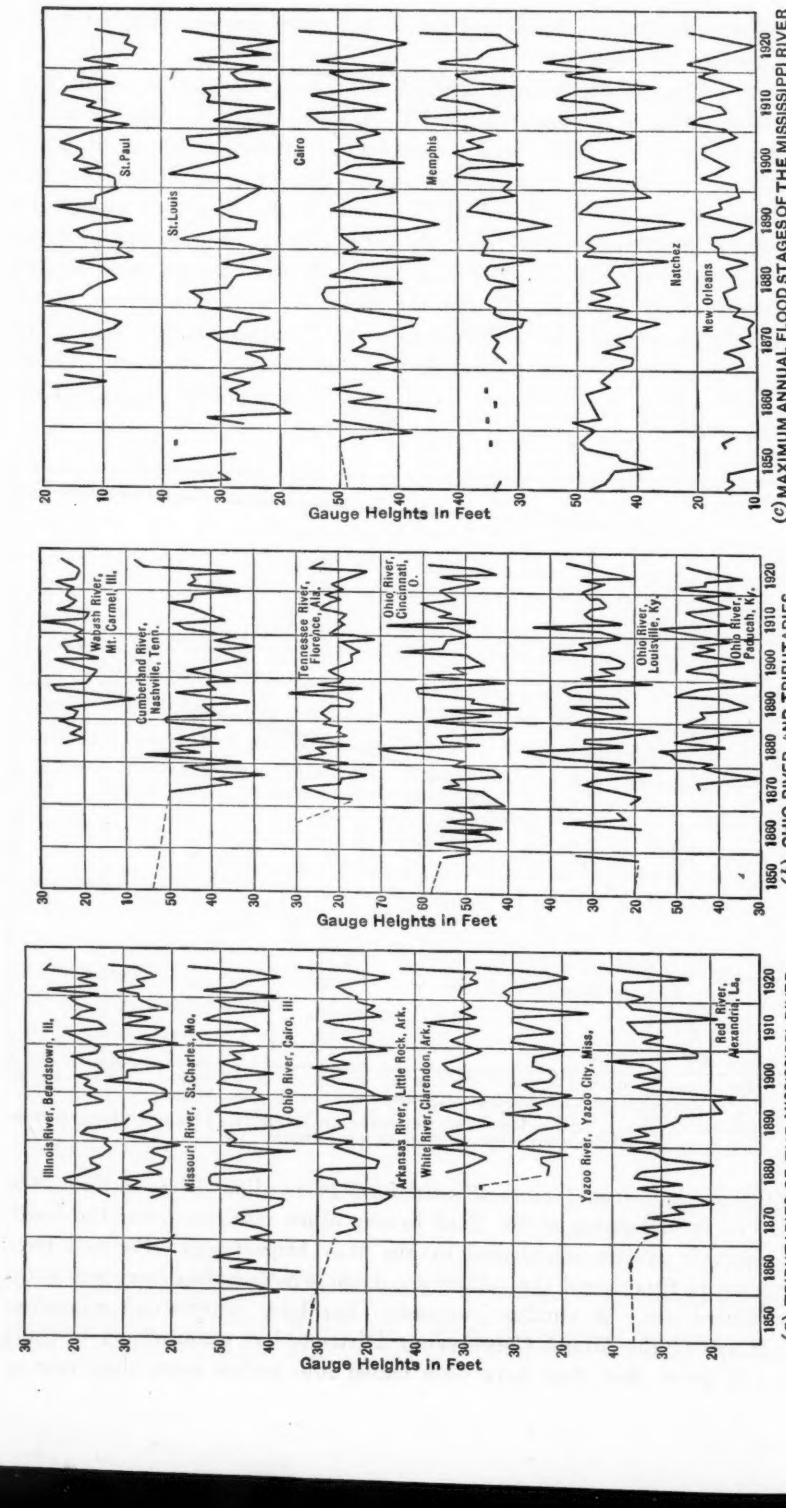

FIG. 45.—HYDROGRAPHS OF MAXIMUM ANNUAL FLOOD-STAGES.

times during the past five years, and have averted material damage in that district, although the recent storms have attained only moderate severity.

This regulation of the flow is not to be measured by nominal storage capacity alone. Factors too often neglected are:

(1) Reservoir storage above the spillway level; ordinarily nearly equal to that which lies below.
(2) The effects of back-water, extending to great distances along the main valleys and retarding surface flow while promoting underground storage.
(3) Percolation into the lateral areas thus temporarily flooded.
(4) The soil moisture capacity reserved below the dam by reason of the delay afforded by the foregoing factors against inundation.
(5) The lag in movement of water outside the main channel, always subtracting from the flood crest and extending the period as well as the losses in transit.

Due to these factors, the actual retardation accomplished by a reservoir with an adequate spillway is a multiple of the apparent capacity. This has been repeatedly demonstrated on various streams, under conditions and control roughly simulating those of the laboratory.

Salvage of Fragmentary Records.—Investigators have been richly rewarded for applying the winnowing process to old records. The observations at Natchez, extending intermittently from 1770 to 1802, and each year thereafter, are outstanding examples. No doubt, there are other data to be uncovered in the reports of the Chief of Engineers, U. S. Army; the Weather Bureau; the Geological Survey; State and municipal engineering departments; power and irrigation companies; and occasionally in private files or libraries.

The Society has a Special Committee charged with the duty of collecting flood-protection data. This able and experienced personnel made good progress so long as financial support was forthcoming; but the work was discontinued, against earnest protest of the Committee, at the worst conceivable time. An unusual opportunity for creditable public service has thus gone by, only partly utilized; and each month of delay makes the task more difficult.

A commendable spirit of co-operation in meeting the immediate demands of various governmental departments recently produced results far beyond original expectations. The foregoing illustrations are representative of a great mass of information hitherto unassembled, but reposing in hundreds of volumes, or in manuscript form.

To accomplish what is required in this broad field, some competent agency such as the Society's Special Committee on Flood-Protection Data must be financed and set to work, with powers commensurate with its difficult task.

After the records have been interpreted and co-ordinated their most essential elements should be reduced to graphic form and summarized in some convenient manner for each station, as has been shown on Fig 42.

The Flood Control Problem Defined.—The problem of flood control is simplified by consideration of the relatively large volume for which pro-

vision has already been made in channels and in some form of storage. The relatively infrequent flood stage attained on large rivers, and the few days or weeks that it continues above the danger line, reduce the difficulties proportionately.

Definite, tangible, and positive control projects are needed immediately. These require popular support which cannot be had by antagonizing, but by recognizing, every commendable theory or movement. Therefore, every reasonable means of improving or maintaining soil structure, fertility, and vegetative covering, stabilizing drainage channels and reducing erosion, promoting the passage of surface water to soil storage, and allowing for its orderly release therefrom, should be encouraged as steps in the right direction. No matter how remote and indirect may be their influence, they benefit the locality; they enlist conscious support and sympathy; they combine free-will offerings with taxation toward a solution of America's foremost problem. They bear the same relation to effective flood control as do war-savings, abstinence, and speeding up of industry to the winning of a foreign campaign.

Checking of soil erosion in Alpine regions, for example, seems to be as imperative in Switzerland as the defense of the fatherland; or perhaps it is one means of defense which is worthy of adoption in other mountainous districts included in American problems.

Only one-fourth of the total rainfall appears as run-off in the Mississippi River. This is accommodated without material damage on an average of three years out of every four. The minor portion of the highest known flood is the destructive element, and that is susceptible of control.

Conclusions.—

1.—"The Mississippi needs more room", as aptly expressed by Col. George R. Spalding, Corps of Engineers, U. S. A., after conducting the campaign of rescue.

2.—Its control is a National problem and should be accomplished as the nation would repel an invasion.

3.—Wherever human life is at stake the protection should be positive and enduring. Where only inconvenience and slight damage would accrue from temporary flooding, this means should be encouraged and insured during emergencies such as the 1927 flood, as a means of dissipating the power of the flood. Such automatic relief is as essential to storage basins and river channels as are safety valves for steam boilers.

4.—Levees and other barriers as a first line of defense, assisted by reservoirs and regulating basins, auxiliary channels and spillways, additional outlets and diversion into adjacent drainage areas, all correlated, under intelligent supervision and control, must be utilized in any effective plan for the mastery of the Mississippi River.

EDWARD H. SCHULZ,* M. AM. SOC. C. E. (by letter).†—The continued and heavy rains of the fall of 1926 and spring of 1927, brought about an extra-

* Col., Corps of Engrs., U. S. A.; Member, Mississippi River Comm., Chicago, Ill.

† Received by the Secretary, December 12, 1927.

ordinary flood in the Mississippi River and its tributaries, from Rock Island, Ill., to the Gulf of Mexico. Rains were exceptionally severe on the watersheds of the Illinois, White, and Arkansas Rivers and in the valley below Cairo, Ill. There was much damage to property, much flooding of land, a large loss of farm animals and crops, and complete interruption of rail and highway traffic. Above all, there was acute suffering among the people, although fortunately not a heavy loss of life, thanks to the relief measures of the Red Cross, of the Army—the Quartermaster, Air Corps, and Engineers —and of other Governmental agencies.

Protection Against Future Floods.—The Mississippi River Commission, with the approval of the Secretary of War, and the President, has been directed by the Chief of Engineers to prepare a revised plan of protection against a probable flood on the main river and its tributaries, as far as they are affected by the back-waters of the Mississippi River floods. Public hearings were held, and investigations of rainfall, gauge heights, and discharges have been made with a view of presenting a comprehensive report, a plan, and an estimate for the action of the Chief of Engineers, the Secretary of War, and for presentation by the President to Congress. Committees of the Commission and the District Engineers in charge of local work have studied and reported on auxiliary means of help, such as dredging, reservoir, diversions, and outlets, as well as on the general subject of levees, including greater free-board and larger sections.

Present Levee Jurisdiction.—The Commission's jurisdiction is limited by law, for levee and other protection work, to the Mississippi River below Rock Island, and the tributaries as far as they are affected by Mississippi River floods. The subject of a general survey of all tributaries, as well as of other rivers of the United States, covering floods, irrigation, water power, and navigation, was authorized by Congress, January 21, 1927, based on a report in House Document 308, 69th Congress, First Session.

Physical Characteristics.—The distances along the Mississippi River and the slopes at low water are as follows:

Rock Island, Ill., to mouth of Missouri River	302 miles,	slope	0.4	ft.	per	mile.
Mouth of Missouri River to Cairo, Ill.	209 "	"	0.6	"	"	"
Cairo, Ill., to mouth of the Red River	772 "	"	0.35	"	"	"
Mouth of Red River to the Gulf	309 "	"	0.02	"	"	"

The alluvial valley begins about 52 miles above Cairo near Cape Girardeau, Mo. From this point to the Gulf the river flows in a valley varying from 20 miles in width near Natchez, Miss., to 80 miles near Greenville, Miss., the average width being about 45 miles. Below Plaquemine, La., the valley merges into the fan-shaped delta with a radius of 60 to 100 miles.

Rock is found not far below the bed at St. Louis. From St. Louis to Cairo, the average depth to rock is about 92 ft., varying from near the bed

to 130 ft. below. The general drop is 105 ft. in this distance. Below Cairo, the dip is rapid. No difficulties with rock would occur therefore in any river or flood improvement.

The Water-Shed.—The area of the whole Mississippi water-shed is 1 240 000 sq. miles, or about 41% of the United States. The extreme dimension in longitude is 1 900 miles, extending from the States of New York to Montana, and in latitude 1 400 miles, from Canada to the Gulf. Of the total area, 390 000 sq. miles are arid, 190 000 sq. miles, semi-arid, and 660 000 sq. miles are humid.

The distance from the head-waters of the Missouri to the Gulf is 4 200 miles, from Lake Itasca, 2 475 miles, and from the head-waters of the Ohio, 2 370 miles.

The six natural divisions of the drainage and normal rainfall are as given in Table 35.

TABLE 35.—THE MISSISSIPPI WATER-SHED.

Division.	Area, in square miles.	Ratio to entire basin.	Normal precipitation, in inches.
Upper Mississippi	165 900	0.13	30.9
Missouri River	527 100	0.43	20.7
Ohio River	201 700	0.16	44.2
Arkansas Basin	186 300	0.15	29.8
Red River	90 000	0.07	38.3
Central Valley	69 000	0.06	48.8
Total	1 240 000	1.00	30.1

It will be noted that the average rainfall is about 30 in. During March and April, 1927, 25 in. of rain fell, and in January to April as much as 35 in. in some parts of the lower valley, more than 23 in. above the normal for those months.

The Delta Basin.—In general, floods on the Lower Mississippi occur from February to May, and low water from August to December. The delta area, normally subject to overflow (except for the levee system), is 29 790 sq. miles in extent. This is equal to about 19 000 000 acres, divided, as follows:

By States:	Square miles.
Illinois	65
Kentucky	175
Tennessee	453
Arkansas	4 652
Mississippi	6 926
Louisiana	14 695

By Basins or Districs:	Square miles.
St. Francis and miscellaneous in Illinois, Kentucky, and Tennessee	6 706
Yazoo	6 648
White	956
Tensas	5 370
Atchafalaya	6 085
Pontchartrain	2 001
Lafourche	2 024
Total	29 790

The flood of 1927 covered a great portion of these areas. The previous large floods occurred in 1828, 1844, 1858, 1862, 1882, 1883, 1893, 1897, 1903, 1912, 1913, and 1922.

Discharges and Gauge Heights.—The maximum discharge at Cairo for 1927, was 1 800 000 sec-ft., the actual height was 56.3 ft. on the gauge, and the confined height was estimated at 58.5 ft. The actual height at Vicksburg, Miss., in 1927, was 58.5 ft., which was 3.65 ft. above any previous flood, with a discharge of 2 278 000 sec-ft.—more than 20% greater than any previous flood.

Three waves passed Cairo in the last flood. The first was early in January with the Cumberland and Tennessee Rivers at high stage; the second early in February, with the Ohio River high; and the third in March and April, with the Upper Mississippi River high, and the Arkansas and White Rivers both at extraordinary stages.

In 1927, during maximum gauge at Cairo, the Ohio River contributed 814 000 sec-ft., whereas in 1913, it gave 1 395 000 sec-ft. One method of arriving at a future maximum flood has been to add the excess Ohio flow of 1913 to the Upper Mississippi flow of 1927. While this may be taken as the maximum possible, the maximum probable may be taken as somewhere between 2 150 000 and 2 250 000 sec-ft. at Cairo.

The average number of days of extreme flood heights is about 9 at St. Louis, 14 at Cairo, 13 at Memphis, 19 near Arkansas City, 24 at Vicksburg, and about 25 at New Orleans. The number of days above ordinary high water is as much as five times this period.

The maximum discharge of the Missouri River is about 1 sec-ft. per sq. mile, that of the Ohio River is about 6½ sec-ft., and that of the Mississippi at Vicksburg, about 2.0 sec-ft. per sq. mile of water-shed.

The extreme range of gauge readings to date at Cairo and below, has been listed in Table 36.

The maximum flow at the latitude of Arkansas City in 1927 was 2 662 000 sec-ft., including overflow, and the actual discharge in the main river was 1 712 000 sec-ft. At Old River, the inflow was 2 010 000 sec-ft. and the outflow 1 461 000 sec-ft. in the main river, and 592 000 sec-ft. down the Atchafalaya River.

The 1927 flood was on the average 1.3 ft. higher from Cairo down, than the previous highest, or 1.7 ft. below levee grade. It exceeded the previous high gauge by 3.4 ft. at the mouth of White River, and was 2.2 ft. below the previous high at Cottonwood Point, Mo.

TABLE 36.—RANGE OF GAUGE READINGS IN MISSISSIPPI RIVER.

Location.	Extreme low.	Extreme high.	Range, in feet.
Cairo, Ill.	−1.0	56.3	57.3
Memphis, Tenn.	−2.65	46.55	49.2
Arkansas City, Ark.	−3.6	60.55	64.15
Vicksburg, Miss.	−6.5	58.5	65.0
Red River Landing, La.	−0.6	57.5	58.1
New Orleans (Carrollton), La.	−1.6	21.27	22.9

Since the Mississippi has no natural reservoirs, it is not surprising that there should be great variations in stage and flow, and that the river heights did exceed the previously expected maximums. The reservoir area of the Great Lakes gives the St. Lawrence a storage of more than 100 000 sq. miles. The Mississippi River has an average flow three times as large as the St. Lawrence, and a maximum flow nearly eight times as large.

The Levee System.—The present levee system had its beginning in the small mounds, about 2 ft. high, built at New Orleans as early as 1717. With each succeeding flood these small protections were built higher and took in more territory, so that gradually a levee system was created. This work was first done by private plantations, then by groups, and, finally, by levee districts and States. In 1859 the levee grade was only about 5 ft. above the banks. The United States as early as 1824 appropriated funds for river snagging, etc., and up to 1879, about $10 000 000 had been spent. It was not until 1879 that formal assistance on a large scale, both for levees and revetments, was given by the Federal Government.

At present, the levee system is about 1 815 miles long, of an average height of 17 ft. and about 472 000 000 cu. yd. in volume, which is about 260 000 cu. yd. per mile.

The expenditures to date have been as follows:

	Levees.	Revetment.	
United States	$71 000 000	$61 000 000	
Local co-operation	167 000 000	3 000 000	
Total	$238 000 000	$64 000 000	
Grand total			$302 000 000

The most recent authorization by the United States was in 1923, the project covering $60 000 000, of which about $30 000 000 had been spent at the beginning of the 1927 flood. The 1914 project was thus about 90% completed. The present local co-operation required by law is one-third of levee costs, and all right of way and maintenance. The United States constructs

all revetment, except on tributaries, where local interests are required to co-operate.

The Mississippi River Commission.—This Commission was organized in 1879 to make complete surveys from the head-waters to the Gulf, and to prepare plans and execute works for navigation and protection against floods from Cairo to Head of Passes, La., including outlets, levees, closing chutes, bank revetment, and dredging. Jurisdiction was later extended to Rock Island, and to include tributaries in the flood back-water of the main river. The levee grade line established in 1914 was placed 3 ft. above the 1912 confined flood, the greatest to that time. The characteristics were: Free-board, 3 ft.; crown width, 8 ft.; river slope, 1 on 3; land slope, 1 on 3, to banquette, and 5 to 8 ft. below crown; and banquette, 20 to 40 ft. wide, with 1 on 10 slope and back slope 1 on 4.

Studies and plans are now being made to provide greater free-board, larger crown width, and flatter slopes. The new section would thus be much greater in volume for this average height.

Crevasses.—There were twelve principal localities where crevasses occurred. Counting the smaller breaks and those on tributaries, there were twenty-four all told under Commission plans. The first occurred at Dorena, Mo., 33 miles below Cairo, on April 16, and the last at McCrea, La., on the Atchafalaya River, on May 23. All breaks occurred on the right bank except two. one at Mound, Miss., on April 21, flooding Greenville, and the other at Caernarvon, La., on April 29, opened by the local interests to reduce the flood heights at New Orleans. There were heavy rains and high winds on April 20, with a rise of 1.1 ft., in 24 hours at Arkansas City. The Mound crevasse opened to ½ mile in 4 hours, and eventually flooded more than 700 000 acres, destroying many farm animals and flooding many towns. Mail placed in the street box at Greenville on April 24, was not delivered in Chicago for more than three weeks, showing the disruption of communication and transportation service. Four breaks occurred on the Arkansas River, and seven on the Bayou des Glaises and Atchafalaya Rivers. Many of the levees were not up to grade, nor under Commission jurisdiction. Some of them were undermined or cut by wave-wash and in spite of all the efforts of the local levee districts and the United States District Engineers, they were overtopped or undermined and destroyed.

The flood has shown that protection must be given for greater floods, that levees in the geographical jurisdiction must be under Federal control, that the problem is National, and that the participation by local interests must be made more liberal.

Caving Banks and Revetments.—The present average bank height is about 44 ft. above low water at Cairo, and about 45 ft. at Red River Landing, with an average slope down stream of 0.35 ft. per mile.

Caving banks are a notable characteristic of the lower river and can be prevented only by revetment. There are about 127 miles of revetment now built, consisting formerly of mattress and rip-rap, but more recently of concrete slab type. Several hundred miles of additional revetment will be

needed. The cost is $300 000 to $400 000 per mile. The caving has at times equalled 1 000 000 cu. yd. per mile, in certain sections, the total amounting to 400 000 000 cu. yd. per year.

The lack of this revetment protection causes the need of expensive setbacks and relocations in the levee line, often costing more than $500 000 per year. At times, setbacks have a benefit of their own in providing more cross-section area for floods. Fixation of the bank line is an important part of regulation and flood control.

Suspended Matter (Sediment).—The amount of suspended matter present depends on the velocity, character of bed and banks, whirls, and eddies. It has been found to equal $\frac{1}{1\,500}$ of the discharge by weight and $\frac{1}{2\,900}$ by volume. There is also material rolled along the bed. The total quantity carried to the Gulf is estimated as equal to a volume of 1 sq. mile, 268 ft. high. The banks of the Southwest Pass have advanced at times at a rate of 1 mile in 20 years. The sediment deposited on the delta is estimated to average 0.01 in. in thickness annually.

Property Values and Damages.—The valuation of properties in the Delta Basin is estimated at $4 000 000 000 to $6 000 000 000; the damages of the 1927 flood at $200 000 000 to $400 000 000, of which $10 000 000 was physical injury to the railroads. The delta was overflowed over 50% of its area. More than 600 000 people were made homeless. The expense of the flood fight, for closing levee gaps and crevasses, and repairs, was more than $7 000 000. In addition, the levee districts spent more than $1 300 000 for the flood fight. The estimated cost of raising the wharves at New Orleans, and of right of way for necessary levee setbacks must also be considered. In addition, a thorough study of the economic considerations should be made at the earliest practicable date.

Future Protection.—Suggested remedies are control of drainage and contour plowing, reforestation, additional levee heights, dredging, reservoirs, outlets, diversions, and spillways.

Contour plowing and land drainage would give but meager help; they are impracticable because there is no law to enforce such a measure, and, as stated, the relief would be insignificant.

Reforestation should be done on its own merits. There is great need for such work. The benefit to flood control is problematic. An examination of the water-shed records in Minnesota shows that forests caused greater floods than cultivated land, but probably less than deforested scrub land or pasture. To include any forestation for flood control would require many years, even if the benefits were certain.

Any material raising of levee grades should be avoided, if possible. To hold in check the probable future flood would require a considerable raise unless auxiliary means can be used. These additional heights not only become very costly, but add a greater menace to the property and people behind the levees.

No particular change of the bed has been noted. If anything, there has been a lowering of 2 to 4 ft. between Cairo and Helena. The average depth

of the thread of the stream at low water, taking both the pools and bars, is 31 ft. at Cairo, 48 ft. at Red River Landing, and 84 ft. at New Orleans. Dredging is necessary to maintain the present navigation project of 9 ft. by 250 ft. at all stages above Red River, and it costs about $600 000 per year. An enlarged channel width is advisable in view of the recent large increase in river traffic.

To dredge for flood control would require several hundred million cubic yards per year at an enormous cost. This would lower the bed only a few feet, and it would be filled again with the next flood.

Cut-offs would reduce the length of the river, but would increase the slope and velocity, and require new levees and revetments in the vicinity. On the whole, they would be a detriment to the river and are decidedly inadvisable.

At first thought, reservoirs seem to offer help. To be of real benefit, they should store water at least three months, from February to May; 1 000 000 acre-ft. of storage will hold back about 5 600 sec-ft. for 3 months, which is equivalent to about 1.07 in. of gauge on the main river. (About 60 000 sec-ft. corresponds to 1 ft. on the gauge.) Reservoirs are naturally divided into those above Cairo, those below Cairo, and those in the valley itself. To take care of 4 ft. of river height will require about 44 000 000 acre-ft. at a cost of more than $500 000 000. Additional storage becomes more expensive, since the most available and economical sites would be used first. The storage nearest the lower valley is the most advantageous, both in rainfall and in control. Storage for local flood contral although simpler is nevertheless very expensive. The Dayton Flood control, as an example, has cost $30 000 000.

In general, it is difficult and at times impractical, except at great cost, to use reservoirs as a direct means of flood control for the lower river. Where power is involved the benefits are necessarily divided, although still valuable especially to local flood control. Further study should be made of sites on the tributaries of the Lower Mississippi and of basin storage along the main valley below Cairo. Here, real benefits may be possible.

Outlets, diversions, and spillways seem the most promising remedies. A diversion channel from the main river near Arkansas City to the Red River is possible, and an increased discharge is practicable through the Atchafalaya Outlet. Also, spillways are feasible above and below New Orleans. Any diversion above Arkansas City becomes much more expensive than the corresponding levee protection. The diversions mentioned will materially reduce levee heights and will reduce the total cost over any plan without such diversions, in addition to giving greater security. It is reasonable and practicable that some solution based on revetment, on levees of proper section and free-board, and on diversion and outlets and possible reservoirs on the lower tributaries will secure the necessary protection.

Conclusions.—Complete protection against floods such as that of 1927, and of future probable floods, is vital to the security and prosperity of the Mississippi Valley. The business and transportation interests, not only of the valley, but of the nation itself, cannot be harrassed by recurring floods.

The cost of the protection may run into hundreds of millions of dollars. The economic conditions should be studied, and a comprehensive plan submitted, based not only on sound engineering, but on considerations of National obligation and of lasting benefit to the whole country.

Q. C. AYRES,* M. AM. SOC. C. E. (by letter).†—Like many another engineer whose livelihood does not depend, directly or indirectly, upon the methods used in the control of the Mississippi River, but whose inclinations lie within the general field of hydrology and hydraulic engineering, the writer has read with intense interest the very valuable and illuminating papers contributed to the recent Flood Symposium.‡ He recalls the many perplexing questions that assailed his mind in 1913, when he took part in the memorable high-water fight of that year at Greenville, Miss., and he notes that most of these questions still remain unanswered. Also, the many months of strenuous activity in connection with swamp surveys in the shadow of the mighty river are not forgotten.

An outstanding feature of this Symposium (as well as of earlier papers) seems to be a sharp difference of opinion, accentuated to an extent not commensurate with the large amount of attention devoted to this subject in the past. A moderate degree of such variance is, of course, a normal, stimulating, and desirable thing, but it would seem that radical differences would have been ironed out before now. If the recognized leaders of the profession, both within and without the Mississippi River Commission, can not agree among themselves on fundamental conceptions of river control, how can they expect anything but exaggerated disagreement among the rank and file? And what can that breed but still more accentuated misunderstanding in the mind of the voting public? Since the Congressional mind is nothing more or less than a composite cross-section of public beliefs, inflamed by diverse political and partisan interests, it would seem that flood legislation in the past has been all that could have been reasonably expected. After all, under the American system of government, no matter how competent and ingenious an engineering body may become, any program that might be devised must be patterned to fit the specifications of purse-holding laymen.

It is the writer's belief that, when all the facts are considered, the record of the Mississippi River Commission in the judicious handling of a very difficult and many-sided problem will measure well up to the standards of contemporary engineering practice.

The writer particularly has in mind Mr. Morgan's paper.§ The case he makes against the Commission seems just a little bit too plausible. For example, he quotes‖ the President of the Commission as having stated in 1912 that $73 000 000 was ample to do all the remaining necessary work on the levee system. This is no doubt a correct statement of majority opinion, but the writer happens to recall that the individual estimate of a civilian member

* Associate Prof., Agricultural Eng. Dept., Iowa State Coll., Ames, Iowa.

† Received by the Secretary, December 14, 1927.

‡ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2451, *et seq.*

§ *Loc. cit.*, p. 2532.

‖ *Loc. cit.*, p. 2533.

of the Commission—C. H. West, M. Am. Soc. C. E.—at that time was $250 000 000.

This circumstance would seem to blunt the point of Mr. Morgan's sweeping denunciation on one score at least and leads to the belief that perhaps other scores would be similarly relieved of harshness if all the pertinent details were known. The impression of blind "ancestor worship" so artfully created by Mr. Morgan as the outstanding trait of the Commission personnel seems subject to discount on its face. Furthermore, it is an anachronism to state that the Commission failed to apply 1927 methods to the problems of the preceding century.

Happily, engineers are agreed upon one matter of transcending importance —the need for accurate and comprehensive data. It would seem almost self-evident that the first step in the solution of any scientific problem is the collection of adequate and indisputable facts; yet this basic phase of the Mississippi question has been undeniably neglected to a large extent, under pressure of restrictive legislation and limited funds. Lack of conclusive information doubtless explains in large measure the divergence of opinion so regrettably apparent, but for this, any indictment of the Mississippi River Commission must be leveled also at the entire profession.

There is reason to hope, however, that the Seventieth Congress, now in session and smarting under the sting of a major National disaster, will enact new legislation substantially in accord with the latest recommendations of the many agencies now at work on the problem. Facilities may thus be provided for the first time to gather a great mass of physical data, to enlarge the vision and diversify the engineering talent at the disposal of the Commission, and to enable many important controversial theories to be definitely proved or disproved. In such an event, the present moment may mark the end of a long period of pioneering with expedient disbursements, and the beginning of a new epoch more in keeping with the ideals of the profession.

H. S. Gladfelter,* M. Am. Soc. C. E. (by letter).†—The paper by Major Godfrey‡ is of much interest in presenting briefly, the extent of the development of modern transportation on the Mississippi River and the extent to which flood protection measures may be aids to navigation.

With the gradual building up of rail-transportation systems, running parallel to the Mississippi River, old-time steamboats gradually disappeared. These old steamboats had no regular schedules, and their rates were whatever the traffic would bear. They offered no insurance that freight entrusted to them would ever reach its destination. No adequate terminals were provided for the exchange or storage of freight after it was delivered. The designs of boats and their mechanical equipment did not change materially in the period from 1850 to 1900. In 1852, when steamboating was at its height, 3 100 boats docked in the Port of St. Louis, and the average load of these boats was only 26 tons.

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* Engr., Mississippi River Comm., Dredging Dist., Memphis, Tenn.

† Received by the Secretary, January 16, 1928.

‡ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2557.

Contrast this condition with that now existing. The Federal Barge Line operates 13 powerful boats that handle tows of 8 000 to 12 000 tons. These vessels are equipped with modern propellers and triple-expansion condensing engines. The American Barge Line (referred to* as the W. C. Kelly Barge Line) with four high-powered Diesel towboats, handles tows of from 3 000 to 4 000 tons, and the giant stern-wheel towboat, *Sprague,* with modern machinery, having a power plant of exceptionally high efficiency, handles tows consisting of 224 000 bbl. of crude oil, or about 40 000 tons.

Major Godfrey refers* to experiments now being conducted with pulverized coal as fuel on one of the boats of the Federal Barge Line. If these tests are successful a new field of cheap fuel will be available for use on the larger vessels. With high labor costs, it is essential, in the operation of river boats, that machinery of the most efficient type be adopted. The old days of inefficient machinery are gone.

The use of modern terminals with machinery for handling the freight is emphasized. Modern terminals have been erected in a number of cities from St. Paul, Minn., to New Orleans, La. These have car interchange facilities. In some cities even store-door delivery is attempted by motor truck, in order to compete with rail transportation. River transportation, to be successful, must give all that rail transportation offers and more.

Considerable interest has been taken in the statement in the message of President Coolidge to Congress that the Federal Barge Line, having demonstrated that it can make money, should be turned over to private interests as soon as possible. With the competition offered the railroads by water transportation interests, the question naturally arises, as to whether the railroad companies will be allowed to acquire control of the water transportation and, in order to protect their rail interests, gradually throttle the water transportation as was done in the past.

The earliest system of aids to navigation consisted of regulation works in the form of permeable dikes. These seem to have been effective for a short time in improving channel conditions, but due generally to insufficient funds, were not properly maintained, and, eventually, all the dikes failed. This system of regulation, if applied to the entire length of the river and properly maintained, would probably have resulted in good channel conditions, but the expense of maintaining a channel in this way would be very much greater than the cost of maintaining the dredged channel, adopted later.

The assistance offered to navigation by dredging, channel-marking, and channel charts, has been effective in providing a channel suitable for present requirements. Under this system, the commerce on the river has increased rapidly. The constantly increasing volume of navigation and the economic size of tows will undoubtedly, in the future, demand a less restricted channel and one of greater depth and width. Dredging alone will not be effective in producing the required channel economically.

The plan of flood control of the Mississippi River presented by the Chief of Engineers to Congress recognizes the limitations of dredged channels and

* *Proceedings,* Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2558.

provides for a system of regulating works similar to that used successfully on the Mississippi River between St. Louis, and Cairo, Ill. This system of regulation would serve the dual purpose of bank protection and channel improvement.

With the work embraced in this flood-control plan, navigation interests are assured that channel conditions will be improved; and while flood control and improvement of navigation are entirely different problems, it will be found that many of the solutions of flood control will also be solutions for providing improved navigation.

Byron E. White,* M. Am. Soc. C. E. (by letter).†—The excellent and authoritative papers presented on this subject have done much toward clarifying the atmosphere of uncertainty which has shrouded the Mississippi River problem. It is fortunate that it has been so fully treated, in its various phases, by eminently qualified engineers.

A careful reading of all the literature bearing on the question, leads to the conclusion that there is no escape from the construction of levees along the lower river, as the main line of defense against floods. It is also quite clear that the raising of levees to greater heights cannot go on indefinitely. Some auxiliary means, therefore, must be adopted to care for the excess flood waters, beyond the safe capacity of the channel formed between the levees.

The only means which has been more or less generally accepted as practicable are spillways and auxiliary flood channels. This is the plan so well described and advocated by Mr. Garsaud.‡ The greatest objection raised to spillways is the formation of bars or shoals down stream therefrom in the main river channel and the resulting reduction in carrying capacity, etc. If such spillways are controlled, so that the flow in the river down stream may be kept at a safe height below the tops of the levees, and only the excess water diverted, such shoaling should be effectually prevented. Colonel Kutz§ states his belief that such will be the case.

Such spillways or sluice-ways (as they may properly be called) will be of unprecedented dimensions and capacity, but, otherwise, not unlike hundreds of similar structures. They should present no unusual difficulties in construction or operation. Their proper use should be of great assistance in maintaining the main river channel during and after floods and minimizing the work necessary to maintain navigation channels after a flood has subsided.

Any doubts as to the proper arrangement, location, and performance of such controlled spillways could be rather quickly, cheaply, and easily proved or dispelled by carefully conducted tests of models in a hydraulic laboratory, such as has been so strongly urged by John R. Freeman, Past-President, Am. Soc. C. E. Many other detail problems of levee construction, control of erosion, and channel control could be studied in such a laboratory. Such immense sums of money spent in construction, and such vast interests, both financial and human, are involved in the success or failure of the plans and

* Gen. Engr., Utica Gas & Elec. Co., Utica, N. Y.

† Received by the Secretary, January 28, 1928.

‡ *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, pp. 2577–2585.

§ *Loc. cit.*, p. 2509.

structures necessary to control the destructiveness of floods, that the relatively small cost of model tests fades into insignificance when the certainty of results and other advantages are weighed against it. The study of these problems in a laboratory should do much to popularize model testing among engineers and executives and aid immensely in putting American design and construction work on a more sound and satisfactory basis, especially where novel designs are involved.

While this may cause some delay in the preparation of final plans and the execution of the work, the certainty of results should more than compensate for it, especially if the existing levees and other structures are promptly repaired and made ready for service before the next flood season. It would be almost criminally wasteful to embark on a radical change in plan, without first making sure of its efficacy in detail. Unless there is a real threat to the stability of the levees, the writer is also strongly in favor of the construction of wide, paved highways along them, for ease of access, inspection, and repair, as suggested by Mr. Jonah.*

Much doubt as to the feasibility of reservoir control as a means of preventing, or even materially reducing the height of floods in the lower valley is expressed by practically all who are familiar with the river. The physical dimensions of a dam which would impound the requisite volume of water might be very great, without exceeding the value of a large regulating capacity. Hence, it is just possible, even if a little improbable, that, on account of the great length of dam which would be involved, some feasible site may have been overlooked in all the investigations made thus far. Another search for such sites may well be in order, before definitely abandoning all idea of partial control by means of reservoirs.

In view of the assertion that only a relatively small part of the lower valley has been topographically mapped, it is not difficult to conceive that some, perhaps several, feasible dam and reservoir sites may have escaped proper attention. A survey by the rapid methods of the U. S. Geological Survey, covering those areas which hold forth any promise, would give quick results and disclose any sites, worthy of further and more detailed investigation. It is, of course, possible that such studies have been made and that there are no potential dam sites which are not known and have not been studied, but the discussion has not shown this to be the case.

The almost infinitesimal effect of reservoirs located on the Upper Mississippi and Ohio Rivers on flood heights at Cairo, Ill., cited by Colonel Kelly,† may seem astonishingly small, yet they are probably not far from the truth. Therefore, any effective reservoir control must be located (as stated by some of the authors) much closer to the region to be protected and must store vast quantities of water and cover immense areas of land. Whether such areas exist, and whethey they are of small enough property value to warrant their use for this purpose, does not appear to have been authoritatively stated.

The study of reservoir possibilities referred to by Colonel Kelly† does not offer much encouragement for the hope that economical reservoir sites,

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 200.

† *Loc. cit.*, December, 1927, Papers and Discussions, pp. 2519–2531.

effective for flood control, may exist in or near the lower valley. It is, of course, possible that in the haste with which the investigations and report had to be made, something of value has escaped attention or has not been properly investigated or evaluated. The case should not be considered closed until every possibility has been completely studied and its value deliberately weighed.

Reforestation has also been frequently advocated as a means of reducing the height of floods on the Lower Mississippi and elsewhere. It is generally recognized that forest cover is quite effective in limiting or preventing soil erosion, as well as helping to retain the water absorbed by the soil. Record floods, such as that of 1927, are the result of prolonged, intense rainfalls, and often (although probably not in this case) the rapidity of run-off is augmented by frozen ground. Under these conditions the effect of forest cover on holding back the water must be very small, especially when the ground is already thoroughly saturated, as was the case in 1927.

The work, time, and expense involved in reforesting, say, 20% of the total drainage area (1 250 000 sq. miles) of the Mississippi, or 250 000 sq. miles, are enormous. At the rate of 1 200 trees per acre, there would be required 192 000 000 000 trees and the cost would be from \$1 500 000 000 to \$2 000 000 000.

It is hardly possible that a much greater area, aside from that already forested, could be devoted to that purpose, without encroaching on lands which are more suitable for agriculture and grazing. When the percentage of the total area available for reforestation is taken into account, together with its inability to greatly limit run-off under exceptional conditions, this does not appear to offer much encouragement as a means of flood control. Since the effect of such reforestation as may be practicable will not be great, it would be advisable to consider it as another line of defense against the possible maximum flood.

These remarks must not be misconstrued as opposing reforestation. The immense value of an adequate timber supply, about 30 to 50 years hence, is of itself sufficient reason for encouraging the reforestation of all areas where trees can be grown, as a crop, to better advantage than anything else. The comparatively small benefit of forests as a means of flood prevention, the percentage of the drainage area available for forest planting, and the length of time and the difficulty of getting an effective reforestation program started and carried to completion, all rule it out as an immediate or major factor in this situation.

The events of 1927 emphasize the desirability and, indeed, the necessity for placing the planning and supervision of the construction of all works for the control of the Lower Mississippi in the hands of a single agency of the United States Government. Further, in order to stem the rising tide of paternalism and the reliance on the Federal Government in all kinds of major projects involving local benefits, the States, localities, and local interests should be required to shoulder a reasonable portion of the costs. To do otherwise will constitute a precedent which will be used as an argument in unnumbered future (as well as present) projects. Once the principle of local sharing

of costs is abandoned, there will be no end to the calls for contributions from the Federal Treasury to do that which is the duty of individuals, communities, and States. While the nation should help, where navigation and national interests are involved, its generosity should not be imposed upon. The loaning of the credit of the nation in an emergency is a different matter.

In regard to Mr. Scheidenhelm's excellent paper,* on "Power as Affecting Flood Control", there are a number of points which might well be amplified.

In the face of the already low and constantly declining cost of fuel-generated energy, the value of stored water for power generation is so small that, unless the cost per unit of volume is exceptionally low, storage reservoirs, exclusively for that purpose, are not economically feasible; that is, unless there exists a large aggregated head down stream from them, which it is economical to develop for power production. In the usual case, this means that, unless the available head, or fall, below the reservoir site amounts to several hundred feet, the cost of the reservoir may be prohibitive. This fact eliminates many apparently promising streams from consideration as power producers. As steam power costs go lower (and further economies are constantly being effected), this effect will be greater and will continue to narrow the choice of streams on which economical hydro-electric plants may be constructed.

In the many cases where suitable reservoir sites are found above large available heads, the head at the reservoir dam itself will usually be found to be a small part of the total involved. If generating equipment is connected to the outlet conduits, in many cases, it will be a producer of secondary energy, only, as and when water and a suitable head are available. When such an installation is a part of a large interconnected system, this arrangement is usually entirely feasible and practical, provided the costs are not too great.

In favorable locations, an additional head, immediately down stream from the storage dam, may often be combined with that at the dam by extending the outlet conduit down stream. When this additional head is equal to, or in excess of, that at the dam, power and energy, with properly designed equipment, may be generated from any water released, down to the practical drawdown level in the reservoir.

In this, as well as in the other case, previously described, a firm power value can only be attached to such power and the corresponding energy, as is produced from the minimum quantity of water released from the reservoir, either continuously, or at all times when firm or dependable capacity is required. If, at any time when firm power is needed, water cannot be released, or the head is so low that no power may be produced, then the connected power plant will have no firm capacity value. The value of such a plant when incapable of producing firm power and energy is much reduced, because it must be based on the value of secondary, or intermittent, energy only. Where steam or fuel-generating plants are connected to the system, the upper limit of this value is the cost of fuel per kilowatt-hour, plus the additional labor and other charges caused by operating the steam plant, less the cost of transmission to the market which would otherwise be supplied by steam power or

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2610.

other alternative source. In may cases there are other conditions or costs which still further reduce the value of the available power and energy.

In the case of reservoirs, where such additional head is not available, and it is desired to draw the water as low as practicable, the installation of a power plant is only justified when the intermittent supply of energy has a value sufficient to pay the charges on the plant, plus a reasonable profit. In such cases, therefore, the installation and operating costs must usually be exceedingly low, if the plant is to be justified. Herein lies the best reason why power plants are not usually found at the foot of storage dams.

Where storage for flood control and storage for power are sought to be combined in a single reservoir, the flood-control requirement will usually intensify the conditions unfavorable to power generation at the foot of the dam; and, where additional head is available immediately below, it will, of course, operate to reduce the power and energy output, and, consequently, the value of a connected hydro-electric plant. Also, where both flood control and storage for power are combined in the same reservoir it is sometimes although not frequently, possible to increase the height of the dam and utilize the upper portion, where the proportion of volume to depth is greatest, for flood control. If, in this, as in the preceding case, the contemplated total drawdown of the reservoir is too great a part of the total head available, the head will finally become so small that no power can be generated. The value as a power site is then greatly reduced, if not altogether wiped out.

Many of the reservoirs at the head-waters of the Mississippi, the Ohio, and other tributaries of the Mississippi, will be beneficial to power development, especially those in the mountainous regions. They will also exert some slight effect on floor reduction in the lower valley, as described by Colonel Kelly.* Those lower reservoir sites, which would exert the greatest influence on flood control along the lower river, will be practically, if not entirely, useless for the production of power. It is unlikely that basins will be found, at proper locations, the capacity of which will be sufficient to accomplish completely the desired purpose, even if designed as retarding reservoirs, like the Miami Conservancy District structures. It will be essential, therefore, to empty them as rapidly as possible, after each filling, in readiness for the next flood. The head will, therefore, be extremely variable, and power will be available only at long intervals, and for short periods of time. Hence, there is slight possibility that power will be a by-product of their construction.

Favorable effects of reservoirs on flood heights are, of course, most noticeable immediately below the dam, and it is possible that the sequence of arrival of flood crests from different tributaries at some point down stream, as, for instance, at Cairo, may be such that the flattening and lengthening of the flood peak above the crest of a storage dam may have unfavorable results far down stream. Hence, reservoir control at distant points does not have the beneficial effect that the same capacity would have, if concentrated near the point where its effects are most desired.

* *Proceedings*, Am. Soc. C. E., December, 1927, Papers and Discussions, p. 2519.

The recent development of large interconnected power systems has made it feasible and economical in many cases to utilize the intermittent power and energy available at the foot of reservoir dams, when conditions for them are favorable, and when within a reasonable distance of existing transmission systems and markets. Interconnection on a large scale has also made it economically possible to combine storage for flood control and for power in many instances, where it would otherwise be uneconomical for either purpose alone. Where such cases exist, the just allocation of costs between these uses is a matter of great importance in order that neither interest may be unjustly treated or taxed. The proportion of the cost which power interests can afford to pay are strictly limited by economic conditions. Expert engineering, legal and business advice, as well as a sense of fairness and open-mindedness, are, therefore, essential in dealing with such a problem. With such an approach, a reasonably harmonious compromise of the conflicting interests of storage for power and flood control, may often be reached, to the benefit of all interested parties.

The degree of conflict between the requirements of storage for power and for flood control, are proportionate to the relative extent of stream-flow regulation which may be effected by the reservoir or system of reservoirs in question. If complete, or nearly complete, regulation may be attained, the interests need not seriously clash. It then becomes merely a matter of draining the reservoirs well down in advance of the usual flood seasons. From this case down to that of the relatively small reservoir, effecting little regulation, the interests become more diverse. Nevertheless, large benefits may usually be obtained by both interests, by the exercise of fairness and common sense. Here, the connection of the power plant or plants to a large interconnecting system may likely be the saving grace which makes the generation of power at all practical and economical.

In general, the writer believes that while the requirements of power and flood control in a reservoir are more or less at variance, unless the necessities of one or both are decidedly fixed, both may be reasonably well accommodated, to the economic benefit of the community and the interests involved. A factor of equal and prime importance, which seems to be often overlooked or forgotten in some current discussions, is the existence of a satisfactory market, capable of absorbing the output in a reasonable time after the supply becomes available. If the anticipated absorption runs into too long a period of time, the interest and carrying charges on the unutilized portion of the investment, which must be added to the capital account, may make the ultimate cost altogether too high. If an adequate and fairly immediate market is not assured beyond a reasonable doubt, there is no justification for the construction of the project; and again, beyond a doubt, the output must compete in cost and dependability, with the other available sources of supply.

In the pressure for reservoir construction, it is often assumed that hydroelectric power supplies will be created which will assume a large proportion or all of the costs. It is sometimes asserted, also, that desirable reservoir or power sites are being allowed to lie dormant for ulterior reasons. It is well

to bear in mind at all times that, with few exceptions, every existing market is adequately supplied, and will continue to be well supplied, up to the moment when the output of a new development becomes available, barring unexpected and unusually large demands. The ancient law of supply and demand, coupled with good business and engineering judgment, may be depended upon to bring about the development of the most economical potential source of available supply.

There are projects of such magnitude, or remoteness from any adequate present market, that any corporation or other authority may well hesitate to embark on their immediate development, and the assumption of the annually mounting carrying charges, which must accrue before their entire output is successfully marketed. These facts and factors, affecting each project, must be carefully and conscientiously weighed before arriving at a final decision, or the results may be unhappy, if not disastrous.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

PAPERS AND DISCUSSIONS

FOUNDATIONS AND DRAINAGE OF HIGHWAYS
Discussion*

By Messrs. Harry Tucker, and Winslow H. Herschel.

Harry Tucker,† M. Am. Soc. C. E.—A careful study of this excellent paper prompts the following comments:

1.—It may be taken as axiomatic that a majority of the failures of pavements—particularly the rigid pavements—are due to unstable foundations.

2.—The unstable condition of the foundation is due largely to the volumetric change of the sub-grade materials.

3.—This volumetric change of the sub-grade material is due to its moisture content.

4.—The equivalent moisture content of a sub-grade material—a measure of the quantity of water which it will take up—depends approximately on its clay content.

Therefore, practically, the problem of obtaining a suitable foundation for a road may be solved by (*a*) providing a sub-grade material which has a low clay content, below 25% if possible; or (*b*) reducing the moisture content of the sub-grade material. The latter requires some form of underground drainage to lower the ground-water level to a point where it will not be injurious either as free water or as water rising through capillary attraction.

Tile and blind drains have been long recommended by highway engineers to improve the condition of the sub-base, and the practice, for certain cases, is fairly well standardized. Their use is open to two serious objections: First, their universal use in all cases where improvement of the sub-base material is necessary would mean considerable increase in the cost of a road project; and, second, they are not always effective, for even where they are utilized serious damage may result from water which enters the sub-base from the shoulders and along the edges of the pavement, before it is carried

* This discussion (of the paper by Albert C. Rose, Assoc. M. Am. Soc. C. E., presented at the Spring Meeting of the Society at Asheville, N. C., on April 22, 1927, and published in January, 1928, *Proceedings*) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Prof. of Highway Eng., North Carolina State Coll., Raleigh, N. C.

off by the underground drains. Such unfavorable conditions would prevail during a wide range of temperature after a heavy rain, snow, or thaw. For these reasons the use of underground tile and blind drains is probably limited to cases where it is necessary to lower the level of free water, and where the question of cost is not so important.

The alternate method is that suggested by Mr. Rose, namely, the use of a sub-base material with a low clay content, less than 25% if possible. If this is done, it is probable that failures of the pavement due to distortion of the sub-base will be infrequent. Is this method practicable?

The method outlined by the author would seem to be the most practicable for a great many of the Southern States. It will be found that the clay content of many of the top-soil roads is 25%, or less, indicating that this material would be ideal for a sub-base. This was pointed out by C. M. Upham, M. Am. Soc. C. E., some years ago in his description of the progressive system of building highways.* In this system all highways were graded to the standard for hard surface construction, but were surfaced with top soil or sand clay. When traffic needs demanded hard surface pavements, and finances permitted it, the top soil was used as the sub-base. In this manner a suitable sub-base material would be obtained without increasing the cost of the road to any great extent. This method requires that the ground-water level be kept at a reasonable depth below the pavement, so that it will not cause damage as "free water".

The use of top-soil material for the sub-base will help in decreasing pavement failures. Mr. Rose has indicated a number of other soils which may be used for the same purpose, and has pointed out how the survey maps of the U. S. Bureau of Soils, together with his tri-axial diagram (Fig. 11†), will enable highway engineers to select a suitable material for the foundation of highways. It is thus a problem for the individual engineer to select a suitable sub-base, consistent with the type of pavement used, the traffic needs, and the cost. It is a fruitful field of research for every highway engineer.

WINSLOW H. HERSCHEL,‡ ESQ. (by letter).§—When one has to deal with a mushy or pasty material like wet soil, to which neither the laws of hydrodynamics nor of elastic solids may be readily applied, it is fortunate when any progress whatever can be made. Mr. Rose is therefore to be congratulated that he has to some extent brought order out of chaos. On account of the complexity of the subject it is no surprise that any proposed method fails under certain circumstances.‖ It may be of interest to consider some possible causes for these exceptions.

The author shows that the clay content is indicative of the field moisture equivalent; that this, in turn, shows the shrinkage value of the soil and it is the shrinkage value which is most important as a criterion for the value of soil to support pavements. He states¶ that "* * * the displacement of

* *Engineering News-Record,* Vol. 88, p. 202.

† *Proceedings,* Am. Soc. C. E., January, 1928, Papers and Discussions, p. 142.

‡ Associate Physicist, U. S. Bureau of Standards, Washington, D. C.

§ Received by the Secretary, February 3, 1928.

‖ *Proceedings,* Am. Soc. C. E., January, 1928, Papers and Discussions, pp. 141–142.

¶ *Loc. cit.,* p. 123.

pavements seemed to be caused principally by the distortion of the sub-grade in one direction—vertically." While this may be admitted, the horizontal movements of the soil should not be overlooked.

In railroad work the phenomenon of subsidence is recognized as important. Material dumped on the right of way may settle out of sight and eventually re-appear at a considerable distance to the side of an embankment, as an "island" in what would be considered solid ground. Evidently, this involves horizontal flow of the soil, and such horizontal flow takes place in the sub-grade soils of many highways, even though not to such an extent, nor in such a spectacular manner. The amount of flow depends on the consistency of the soil, and it is questionable whether any test mentioned by the author, including the more elaborate tests of compressibility, elasticity, and permeability,* will adequately define the consistency of the soil. Mechanical analysis and the determination of the clay content are inadequate. Clays differ in many respects, including the percentage of colloidal matter which they contain and the state of flocculation of this colloidal matter, and these factors are not without influence on the consistency of the soil.

Consistency may be defined as the property which determines the distortion-force relation. In a simple viscous liquid the rate of distortion or flow is proportional to the force or pressure which causes the flow, and the viscosity may be expressed by a single numerical value. With a plastic material like soil, there is no such proportionality and at least two quantities are required to define the more complicated flow-pressure relation, that is, to define the consistency of a plastic material. Space does not permit a detailed explanation of the methods of measuring and expressing the consistency of plastic material,† but for the purposes of this discussion it will be assumed that consistency depends roughly on two quantities, namely, A, the pressure required to start the flow; and B, the rate of flow after motion has begun.

If it is admitted that consistency should be considered in tests of sub-grade soils, and that it is better to measure this composite property directly rather than to infer it from other tests, then it is a question whether information is required regarding Quantities A and B, or whether one alone will give adequate, even if not complete, information.

Perhaps in the case of asphaltic surfacing for highways, and surely in the case of soil under bridge piers, where an initial distortion may be as bad as complete failure, knowledge of Quantity A alone is sufficient. When, however, the soils under pavements are considered, from an economic standpoint it is impracticable to make all roads as solid as bridge piers. Rather is it desirable merely that the yielding of the sub-grade soil shall be small and gradual enough so that resurfacing will be necessary only on account of wear of the road surface, and not because the lateral flow of the soil, combined

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 143.

† F. P. Hall, "Methods of Measuring the Plasticity of Clays," *Technologic Paper No. 234*, U. S. Bureau of Standards, 1923; W. N. Harrison, "Controlling the Consistency of Enamel Slips," *Technologic Paper No. 356*, U. S. Bureau of Standards, 1927; Winslow H. Herschel and Ronald Bulkley, "The Measurement of Consistency as Applied to Rubber-Benzene Solutions," *Proceedings*, Am. Soc. for Testing Materials, Vol. 26, Pt. 2, pp. 621–633, 1926; and Winslow H. Herschel, *Proceedings*, 6th Annual Meeting, Highway Research Board, pp. 413–416, 1927.

with its volumetric changes, takes away the support from the surfacing so that it goes to pieces under heavy traffic.

The rule to keep water out of the foundations is as good as ever, but again from economic reasons, perfection cannot be reached, and it is desirable to spend the available money where it is most needed and will do the most good. What appears to be necessary is the determination of the consistency of sub-grade soils with various percentages of moisture, that is, curves showing the variations of Quantities A and B with the percentage of moisture for each type of soil. Then the problem becomes one of finding at what percentage of moisture a dangerously "soft" consistency occurs, and in adopting such types of design as will maintain the moisture content below this value.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

DESIGN AND CONSTRUCTION OF CONCRETE PAVEMENTS

Discussion*

By Messrs. R. T. Giles, and Elmer G. Hooper.

R. T. Giles,† Esq.—The author has very ably covered one of the most important features to be considered in connection with the building of concrete roads.

However, with the sub-grade and the design of the slab taken care of properly, there is still the problem of designing the concrete and the execution of the work in such a way as to be sure to reproduce in the slab a concrete which will compare favorably with the mix designed.

There are several theories on the strength of concrete, and after breaking a large number of specimens, the speaker has concluded that each of the different theories is an important factor and that the strength of the concrete is controlled by applying a combination of them.

The failure of the engineer to specify, and secure on the job, satisfactory measuring equipment, as a means to insure the proper control, causes more unsatisfactory concrete than faulty design of mix. Without accurate measuring means, the design of the concrete is upset and a constant uniformity is practically impossible.

The cement, being finely ground and lacking in moisture, is probably the most accurately proportioned material entering into concrete. With reasonable inspection under existing conditions, the quantity of cement can be kept within 5% of that specified.

The accurate measurement of fine aggregate on a construction job introduces problems not encountered in the laboratory. In some experiments with the measurement of dry sand it was found that the volume could be varied very little by the change in the distance of fall. In the case mentioned, the

* This discussion (of the paper by Clifford Older, M. Am. Soc. C. E., presented at the Spring Meeting of the Society at Asheville, N. C., on April 22, 1927, and published in January, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Pittsburgh, Pa.

rate of compaction was about 0.1% per ft. of drop. It is not felt that drying sand would solve the problem because it might introduce such difficulties as "flash" set in warm weather and the possible necessity for longer mixing time.

Measuring fine aggregate in a saturated condition has proved extremely efficient, and it is felt that for many kinds of construction work this method will be found highly satisfactory. The inundation method not only gives a constant volume of sand, but regulates the amount of moisture. With these factors regulated, the most important steps in the control of the concrete have been taken. However, it will be necessary to control the grading of the coarse aggregate and use an accurate measuring device for the added water in order to obtain the full benefits of inundation.

A number of organizations are proportioning fine aggregate by weight. Under favorable conditions, this is no doubt an advanced step over volumetric measurement.

Weighing introduces the need of determining the unit weight and the moisture content of the material for accurate control. This makes it imperative to use a certain amount of laboratory equipment on each job. While this is not necessarily a handicap, it is a point that must be considered and it should be recognized that the personal element of the inspector will be reflected in the results secured. It means more details for the inspector to look after, and in many cases he is occupied in so many other ways that the actual inspection is neglected.

Volumetric proportioning of fine aggregate is fast disappearing and rightly so. This method requires the services of an alert and energetic inspector who is willing to make innumerable tests. The personal element necessarily encountered is very great and wherever possible should be displaced by other means which automatically control the variables.

While much has been written about the variable introduced in concrete by the bulking of sand, that introduced by segregation of the coarse aggregate has not been given the attention which it deserves. Accurate measurement of coarse aggregate is not as difficult as that of the fine aggregate. On the other hand, however, the segregation, when large sized stone is used, is a serious factor. There are two positive ways to control the segregation: First by limiting the maximum and minimum size so that there will not be more than 1 in. between the two sizes specified; and, second, by using several different sizes of coarse aggregate and measuring the required amount of each size. It is questionable whether the first method is economical, as the material plants would have to produce one grade of stone only for concrete work. Splitting the stone into separate sizes has been successfully done on a considerable mileage of road work.

Most roadway specifications limit the maximum size of coarse aggregate to 2½ in. With the segregation eliminated by separating into three sizes, it should be possible to increase this maximum size to possibly as much as 3 in. With the increase in maximum size, the fineness modulus of the aggregate is raised, and this should increase the strength of the concrete and

particularly the flexural strength, which is of extreme importance in roadway slabs. With separated sizes of coarse aggregate, or the range of size limited to 1 in., the measurement becomes simplified and either volumetric or weight proportioning should prove highly satisfactory.

Water-measuring devices, particularly on paving mixers, have not been entirely satisfactory. However, the mixer manufacturers are devoting serious thought to this problem and no doubt the new mixers will see some decided improvement.

Uniform concrete, economically produced, is the goal to which all engineers are working and as non-uniform concrete cannot be economical, there is a double reason for accurate control. Cores from roads that were built by the method of volumetric proportioning showed the strength to vary as much as 150% in many cases. Part of this variation is probably due to testing, but a large part is due to proportioning.

While it will be impossible to produce a uniform concrete on roadway work, because of the variable quantities of water taken from the concrete by the sub-grade, a great improvement is possible and highly desirable.

Elmer G. Hooper,* Assoc. M. Am. Soc. C. E. (by letter).†—In this paper Mr. Older has shown his usual clearness of perception and has added materially to his already large contribution to the knowledge of scientific methods in design of concrete pavements. He has fairly given credit to H. M. Westergaard, M. Am. Soc. C. E., for the development of more comprehensive formulas for the determination of stress in concrete slabs, and he gives a very acceptable brief of their origin and use. The writer feels, however, that Mr. Older has made a broader statement than he intended when he states,‡

"Inasmuch as his [Westergaard's] equations take into account all essential factors, including the effect of sub-grade support, they provide a means for predetermining the stresses that may be expected under any given set of conditions."

It is quite obvious that, in any structure, stresses in any part are the resultant of a number of factors, or influences, and accurate design for fixed or safe stresses requires consideration of all of them. At any one time there may be cumulative increments due to the magnitude and position of the load, distribution of the vertical reaction of the subsoil, eccentricity of subsoil friction and slab resistance, variation in temperature through the depth of slab, variation in moisture through the slab, seasonal temperature variations, and uneven heaving of the sub-grade, which would produce extensive bridging. Professor Westergaard makes reference to some of these in the final paragraph of the paper§ mentioned by Mr. Older. With due regard for these very important factors, the formulas proposed provide a better, although not complete, guide for determining pavement stresses, because they introduce the very essential factor—modulus of elasticity of the pavement material—a factor which has previously been only indirectly considered.

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† Received by the Secretary, January 9, 1928.

‡ *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 147.

§ "Stresses in Concrete Pavements Computed by Theoretical Analysis", Fifth Annual Rept., Highway Research Board, National Research Council, *Public Roads*, April, 1926.

The writer recommends that these formulas be used only after a complete analysis of the report referred to, giving special consideration to the importance of the neglected factors enumerated in its last paragraph. Professor Westergaard did not overlook the existence or the effect of these factors, although he has not introduced them in any way into the equations. It is unwise practice to use formulas for design of important structures without complete knowledge of their origin and limitations.

As an illustration and to emphasize the correctness of the last statement, the writer, in the summer of 1926, observed the construction of a stretch of concrete pavement in an Eastern State. The finished pavement width was approximately 18 ft., made up of two abutting 9-ft. slabs, with an undoweled center joint, and doweled transverse joints, rather heavily reinforced with bar mat, the thickness of the edge and center being about 10 in. and 7 in., respectively. The design had evidently been influenced by the published conclusions and recommendations of Mr. Older in his report* on the Bates Road tests, and of others who had the advantage of corroborative experience. The writer considers that a mistake was made in adopting the 70% center thickness without complying with all the essential conditions, one of which was that dowels or other means should be provided to transfer part of the load across the joint. Without cross-connection the center-line joint becomes a free edge and, as such, needs the benefit of extra thickness. Center-line edges get far more repetitions and closer applications of the heavy loads than the outside edges and, therefore, should receive the greater strengthening. Any one who observes the usual path of heavy trucks on pavements will be convinced of the truth of this statement. It is well known that drivers of heavy vehicles leave a wider margin toward the shoulder of the road than at the center as a factor of safety against dropping off the pavement. The writer believes that this particular pavement design is uneconomical and unsound because there is either too much material on the outside edges or too little on the center.

The point to be made is that some, or some part, of the excellent recommendations were adopted without considering the restrictions or limitations which were quite clearly indicated in the original publications, and the neglect to consider them will surely nullify all expected benefits. Intelligent use of the formulas mentioned by the author will no doubt prove of great help in the design of rigid pavements.

Mr. Older recommends that steel be used in sufficient quantities to take care of temperature stresses.† A word of caution in regard to this may be in order. The statement itself gives the impression that temperature stresses may be separated from all the others and may be taken care of by the steel independently while concrete is carrying stresses from all other sources. This is not the case, and that may explain the reason why questions arise as to the economy of using steel for pavements in smaller amounts than are required to take up all tension.

Concrete and steel act together as a single unit after the pavement has set up. There is probably some tension in the concrete and some compression

* "Highway Research in Illinois" (Bates Test Road Report), *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1180.

† *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 153.

in the steel due to the shrinkage of concrete in curing. Stress in any elastic material develops only as the result of deformation. Any increment of deformation in the slab caused by refusal of that slab to move on the sub-grade is shared by steel and concrete alike. It is necessary that there be no cracks in the slab at critical sections if any of the proposed formulas for stress are to hold. Therefore, no deformation in the steel is permissible greater than that producing a safe stress in the concrete. Where bending stresses are being developed the extreme fiber deformation is greater than any other in the cross-section. The position of the steel is, therefore, of vital importance in determining its allowable deformation and the stress it may take. Stresses determined by Professor Westergaard's formulas are approximate for the extreme fibers. For the section directly under the load in mid-slab, tension from that load occurs on the bottom. There may be maximum safe deformation in the bottom fibers, but there is no deformation in fibers along the neutral axis. If a direct pull due to lowering temperature exists at the same time each fiber across the section will receive an increment of "stretch" which produces greater tension in the bottom and reduces compression in the top.

Fig. 2 shows approximate vector deformations, separately and combined. $N' O' C$ represents the compression deformation due to the load, P; $N' O T$ is tension deformation due to the same load; $T C S' S$ is deformation due to temperature pull; $N S' O'$ is the net compression deformation; and $N O S$ is the net tension deformation due to the load, P, and the drag of the sub-grade. If the vector, $O S$, represents the maximum safe fiber stretch, it is obvious that steel placed near the extreme fiber can stretch no more than that; if placed half-way between O and N, it can stretch only one-half as much; and if placed at the neutral axis, N, there can be no deformation, and hence no service.

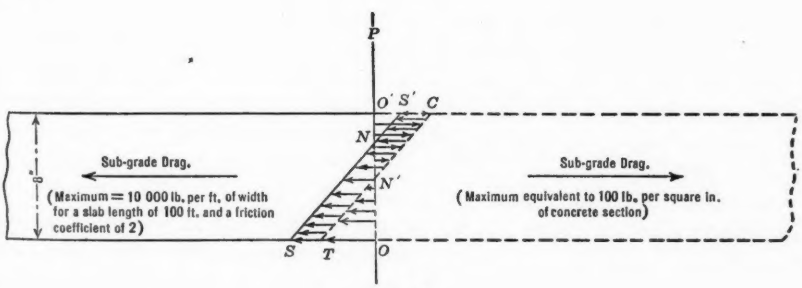

FIG. 2.

Since P is a moving load, the bending stresses will reverse both before and immediately after the load passes the section. This re-arranges the deformations and changes the neutral axis so that steel in any one position will not be uniformly effective under all conditions.

Assume, for example, that E for concrete is 3 500 000 lb. per sq. in.; E for steel is 30 000 000 lb. per sq. in.; and the safe stress in the concrete is 350 lb. per sq. in. The maximum deformation of concrete will approximate 0.00001 in. The corresponding stress in steel will be 3 000 lb. per sq. in.; half-way to the

neutral axis it would develop only 1 500 lb. per sq. in., and on reversal of bending stresses that position might approximate the new neutral axis, when it would carry no load. At 1 500 lb. per sq. in., it would require 6.67 sq. in. of steel per ft. of slab referred to (Fig. 2), which obviously is unreasonable. For a length of 50 ft. and a friction coefficient of 1 the steel percentage would still be high, even for balanced reinforced concrete.

It is apparent then that a different interpretation, probably the one intended, should be placed on Mr. Older's statement about steel for temperature stresses. When the concrete has cracked because of excessive deformation, the steel may work effectively up to 30 000 lb. per sq. in., or more, and may hold broken edges closely enough together so that the load will be transferred across and insure that a reasonable degree of continuity be maintained in the pavement. This is obviously the only way in which reinforcing metal in the usual small percentages used in concrete pavements can give adequate return for the money invested, considering temperature effects only.

The writer's purpose in discussing at such length these two items of this paper is to emphasize the need for extreme care in interpreting new suggestions, new formulas, and new methods for the design of engineering structures. The originators may endeavor to make themselves clear, but inevitably they take for granted certain knowledge and experience on the part of the recipient, which may or may not be justified. "Short-cuts" and convenient assumptions, therefore, may become serious stumbling blocks to the successful application of highly meritorious new ideas.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS
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THE ROLE OF THE CIVIL ENGINEER IN POWER DEVELOPMENT

Discussion*

By Messrs. P. Junkersfeld, Geo. A. Orrok, Joel D. Justin, and F. W. Scheidenhelm.

P. Junkersfeld,† M. Am. Soc. C. E.—The function of the civil engineer in the field of power development has been well stated in this paper. Although his work is often less striking than that of others engaged in this field, it is none the less important.

Engineering and business sense and vision are required in the conception and development of successful power projects. Take, for example, the larger problems involved in the development of steam power stations.

Success for such a project requires a location advantageous for the distribution of output as well as for input of raw materials, which, in this case, are principally fuel and condensing water. The ideal location, considering output alone, is that which permits radiation of feeders approximately equal in all directions, considering not only existing conditions, but future development for the sale of power. The slogan of the St. Louis Chamber of Commerce, "Ship from the center—not the rim", expresses this consideration concisely. The ideal location, with respect to raw materials, calls for a location near coal mines, or other source of fuel supply, advantageous from the standpoint of quality and price. It should also be a place affording an ample and dependable source of condensing water.

Sites combining these requirements, each in the maximum degree, are seldom, if ever, available, and experience and judgment are required in reaching a compromise which will give the best over-all result at more or less sacrifice of the minor considerations.

* This discussion (of the paper by I. W. McConnell, M. Am. Soc. C. E., presented at the meeting of the Power Division, New York, N. Y., on January 20, 1927, and published in January, 1928, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Pres., McClellan & Junkersfeld, Inc., New York, N. Y.

The man who co-ordinates these major and dependent factors for a successful whole must be broad and impartial. He deals with broad engineering problems, and does not necessarily bear the exclusive label of any particular branch of the Engineering Profession. He must be capable of directing, effectively, the activities of specialized engineers, including the civil engineer, and he must have a balanced grasp of all the problems entering into the project. Intense concentration on any particular feature of the project at the expense of other considerations will lead to an unbalanced whole. For example, if the civil engineer is in a position, and is inclined, to force a location or design that will benefit and cheapen the structural design at the expense of mechanical and electrical work, the whole project will suffer, especially in subsequent performance.

The directing head of a power project discharging the important duties outlined may be a man of civil engineering training as well as of training along other engineering lines. He must have initiative, ability, and experience.

Some power projects, due to their nature, offer peculiar advantages to the executive of civil engineering training, because it is undeniable that engineering training is of great advantage to an otherwise properly qualified executive.

TABLE 16.—Percentages of Investment Costs.

Items.	Composite (all stations).	Limits.	
		High.	Low.
Land	1.0	2.6	0.1
Structures	36.0	43.9	26.0
Boiler-plant equipment	27.9	35.4	19.4
Turbo-generator equipment	20.4	28.2	15.7
Electrical equipment	13.9	18.7	9.9
Miscellaneous equipment	0.8	3.5	0.2
Total	100	....	....

Mr. McConnell has presented figures which show the importance and, in some cases, the predominance of specialized work involving power developments that comes under the direction of the civil engineer. Table 16, based on recently reported costs and corresponding capacities of fifteen American steam power stations, supplements the author's figures and confirms his estimate of the important rôle played by the civil engineer in the development of such projects. All the stations considered in this tabulation are of a capacity exceeding 40 000 kw. and three of them are of 200 000 kw., or greater capacity. They cover a wide range of (1) physical and economic conditions; (2) electrical requirements in transmission and distribution; and (3) solutions of the corresponding and progressively changing problems individually and as a complete project for dependable and economical service.

Considering the opportunities of the civil engineer in power development and distribution, it may be said that strictly engineering training and ability are not sufficient in themselves to insure success in the most responsible positions. As in other fields of endeavor, success depends largely upon the extent to which the individual grasps the larger aspects of his problems and frees himself of a too narrow conception of his duties. Particular study is necessary to provide a power plant suitable to the economic conditions under which it will operate, as expressed in part by the terms "load factor", "use factor", and "capacity factor". While these terms express a fundamental economic condition, and are of use primarily in the basic conception of the entire plant, they also have an important application to the work of the civil engineer.

Mr. McConnell's paper has given civil engineers food for thought, and should encourage them to a proper realization of their part on steam power projects, and also on water power developments, in which it is even more important than in steam power development.

GEO. A. ORROK,* M. AM. SOC. C. E.—The author has asked a question and then demonstrated a thesis. Of course, the answer and demonstration must depend on the definition of the terms used and in this particular case on the line of demarcation between the hyphenated engineers—civil, electrical, mechanical, operating, constructing, transmission, and efficiency—with the geologist, chemist, and metallurgist in the near background. Fortunately, these lines of demarcation cannot lightly be drawn and are daily becoming more difficult to define. In 1780, the civil engineer was distinguished from the military engineer by his employment. In 1880, the civil and mechanical engineers were fairly well delimited, but in 1927 the definitions have large loopholes. Where, to-day, would Watt, Stephenson, Trevethick, Perkins, Bourne, Rennie, or Hodgkinson be classed? Does the ability to design and construct static structures as against dynamic machines constitute a difference? Where shall Holley, George Westinghouse, John Fritz, or such men as John Ericson, Col. B. S. Church, J. A. Bensel, Charles E. Emery, or Fred S. Pearson be placed?

The power engineer must not specialize, however, or the power plant will not be a harmonious whole. He may use specialists in heavy foundations, sea-walls, structural steel, architecture, combustion, boiler construction, turbine construction, piping, dynamo construction, switchboards, reactances, cable-making, naval architecture, and materials handling, but the design must be his and not theirs, or the 1 lb. of coal per kw-hr. used as an objective will be 1.4, or perhaps, 1.6 lb., and the station will have to be reconstructed. It must be so designed and constructed that it can be operated economically with a labor and maintenance account of reasonably small proportions. Above all it must be designed, not for the present, but for three to six, or perhaps ten years ahead, when twice or three times as much power will be required possibly within the same physical space limitations.

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Mr. McConnell compares the work of the mechanical and electrical engineer who buys standard types of apparatus offered by the manufacturers. The material so purchased is then tied together by the works of the civil engineer, who deals with man-hours, yards of excavation, sand, stone, lumber, cement, steel, etc. This gaily painted picture is what is known to the trade as "catalogue engineering" and the product cannot compete even in price with the well-designed and constructed plants which the power engineer has been putting into service ever since 1882. Each succeeding plant has used a little less fuel than its predecessor, turning out a few more millions of kilowatt-hours with a little more certainty at a little less cost.

In these designs, the bricks, concrete, rolled sections, boiler-tubes and pipe material, the house service switches, and small motors are standard and bought from the shelf, but everything else is especially designed for that particular station. The manufacturers complain that they are not allowed to standardize, that major improvements are insisted upon with each new order. The main units in a plant are not duplicates. They increase in size and decrease in water rate as the station is filled and the engineer's knowledge of combustion, boiler design, and operation keeps pace with the increasing load demands in that locality.

Mr. McConnell has given some very interesting figures out of the large fund of his experience, showing that, of his three examples of steam plants (Tables 1,* 2,* and 3†), the part which he terms "civil" is roughly 26% of the station prime cost. The speaker's records of about 100 stations average 25.4 per cent. Table 17 shows the percentage costs at four different epochs from averages of a number of good examples.

TABLE 17.—Percentage Cost of Power Plants.

Items.	1900.	1908.	1914.	1920-26.
Building and foundations	14.5	15.5	15.0	36.5
Boiler plant	45.0	40.0	37.0	26.6
Turbine plant	35.0	39.0	40.0	22.5
Electric plant	6.0	5.5	8.0	15.4
Total	100.0	100.0	100.0	100.0

Since 1890 the average cost of the power stations, for which the speaker has reliable figures, has been about $100 per kw. Averaging the costs by 10-year periods, the average in each decade has been about $100; and in these thirty-five years are included records of cost as low as $50 and as high as $225. It appears that the total cost of power stations over this period has been singularly constant. During this time the coal rate has been reduced from about 10 lb. of coal per kw-hr. in 1890 to less than 1½ lb. per kw-hr. in 1925. In 1927 there are at least three stations reporting 1 kw-hr. on about 1 lb. of coal. In 1890 engineers were content with an evaporation of about

* *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 157.
† *Loc. cit.*, p. 158.

3 lb. of water per sq. ft. of boiler surface. To-day, 10 lb. is common and 20 lb. per sq. ft. of surface is, perhaps, the peak capacity. In 1890, 400 kw. was the largest sized machine, while, to-day, an 85 000-kw. generator is under order and 100 000-kw. generators are being contemplated. Two-shaft machines of 160 000 kw. and three-shaft machines of 210 000 kw. are being ordered. In 1890 the engines ran non-condensing, or, at best, a jet condenser was used giving vacua of 24 to 26 in., while, to-day, surface condensers with 75 000 sq. ft. of surface, giving 29 in. of vacuum are contemplated. On the electrical side, generator efficiencies of 65% have been increased to 97.5%, while switches which could be safely trusted to break only a few hundred kilowatts without burning up, now have rupturing capacities of more than 1 500 000 kw.

What has been the progress in the civil engineer's part of the design and construction? The chemist has given him Portland cement instead of lime mortar, or Rosendale; the metallurgist has presented him with steel sections instead of piled wrought-iron beams and the Phœnix column. The mechanical engineer has given the metallurgist the universal mill and Grey mill, as well as the gang punch. The pneumatic hammer, the concrete mixer, and the reinforcing bar have been added to the civil engineer's equipment. The speaker placed in his first concrete foundation and his first reinforcement in 1893, and the pioneers were only a year or two before that, even if Roman and Carthaginian concrete is still in existence, more than 2 000 years old. The steam shovel, conveying machinery, and motor trucks handle the excavation; Kraft and Someyer developed the compressor and rock drill and the chemists produced dynamite and giant powder. Bricks and the trowel remain as in Solomon's time, with the plumb and square. These facts are reflected in Table 17.

It will be noticed that despite the increase in the cost of labor (about 200% since 1890), the costs of boiler plant have been reduced nearly one-half and the prime movers, by about one-third, and this despite the astonishing increase in complexity, which has accompanied the better economy. Electric plant has increased in cost $1\frac{1}{2}$ times; but this is due to the excessively large apparatus demanded by aggregations of power, ranging from 200 000 to 1 000 000 kw., which are now tied into one system instead of 7 000 to 8 000 kw. as at the end of the Nineteenth Century.

Column and floor loads are no heavier than they were in 1900. To-day, three boiler floors are not built as they were at 96th Street, New York, N. Y., nor are the boilers put on the third story with the engines on the ground floor as at Duane Street, New York. Building laws require no thicker walls than in 1900 and designers still are allowed 20 tons on a pile when such foundations are required. To crown all, there are more kilowatts per square foot of ground and per cubic foot of building than ever; yet the part which Mr. McConnell calls "civil" has increased about $1\frac{1}{2}$ times.

This increase cannot be due to the steel structure to any great extent, since the cost of such structure, while about 25% higher, represents about 40% of the building cost in 1900 and only about 25% in 1925. Cement has fluctuated from $4 in 1890 to $1 in 1903, and now is about $2. Bricks

which sold at $10 in 1900, now cost $20. The price of concrete has been raised, in spite of modern methods, from $5 per cu. yd. in 1900, to $12 in 1925. With all the labor-saving devices, steam shovels, concrete mixers, conveyors, elevators and troughs, hanging scaffolds, pneumatic riveters, automobile trucks, and modern construction organization, the building, which is the static part of the power house, has increased in cost from about 12 or 15 cents per cu. ft. to more than 50 cents. This increase has just been about enough to absorb all the construction and design economies which the power engineer has been able to secure by taking advantage of every little saving that progress in scientific research has made possible.

Fixed charges per kilowatt have thus remained constant, while savings have been made through the coal pile and by an increased load factor. That these savings have been large is well known, and the public has benefited greatly. At the end of 1898 the maximum price in New York was 20 cents per unit, which, by 1915, was cut to 8 cents, while coal had increased in cost 100 per cent. To-day, the cost of coal is three times what it was in 1898, but the maximum price of service is 7 cents per kw-hr., with a slight coal-adjustment addition, and this in the face of steadily increasing costs of constructing underground distribution systems.

It is interesting to note that Mr. McConnell very closely approximates the figures given by Sir John Snell, Chairman of the Electricity Commission of Great Britain, for the apportioning of total costs in steam-station operation. Mr. McConnell gives the proportion of fixed charges as 48%, fuel, 36%, and operation and maintenance, 15 per cent. Sir John Snell uses 40% for the fixed charges, 46% for fuel, and 14% for operation and maintenance. These differences, perhaps, are accounted for by the lower interest charges in Great Britain and the higher cost of fuel.

JOEL D. JUSTIN,* M. AM. SOC. C. E.—On a hydro-electric project mechanical and electrical engineers, in the main, confine their work to the equipment. They are specialists within their respective spheres and the civil engineer accepts their guidance within the limiting conditions laid down by him.

The controlling factors which make one hydro-electric project differ from another are, in general, hydrology, hydraulics, topography, foundation conditions, load conditions, and the resulting economics of the situation. These factors, and also the actual building of the structures required, are all within the realm of the civil engineer, and when he has determined them, it becomes the duty of the specialists—the mechanical and the electrical engineer—to design and furnish the equipment which will perform the function required.

It is only the civil engineer, as defined by Mr. McConnell,† who has the necessary over-all experience to direct the job. Consequently, in hydro-electric work, the man who is responsible for the entire project and supervises its progress from the time it is a dream in the mind of the promotor to the time it is turning out energy, is the civil engineer.

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† *Proceedings*, Am. Soc. C. E., January, 1928, Papers and Discussions, p. 155.

F. W. SCHEIDENHELM,* M. AM. SOC. C. E.—There is quite a distinction between the hydro-electric power development field and the steam-electric field. By and large, in the steam-electric project (once it is decided that there is to be such a development), it is the mechanical engineer who is first on the ground authoritatively, so to speak. He decides what kind of equipment is to be installed and, thereby, to a large extent, moulds the kind of development that is to take place. Perhaps this is more true with regard to the boiler-room than anything else, and the boiler-room is no small part of the entire steam-electric power station.

In the hydro-development, on the other hand, it is generally the civil engineer who is on the ground first. In fact, the question as to whether there is to be a development at all is largely one of civil engineering.

To enlarge a little on the relation of the civil engineer to hydro-electric developments, it is well to begin with the basic phase, that of economics. Presumably no specialized branch of engineering, such as civil, mechanical, electrical, or mining, has any monopoly of the subject of economics; but this is one of the principal factors entering into a water-power development. Especially is this true in the Southern Appalachian Mountains, where the competition of coal with water is so pronounced. The civil engineer, then, enters into that phase fully as much as do the engineers of the other branches.

The same is true, the speaker believes, of the studies of load or power market. Nowadays, many hydro-electric developments feed into large power systems, whereby it is possible for such a development, despite low stream flow, to carry relatively high peak loads, provided it is supported by some storage. Thus, it is very important to study load characteristics. On this phase the mechanical, the electrical, and the civil engineer have equal opportunity, and it is a matter of specialization by the individual rather than a province of any one branch of the profession.

The items outside the power stations proper are generally more numerous and more important in hydro-electric than in steam-electric developments. Usually, in the latter, the only external problem of importance is that of tying the electrical power output of the station into a transmission system, —although in special cases the problem of providing fuel may extend to, and even include the collateral development of, a coal mine. Hydro-electric developments, on the other hand, usually necessitate the preparation of the pondage or reservoir area. Sometimes, indeed, there are reservoirs serving merely storage purposes. The preparation of such areas usually involves relocation and reconstruction of lands, bridges, and railroads, all of which belong in the civil engineering category. Then there are, in some instances, considerations, usually of hydraulic interest, which affect power users and riparian properties down stream from the proposed hydro-electric development. Various of these items require close co-ordination of the work of the civil engineer with that of the lawyer.

The subject of water supply, that very important phase of a hydro-electric development, falls quite clearly within the province of the civil engineer,

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because hydraulic engineering, with its study of stream flow, appears to be primarily a sub-division of civil engineering.

The questions of physical safety, that is, the suitability of the dam site, the water-tightness of the reservoir site, and the safety of the dam itself, all fall in the civil engineering field. Incidentally, the science of geology, so much concerned in the matter of dam and reservoir sites, is more closely related to civil engineering than to either mechanical or electrical engineering.

After all, it is in the matter of structures that the work of the civil engineer in the hydro-electric field stands out most prominently. Probably the proportion of civil engineering work in a hydro-electric development increases in proportion to the head.

MEMOIRS OF DECEASED MEMBERS

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THOMAS HOWARD BARNES, M. Am. Soc. C. E.*

DIED NOVEMBER 15, 1927.

Thomas Howard Barnes was born in Waltham, Mass., on December 15, 1860, the son of Phineas H. and Elizabeth Howard (Miles) Barnes. His childhood was spent on a farm and his early education was obtained in the public schools of Waltham. In 1877, he entered the Massachusetts Institute of Technology, with the Class of 1881, but lacked sufficient means to continue after the second year.

His first professional position was as an Assistant Engineer in the office of the City Engineer at Cambridge, Mass. After five years in this position, he returned to Waltham as Inspector of Plumbing for the Board of Health. He was also engaged in private practice as a Civil Engineer.

In 1887, Mr. Barnes went to Alabama to lay out and plan the Town of New Decatur. The following year he went to New Birmingham, Tex., where for four years he was employed in similar work.

After a brief period in the City Engineer's office, in Newton, Mass., he again engaged in private practice in Waltham. While there, he was appointed Engineer of the Medford, Mass., Sewerage Commission, in charge of the design and construction of the sewers of that city. This was followed by his appointment as the first City Engineer of Medford. In 1900, he resumed private practice, this time in Boston, Mass., where for the next four years he specialized chiefly in sewerage and water-works design and construction.

In 1904, Mr. Barnes made his first trip to tropical countries, in which field he was later to become an authority. His mission included the study and design of water supply and sewage disposal systems in the Cities of San José and Puerto Limon, Costa Rica. He then went to Bocas del Toro, Panama, to undertake the filling and sanitation of that port. The site of the town was a mangrove swamp, which made it necessary for all buildings to be erected on piles; elevated plank walks served as streets, and the ebb and flow of the tides was the sole and dubious means of sewage disposal. A sea-wall was built around this town, sewer pipes were laid, and sand, which had been dredged from the harbor bottom, was pumped in. Thus, the transformation from an unlivable jungle site to a healthy town was complete. Mr. Barnes later served as engineer on other projects of a similar nature.

While engaged in the work at Bocas del Toro, Mr. Barnes was retained by the United Fruit Company to design and construct a wharf at Almirante, Panama, which town was the outlet for the banana trade from the Company's entire Changuinola Division. This wharf, it is believed, was the first to be built of reinforced concrete, and, until its completion and a thorough demon-

* Memoir prepared by C. MacCallum, Esq., New York, N. Y.

stration of its reliability, was a source of concern to many engineers and practical construction men. Time demonstrated the soundness of the design, and to-day the wharf stands as an example of resistance to deterioration under unfavorable tropical conditions. Other wharves of a similar type were designed and built in various ports in Central America, many of them by Mr. Barnes, until he came to be recognized as an authority on tropical wharf construction.

From 1908 to 1913, Mr. Barnes served both the United Fruit Company and the International Railways of Central America. His work included the design and construction of wharves, hospitals, warehouses, office buildings, water-works, sewerage plants, and other projects in Guatemala, Salvador, Honduras, Panama, Colombia, and Jamaica. From 1913 to 1918, he acted as Consulting Engineer for these two companies, with headquarters in New York, N. Y., with frequent trips to Central America and the West Indies.

In 1918, he re-entered private practice as a Consultant on Tropical Engineering. During the ensuing nine years, he served such companies as the International Railways of Central America, the Demerara Boxite Company, the Cuyamel Fruit Company, and the International Products Corporation, as well as several of the large oil companies, and various Latin-American municipalities and governments. His more important work included a \$3 000 000 beef-packing plant and dock facilities at Covenas, Colombia; hospital designs for Santo Domingo, Dominican Republic, Maracaibo, Venezuela, Cartagena, Colombia, and Puerto Cortés, Honduras; railroad shops in Salvador and Guatemala; wharf construction at Puerto Barrios, Guatemala; work for the American Legation at San Salvador; and a harbor and dock improvement project of great magnitude in Callao Harbor, Peru, on which he was engaged at the time of his death.

During his twenty-three years' practice in tropical engineering, Mr. Barnes visited virtually every country and every important town in Central America and the West Indies. His grasp of the human and economic, as well as the strictly technical, aspects of tropical problems, gained for him the respect of the many important companies which he served. Time and again, it was necessary first to plan and provide sanitary measures, living quarters, water and food supply, and to import labor and the most elementary building material, before actual work on the project in hand could be started.

His consideration for the physical and mental well-being of his employees, high and low, and his unswerving fairness and loyalty to them, won for him, not only respect, but unalloyed affection. His knowledge of the Spanish language and the Spanish temperament, combined with a singular mixture of uncompromising uprightness and firmness of purpose, and his realization of the devious circumlocution necessary in negotiating with Latin-Americans, commanded for him widespread respect and prestige, and a measure of personal affection seldom accorded a "gringo".

The following tribute comes from his Class, Massachusetts Institute of Technology, '81:

"Howard was one among us who seemed to have discovered the fountain of perpetual youth. His sturdy figure and pleasant smiling countenance

underwent little apparent change as the years rolled by. Youthful enthusiasm and the joyful zest for living were undiminished, and whenever he returned from the South Countries—after long or short absence—he slipped into his accustomed seat by the fireside of our friendship, quite naturally and without undue commotion.

"And so—for all his friends, men who knew him for a generation, or for some shorter period—Howard has gone back to his Far Country, but the seat by the fireside shall be his always."

On April 30, 1890, Mr. Barnes was married to Martha Middleton Simmons, of Rusk, Tex., who with a son, Harold Simmons, of Upper Montclair, N. J., and a daughter, Eleanor, of Yonkers, N. Y., survives him.

Mr. Barnes was a member of the Boston Society of Civil Engineers; the New England Water Works Association; the American Water Works Association; the American Public Health Association; and the American Association of Port Authorities. His clubs were the following: The Technology Club of New York; the Park Hill (Yonkers) Community Club; the Whitehall Club; and the Surf Club, of Cartagena, Colombia. He was a Trustee of the Church of the Divine Paternity (Universalist), New York, N. Y., and he was also a Free Mason. On numerous occasions, he contributed technical articles, which were of genuine merit, to the various Engineering Societies.

Mr. Barnes was elected a Member of the American Society of Civil Engineers on October 4, 1899.

ROBERT FRANCIS EASTHAM, M. Am. Soc. C. E.*

DIED FEBRUARY 10, 1926.

Robert Francis Eastham, the son of Virginia (Fisher) and F. Dabney Eastham, was born in Flint Hill, Va., on August 24, 1882. He was educated in the public schools of Virginia and at the Virginia Military Institute, Lexington, Va., and was graduated from the latter as a Civil Engineer in 1902.

Soon after his graduation, and for a period of about three years, Mr. Eastham was engaged by the Whitebreast Fuel Company, of Illinois, in making surveys for the development of mines in Illinois and Iowa. In the fall of 1905 and in 1906, he was employed as Engineer in charge of construction of 36 miles of electric railroad in Northern Illinois.

In June, 1907, Mr. Eastham decided to engage in Railroad Engineering and started with the Baltimore and Ohio Railroad Company as Instrumentman on the construction of its double-track work west of Pittsburgh, Pa. Later, in the same year, he was engaged with the Board of Supervising Engineers on the reconstruction of the Chicago, Ill., City Railways.

During 1908, Mr. Eastham was employed by the Grand Trunk Railroad Company in charge of the construction of its shops at Battle Creek, Mich. Afterward, he became General Superintendent of the General Contracting Company, of Toronto, Ont., Canada, and was engaged principally in paving and reinforced concrete construction. In March, 1909, he was employed as

* Memoir prepared by Vernon M. Peirce, M. Am. Soc. C. E.

Assistant Engineer on the location of the Illinois Central Railroad from Birmingham, Ala., to Jackson, Tenn., and, later, was made Resident Engineer in charge of the reconstruction of the yards of that Company at Centralia, Ill.

In 1910, Mr. Eastham returned to his home in Virginia and became interested in the improvement of highways in Rappahannock County. Through this work he became interested in Highway Engineering as a profession, and immediately associated himself with the Virginia State Highway Commission as Resident Engineer in charge of the construction of highways in several places in the State during the next four and one-half years.

In October, 1914, he resigned his position with the Virginia State Highway Commission to become Highway Engineer with the United States Bureau of Public Roads. Soon after Congress established Federal Aid in road building for the States, Mr. Eastham was selected to have immediate charge of such work in Maryland and Delaware, which position he retained until his death.

Mr. Eastham was married on February 22, 1913, to Mary B. Browning, of Flint Hill, Va., who, with four children, Robert, William J., Lucy Beale, and Frances, survives him.

He was of a most agreeable, courteous, and kindly disposition, always cheerful, with nothing delighting him more than to hear a good anecdote and to relate one. He will be greatly missed by all those who were acquainted with him.

Mr. Eastham was elected a Member of the American Society of Civil Engineers on September 10, 1923.

JOHN DEVEREUX O'REILLY, M. Am. Soc. C. E.*

DIED NOVEMBER 6, 1927.

John Devereux O'Reilly was born at Denver, Colo., on May 4, 1884, the son of Anthony J. and Fredrica (Devereux) O'Reilly. His early youth was spent in Colorado, but by the time he was of college age, his family had moved to New Orleans, La., where Mr. O'Reilly attended Tulane University. In 1904 he matriculated at the Virginia Military Institute, at Lexington, Va., and was graduated from that institution in 1907, having completed the course in Civil Engineering.

Immediately after graduation, he accepted employment as Instrumentman with the Asheville and Hendersonville Interurban Railroad Company. Later, he returned to New Orleans where he was engaged on various engineering projects from 1909 to 1915.

In May, 1915, Mr. O'Reilly was appointed Chief Engineer of the Board of Commissioners of the Port of New Orleans, in which position he continued until 1919. During his connection with this Board, which, as an agency of the State of Louisiana, owns and operates the port faciliites of New Orleans, a number of very important projects were undertaken. Notable among these were the Public Cotton Warehouse, the Public Grain Elevator,

* Memoir prepared by Samuel McC. Young, M. Am. Soc. C. E.

and the Inner-Harbor Navigation Canal, all of which were undertakings of great magnitude.

During the World War, President Wilson appointed Mr. O'Reilly as a member of the Council of National Defense, which had headquarters at Washington, D. C. As a member of this Council, his duties included the preparation of civil and military defenses against possible hostile attacks in the Harbor of New Orleans and in other United States ports.

In 1919, he severed his connection with the Board of Port Commissioners and resumed the private practice of engineering and contracting in and around New Orleans, and he was engaged in this work to the time of his last illness.

A man of pleasing personality who made friends readily, he was well known socially in New Orleans. He was a member of the Louisiana Club, the Delta Kappa Epsilon Fraternity, and of other organizations.

On June 30, 1908, Mr. O'Reilly was married to Beatrice Gilmore, who, with five children, four girls and one boy, survives him.

Mr. O'Reilly was elected a Member of the American Society of Civil Engineers on August 31, 1925.

MERLE WILLIAM ROSECRANS, M. Am. Soc. C. E.*

DIED OCTOBER 9, 1927.

Merle William Rosecrans, the son of William E. and Florence S. (Cornwell) Rosecrans, was born at Ruthven, Iowa, on September 1, 1889. He received his Grammar School education in the schools of Bridgewater, S. Dak. and his High School training at Canton, S. Dak. He was graduated from the Iowa State College, at Ames, Iowa, in 1912, with the degree of Bachelor of Science in Civil Engineering.

During his undergraduate days, Mr. Rosecrans spent his summers with various survey parties in South Dakota and Iowa. On graduating from college he entered the general contracting business with his father, Mr. W. E. Rosecrans, and they built a number of water-works and sewerage systems for various cities in Iowa. In 1913, Mr. Rosecrans went to Oregon as Assistant Engineer for C. W. Woodruff, Consulting Engineer, of Portland, but resigned in 1915 to again enter into business with his father. He was engaged in the saw-mill and logging industry until 1918, when he closed out his interests and become Assistant Professor of Civil Engineering at the Oregon State College. In 1919, he left the College and became Assistant Bridge Engineer for the Oregon State Highway Commission, which position he held until his death in the Salem General Hospital on October 9, 1927, after an illness of one month.

Mr. Rosecrans was admirably fitted for the positions of trust and responsibility which he held. Entering the Highway Department, as he did, at an

* Memoir prepared by L. P. Campbell, M. Am. Soc. C. E., and G. S. Paxson and C. B. McCullough, Assoc. Members, Am. Soc. C. E.

early period in its organization, his keen insight into human nature and his knowledge of construction did much to smooth out the many troubles incident to a young and rapidly growing organization. His executive ability and unquestioned fairness were great factors in binding the Department together and co-ordinating its work to the high standard of efficiency in which he left it. Beyond every other trait which he possessed stands out that priceless quality of loyalty, not only to his work and to his superiors, but also to those subordinate to him. It was this quality that endeared him to all who were so fortunate as to be associated with him. During his eight years of faithful service with the Highway Department, Mr. Rosecrans handled contracts amounting, in the aggregate, to many millions of dollars, and it is a remarkable commentary on his sterling character, kindly personality, and keen appreciation of his responsibilty that he was generally regarded, not as a stern task master and uncompromising public official, but rather, as a sincere friend and valued adviser by all with whom he worked.

Mr. Rosecrans was prominent in fraternal circles. At the time of his death he was Master of Pacific Lodge No. 50 A. F. and A. M. He was also Secretary of the Northwestern Society of Highway Engineers, and he was a member of Tau Beta Pi.

He was married in 1922 to Margaret Hodge who, with his father, William E. Rosecrans, and his brother, Richard Rosecrans, survives him. It can be truthfully said of him that he left a place in the hearts of his relatives, friends, and business associates that can never be filled.

Mr. Rosecrans was elected an Associate Member of the American Society of Civil Engineers on October 11, 1920, and a Member on October 1, 1926.

ALLEN NEWHALL SPOONER, M. Am. Soc. C. E.*

Died January 2, 1928.

Allen Newhall Spooner was born at Jersey City, N. J., on October 2, 1864. He was the son of Edward A. and Angela (Newhall) Spooner, a grand-nephew of William Howe, the inventor of the Howe truss bridge, and also, a second cousin of Elias Howe, Jr., the inventor of the sewing machine. Mr. Spooner's earlier education was obtained in the public schools of Jersey City. In 1882, he entered the School of Mines, Columbia University, New York, N. Y., from which he was graduated in 1886 with the degree of Civil Engineer. He was a member of the Psi Upsilon Fraternity, and active in behalf of his Alma Mater, both as an undergraduate and as an Alumnus.

Immediately after his graduation, Mr. Spooner entered the employ of the Pennsylvania Railroad Company as a Rodman and Draftsman, and, in 1888, he was appointed a Hydrographer in the Department of Docks of the City of New York. From 1889 to 1906 he was Assistant Engineer in charge of construction work, and built most of the granite bulkhead wall along the Manhattan shore of the East River. In 1906, he was appointed Commissioner

* Memoir prepared by F. R. W. Cleverdon, M. Am. Soc. C. E.

of Docks under Mayor McClellan, and it was under his régime that the Chelsea Piers were completed, the South Brooklyn water-front developed from 24th to 39th Streets, and the Staten Island and 39th Street, South Brooklyn, Ferry Terminals put into operation. Previous to his appointment as Commissioner of Docks Mr. Spooner was retained as Consulting Engineer and expert on water-front real estate valuation by such well-known companies as James Shewan and Sons, Incorporated, the North German Lloyd and Hamburg-American Steamship Lines, the New York Dock Company, and the Standard Oil Company of New Jersey.

From 1910 to 1913, Mr. Spooner was Chief Engineer of the New York Submarine Contracting Company and successfully handled the difficult work of placing two 36-in. submarine gas mains across the Harlem River at 210th Street, and one 48-in. main at 129th Street. In 1914, he organized the contracting firm of Allen N. Spooner & Son, Incorporated, and through his indefatigable efforts, his power in handling men, and his exceptional executive ability, during the period of thirteen years prior to his death, he, as President of the Company, brought it from a position of the smallest to one of the largest and best organized water-front contracting organizations in New York Harbor.

Mr. Spooner's career was cut off when he was about to realize the rewards of a life devoted to business; at a moment when the responsibilities of a long period of both public and private service were to be laid aside for the quiet pleasures of retirement. His rare qualities of character were apparent to all who came in contact with him. He was most charitable and tolerant, profoundly sympathetic, and an accurate judge of human nature. He had unusual powers of organization and exceptional executive ability which made it possible for him to secure the loyal co-operation of subordinates. Added to a modesty and generosity of spirit, was an inflexible integrity in behalf of the highest business and personal standards. All these qualities stand out as an inspiration to those in the profession, and a rich memory to his family and to all who had the privilege of his personal friendship.

He died at St. Luke's Hospital, in New York, after a major operation, on January 2, 1928. He is survived by his wife (*nee* Bertha Klaproth) and three children by his first wife (*nee* Emma F. Browne), Mrs. Violet Langsford Sears, and John Irving Spooner, Affiliate Am. Soc. C. E., and Ray Newhall Spooner, M. Am. Soc. C. E.

Mr. Spooner was elected a Member of the American Society of Civil Engineers on December 5, 1900.